Analytical evaluation of average delay and maximum stable throughput along a typical two-way street for vehicular ad hoc networks in sparse situations

Adel Javanmard, Farid Ashtiani *

Advanced Communications Research Institute (ACRI), Department of Electrical Engineering, Sharif University of Technology, Azadi Street, Tehran 11155-9363, Iran

ARTICLE INFO

ABSTRACT

Intermittent connectivity is an intrinsic feature of vehicular ad hoc networks (VANETs) in sparse situations. This type of network is in fact an example of delay and disruption tolerant networks (DTNs). In this paper, we focus on a typical two-way street and analytically evaluate the maximum stable throughput and the average delay for packet forwarding along the street. To this end, we map the mobility patterns of the vehicles with different speeds onto suitable parameters of a BCMP queueing network and derive the location density of vehicles. Then, we employ another queueing network in order to model opportunistic multi-hop packet forwarding along the street with respect to the specifications of MAC and routing schemes. We propose a two-mode MAC scheme suitable for DTNs with predictable mobility patterns. We also consider the effect of vehicles' velocities and opportunistic relaying for routing schemes. In our analysis, we evaluate the average delay and the maximum stable throughput for the proposed MAC and routing schemes. In the last part of the paper, we show the efficiency of the proposed analytical approach by some numerical results and confirm our analysis by simulation.

1. Introduction

Vehicular ad hoc networks (VANETs) are essential parts of Intelligent Transportation Systems (ITSs). In a typical VANET, vehicles are mobile nodes comprising an ad hoc network. Also, some fixed nodes may exist. They facilitate the packet transmission process. Then, a typical VANET is comprised of two types of nodes, fixed and mobile (vehicles), in general. Usually the number of fixed nodes is not so much to remove the necessity of multi-hop packet transmission among mobile nodes. We consider an example of this type of VANET in this paper.

The number of vehicles in a typical street is a random process that is strongly dependent upon the arrival rate of vehicles as well as the mobility patterns of the vehicles at that street. In sparse situations, where the number of vehicles is low, connectivity among vehicles is not preserved. Then in this case, the street, as a part of a typical VANET, is actually a delay and disruption tolerant network (DTN). Up until now several papers focusing on DTNs have been published in the literature. A major part of them is concerning about suitable routing algorithms in DTNs [1–7]. Some of them have focused on increasing the delivery ratio by several versions of epidemic routing [4,6]. Some of the others have considered routing schemes with respect to energy and memory concerns [2]. In

this respect, a few papers have evaluated the level of cooperation [3]. And some research works have focused on throughput maximization by inserting special nodes [8].

The major parts of the above works have considered DTNs in general and not in special situations. In some applications some of the concerns considered in previous works are not important. For example, in VANETs energy and memory are not the main concerns. Furthermore, the mobility patterns of the vehicles are road restricted.

On the other hand, many papers have focused on different aspects of VANETs. Some of them have considered connectivity issue and mobility modeling [9–13]. Some research works have focused on efficient flooding and dissemination algorithms [14,15]. And many research works have focused on designing new routing algorithms in different situations [16–19]. One of the main concerns of the routing algorithms in VANETs reverts to decisions at intersections. This is of more crucial importance in sparse situations. In fact, an efficient packet forwarding technique for VANETs in sparse situations is carry and forward technique [19] as well as opportunistic multi-hop packet transmission. In other words, a mobile node or vehicle stores the packet and carries it by itself until it finds a suitable vehicle. Then, the packet is sent to the more suitable vehicle. This attribute is very effective and viable because of mobility of the vehicles.

In this paper, we focus on a part of a VANET in sparse situations that is a good example of DTNs. In this respect, a typical two-way street is considered and we evaluate the average delay and the

* Corresponding author. Tel.: +98 21 6616 5924.
E-mail addresses: adeljavanmard@yahoo.com (A. Javanmard), ashtianimt@sharif.edu (F. Ashtiani).

maximum stable throughput for the packets generated along one way of the street. In fact, in a typical VANET, two types of services have been proposed, i.e., emergency services and conventional communication services. The key characteristic of emergency services relates to delay but for conventional services throughput plays a key role as well. Moreover, in several routing algorithms proposed for VANETs, the average delay along a typical street is a crucial parameter in decision-making at intersections [19,20]. However, to the best of our knowledge an analytical evaluation of this parameter has not been proposed in the literature, especially in sparse situations. One of the main issues focused in this paper is proposing a solution to this problem. In this respect, we evaluate the average delay as well as the maximum stable throughput versus arrival rate of vehicles in the street. The arrival rate is sufficiently low such that the connectivity among vehicles is intermittent. In this respect, we consider two static nodes at two contiguous intersections, i.e., at the ends of a typical two-way street. The packets generated at one intersection (i.e., first static node at our VANET scenario) are marked and sent towards the other end of the street. We also consider conventional (unmarked) packets generated at the vehicles along the street. The destination of both types of the packets, i.e., marked and conventional ones, is at the other intersection (i.e., the second static nodes at our VANET scenario). In our analyses, we evaluate the average delay for the marked packets as well as the maximum stable throughput for all packets. It is worth noting that in a real-life scenario, the static nodes at intersections can be active public information displays (e.g., electronic billboards) that contain helpful information for all vehicles arriving at the street and also play the role of fixed relay stations [20]. Moreover, the marked packets can be some emergency packets, e.g., the status of probable crash or traffic load in the street. Furthermore, conventional packets include advertised broadcast packets from the stores and malls at the roadside of the street and the packets corresponding to communication services (e.g., VoIP, web browsing, etc.).

In our analytical approach, we map different components of the network, e.g., mobility patterns and the effective factors in multi-hop packet transmission process (i.e., MAC and routing) onto the components of several queueing networks [21]. By solving the related traffic equations we derive the desired performance metrics, i.e., the maximum stable throughput and the average delay. We also propose a two-mode MAC scheme and evaluate analytically the proposed MAC scheme as well as several routing schemes in view of the desired performance metrics. We will confirm our analytical results by extensive simulations.

Following this introduction, in Section 2, we discuss about the mobility modeling. Analytical modeling of packet forwarding process including MAC and routing schemes has been discussed in Section 3. We evaluate the average delay and the maximum stable throughput in Section 4. After several numerical results and simulations confirming the proposed analytical approach in Section 5, we conclude this paper in Section 6.

2. Mobility modeling for vehicles along a typical two-way street

In order to include the mobility patterns of the vehicles we consider a two-way street such that at each way (direction) three lanes exist. It is assumed that vehicles arrive at each lane with Poisson distribution and vehicles' velocities at each lane have distinct distribution, e.g., uniform distribution at the tth lane, U$[v_{lt}, v_{ht}]$. We divide the length of each lane into subregions of length L. And we assume that each vehicle may change its lane and speed after each subregion (see Fig. 1). Actually, we spatially quantize the mobility patterns of the vehicles. In general, we assume that each vehicle is interested in maintaining its lane without any deviation. We are able to consider different options for lane transition according to different mobility patterns.

As it is observed in Fig. 1, for each ith subregion we consider $i' = \lceil \frac{i}{6} \rceil$ as the column indicator, where $\lceil x \rceil$ denotes the smallest integer larger than x. For each column i', we consider i'' as the lane indicator. So, it is a number between 1 and 6. Lanes 1–3 denote the first direction (side) of the street and lanes 4–6 indicate the second direction (opposite side) of the street. The main direction of packet forwarding in the scenarios considered in this paper is the first one. In sparse situations, the mobility patterns of the vehicles are considered to be independent [12]. Thus, similar to [22] we consider
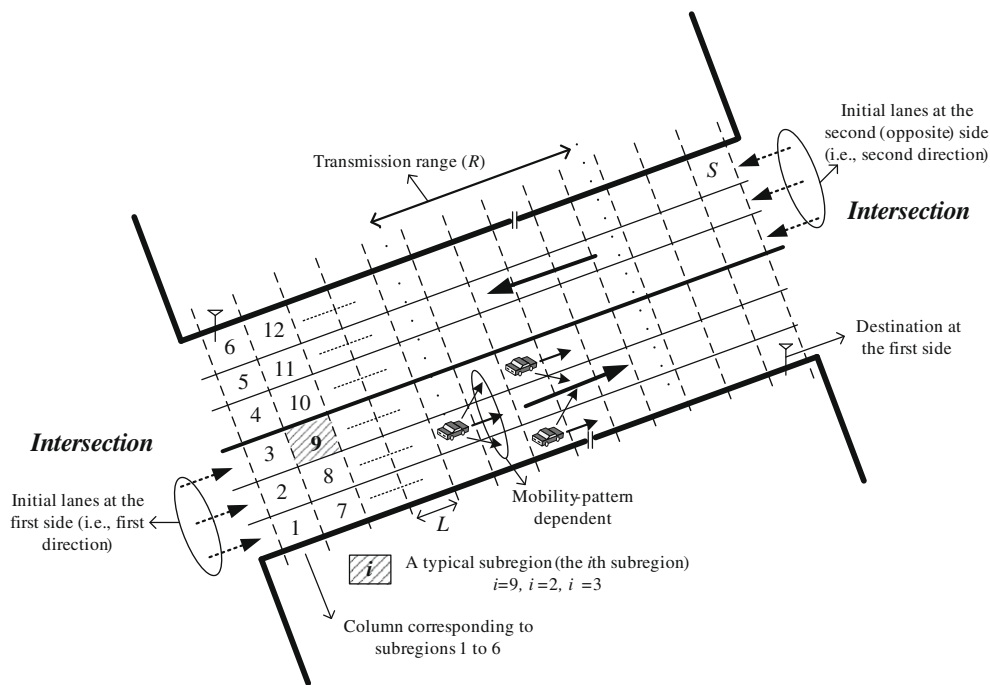

Fig. 1. Mobility patterns of the vehicles and the enumerating method for subregions and lanes in the street.

each vehicle as a customer and each subregion as an $M/G/\infty$ queueing node, leading to a BCMP queueing network [21]. The manner of lane transition is mapped onto the routing probabilities in the queueing network. We also map the sojourn time of the vehicles at each subregion onto the service time of the customer at the corresponding queueing node. Obviously, it depends on the distribution of vehicles' velocities at different lanes. For example, the average service time of a typical vehicle at a typical subregion with length L located at the tth lane with velocity distribution $U[v_{lt}, v_{ht}]$ equals $\frac{L}{v_{ht}-v_{lt}} \ln \left(\frac{v_{ht}}{v_{lt}} \right)$.

It is worth noting that the number of lanes is not an important and effective parameter in our analysis. In fact, the existence of several lanes intensifies the assumption of independent mobility patterns in sparse situations, because the vehicles can overtake each other by simple lane transitions without any hindering effect and any need to speed change. However, the most important factors in our analysis are the distribution of the speeds for the vehicles at the arrival instants and their mobility patterns along the street. These factors strongly affect the number of vehicles at the street in the steady state.

According to the independent mobility patterns in sparse situations, in order to derive the location density of a typical vehicle arriving at the tth lane, we consider a closed BCMP queueing network comprised of several queueing nodes (i.e., subregions along the street, see Fig. 1) and one customer. We assume that when the vehicle (i.e., the customer at the closed queueing network) exits the street it arrives again at the street with the same initial speed category, corresponding to its initial lane. By such a wrap-around technique, we are able to obtain the location density of a typical vehicle arriving at a specific lane of the street. In other words, we exactly obtain the probability that a typical vehicle belonging to the tth lane initially, is at different subregions. In this respect, we have the following traffic equations and the spatial probability distribution corresponding to a vehicle arriving at the tth lane of the street:

$$\alpha_{it}^{mob} = \sum_{\substack{j=1, \\ j''\leqslant 3}}^{S-3} \alpha_{jt}^{mob} r_{jt,it}^{mob}; \quad \sum_{\substack{j=1, \\ j''\leqslant 3}}^{S-3} r_{it,jt}^{mob} = 1; \qquad t = 1,2,3,$$
$$\forall i; \quad 1 \leqslant i \leqslant S-3, \quad i'' \leqslant 3, \tag{1}$$

$$P_{loc}^{t}(i) = b_n \frac{\alpha_{it}^{mob}}{\mu_i^{mob}}; \quad 1 \leqslant i \leqslant S-4, \quad i'' \leqslant 3,$$
$$P_{loc}^{t}(S-3) = 1 - \sum_{\substack{i=1, \\ i''\leqslant 3}}^{S-4} P_{loc}^{t}(i), \tag{2}$$

where S is the number of subregions (in the three lanes along the street at both ways (directions), see Fig. 1), and b_n is a normalization constant. Also, α_{it}^{mob} is the total arrival rate at node i (that is equal to the departure rate from the same node in stable conditions), μ_i^{mob} is the inverse of the average sojourn time at the ith subregion (independent of t), and $r_{jt,it}^{mob}$ is the routing probability from queueing node j to node i. Furthermore, $P_{loc}^{t}(i)$ indicates the probability that a typical vehicle arriving at the tth lane occupies the ith subregion in the steady state. Since the vehicles cannot move from one side of the street to the other side, in (1) we only consider the subregions corresponding to the first side (see Fig. 1). It is worth noting that $r_{jt,it}^{mob}$ is determined according to the mobility pattern considered for vehicles. In other words, it determines the manner of lane transition after traversing each subregion (see Fig. 1). If i, j belong to two subregions such that $|j' - i'| \neq 1$ then $r_{jt,it}^{mob}$ equals zero except the case they belong to the first and the last columns of subregions (due to wrap-around technique), respectively. In this case, $r_{jt,it}^{mob}$ equals one if the ith subregion belongs to the tth lane, otherwise it equals zero.

The number of vehicles at each street is a stochastic process. Similar to previous discussion, we consider an open queueing network such that the vehicles are customers and each subregion at each lane is mapped onto a queueing node. Since in this case we may have arrival rates at each subregion corresponding to vehicles with different initial lanes, we have a multi-class open BCMP queueing network comprising $M/G/\infty$ nodes. Thus, we have the following traffic equations and spatial traffic distribution corresponding to this queueing network:

$$\alpha_{it}^{mob} = \lambda_{it}^{mob} + \sum_{\substack{j=0, \\ j''\leqslant 3}}^{S-3} \alpha_{jt}^{mob} r_{jt,it}^{mob}; \quad \sum_{\substack{j=1, \\ j''\leqslant 3}}^{S-3} r_{it,jt}^{mob} + r_{it,0}^{mob} = 1;$$
$$t = 1,2,3, \quad \forall i; \quad 1 \leqslant i \leqslant S-3, \quad i'' \leqslant 3, \tag{3}$$

$$P_{loc}^{t}(i;n) = e^{-\rho_t^{mob}(i)} \frac{(\rho_t^{mob}(i))^n}{n!};$$
$$P_{loc}(i;n) = e^{-\rho^{mob}(i)} \frac{(\rho^{mob}(i))^n}{n!}; \quad 1 \leqslant i \leqslant S-3, \quad n \geqslant 0,$$
$$\rho_t^{mob}(i) = \frac{\alpha_{it}^{mob}}{\mu_i^{mob}}; \quad \rho^{mob}(i) = \sum_{t=1}^{3} \rho_t^{mob}(i),$$
$$\alpha_i^{mob} = \sum_{t=1}^{3} \alpha_{it}^{mob}, \quad \lambda_i^{mob} = \sum_{t=1}^{3} \lambda_{it}^{mob}, \tag{4}$$

where class t represents vehicles belonging to the tth lane at the arrival instant in the street. Moreover, α_{it}^{mob}, α_i^{mob}, μ_i^{mob}, $\rho_t^{mob}(i)$, $\rho^{mob}(i)$ denote total arrival rate of vehicles of class t, total arrival rate of vehicles of all classes, the inverse of average sojourn time, the traffic intensity of class t and the total traffic intensity, respectively, corresponding to the ith queueing node. In (4), $P_{loc}(i;n)$ indicates the probability with which n vehicles exist in the ith subregion in the steady state. Furthermore, λ_{it}^{mob} denotes the arrival rate from the outside of the street at the ith subregion belonging to the tth lane, so it is zero for internal subregions. And μ_i^{mob} is dependent upon the distribution of velocities at the lane containing the ith subregion and the length of each subregion as well. In addition, the 0th queueing node denotes the outside of the street and $r_{jt,0}^{mob}$ is non-zero only if the jth subregion belongs to the last column of subregions in the street. Since the number of customers at each $M/G/\infty$ node is a Poisson random variable (RV) [21], we obtain the number of vehicles at each subregion as a Poisson distributed random variable with parameter $\rho_i^{mob} = \frac{\alpha_i^{mob}}{\mu_i^{mob}}$. In this paper we consider a symmetric situation for the mobility patterns corresponding to the vehicles at both sides of the street. Then, the above results are also applicable for vehicles at the opposite side (i.e., $i'' > 3$).

Because of sparse situations, the parameter $\rho^{mob}(\cdot)$ for each subregion will be small such that the probability of having two or more vehicles simultaneously at each subregion is negligible. Thus, we approximate Poisson distribution for the number of customers at each subregion with a binary random variable. If we consider L, the length of each subregion, sufficiently small, this assumption is completely matched with the real situations because of size of the vehicles. However, if we consider L large the parameter μ^{mob} decreases and the parameter $\rho^{mob}(\cdot)$ increases, then the binary approximation will not be valid anymore.

It is necessary to mention that there is a nice difference between $P_{loc}^{t}(i)$ in (2) and $P_{loc}^{t}(i;1)$ in (4). The former determines the steady state probability that a typical vehicle arriving at the tth lane of the street occupies the ith subregion. However, the latter determines the probability that the ith subregion is occupied by a vehicle arriving at the tth lane of the street. It is worth noting that the former is independent of the arrival rate of the vehicles at the street, λ_{it}^{mob}, but the latter is strongly dependent on it.

With respect to Poisson distribution for the number of vehicles at each subregion (see (4)), the number of vehicles at the street and with a specific initial lane is another Poisson distribution derived by the summation of detailed independent Poisson random variables. The independency is due to the openness and quasi-reversibility of the BCMP queueing network [21] in the proposed mobility model. In sparse situations, the Poisson distribution of the whole number of vehicles at the street and with any initial lane is a distribution with small variance (variance and mean of each Poisson distribution are equal). Then, we estimate the number of these vehicles with its mean value according to the law of large numbers with high accuracy. We confirm this approximation in our analyses by simulation in Section 5.

3. Opportunistic multi-hop packet forwarding along the street

One of the intrinsic attributes of DTNs is opportunistic forwarding of the packets. In this paper, we consider a multi-hop unicast packet transmission such that the nodes (vehicles) do not store the copy of the packets after they send them successfully. And the main goal is to send the packets as soon as possible to the fixed destination placed at the intersection. In order to model the multi-hop packet forwarding process, we employ an open queueing network consisting M/G/1 nodes. Each queueing node corresponds to a vehicle and the packets are mapped onto customers. We also map the specifications of PHY, MAC and routing schemes onto different parameters of the queueing network, i.e., the service time of the packets at the queueing nodes and the routing probabilities among the nodes. For the physical layer, we consider the Protocol model [23], in which a successful transmission is possible provided that there is not any other transmitter in the interference range of the receiver. Moreover, the propagation conditions (e.g., fading) have not been considered. It is usually assumed that the interference range is double the transmission range. These assumptions are considered in computing the collision probability in the following. We consider a fixed transmission range equal to R for each transmission such that the vehicles in this range receive the transmitted packet, but only one of them will store it. The details will be discussed in the next parts of this section.

One of the effective factors in packet transmission reverts to MAC scheme. Without considering the effect of MAC scheme we are not able to have an exact estimate of the desired performance metrics, i.e., the average delay and the maximum stable throughput. However, most of the previous works on DTNs, esp., those focusing on routing algorithms have not considered the effect of MAC scheme [1–4]. Since the opportunities are rarely obtained and we consider a sparse VANET, it appears that a random access is suitable in order to reduce collision probability within the provided rare opportunities. Otherwise, the opportunities may be easily spoiled. However, the random access should not be such that a large delay is incurred. On the other hand, since one of the features of the mobile nodes in a VANET (i.e., vehicles) is predictable and street-constrained mobility pattern, we should exploit this knowledge in designing MAC schemes and mitigating extra unnecessary transmissions. To this end, we propose a two-mode MAC scheme. First mode is a random access delay in order to mitigate the probable collisions in opportunities provided rarely. And the second mode is in fact a long deterministic delay corresponding to the disconnection situation. Thus, in this mode, the transmission is postponed for a large (compared to random delay in the first mode) time interval. Intuitively, if the packet transmission is not successful due to not finding a suitable vehicle, the opportunity for packet transmission is not obtained in the near future with high probability. So, packet transmission after another random wait (similar to the usual case of random access MAC schemes) is not useful in this

case. Hence, the second mode of MAC scheme corresponding to a long wait (deterministic and not random) is applied. This waiting time is determined with respect to the average time needed for a fast vehicle, i.e., vehicle at the third lane, to traverse a subregion. It is actually the minimum time interval in average that the status of vehicles changes. Following each long deterministic wait a short random wait, i.e., the first mode of MAC scheme, is applied. Then, if the status of the vehicles is not changed, i.e., the disconnection status is maintained, the transmitted packet is not received successfully again and another long wait will be commenced.

For the sake of simplicity we consider a simple slotted Aloha for the first mode of MAC scheme such that the vehicle selects a random number between $0, W - 1$ before a transmission. By assuming the fixed size packet transmission time equal to a time slot and including it in the random wait, we consider the random number of slots between $1, W$. Then, in the steady state, the packet transmission occurs once at each $(W + 1)/2$ slots in average. In other words, the transmission probability at a typical slot (P_{tr}), equals $2/(W + 1)$. However, this probability is true if there is any packet for transmission. Since we map the waiting time corresponding to two modes of MAC scheme onto the service times of the customers, the probability that a packet for transmission exists equals the steady state probability of existing packets in the first mode of MAC scheme. Thus, the probability that the uth vehicle is a transmitter one at a typical time slot is computed as in the following:

$$Transmission\ at\ a\ time\ slot = \frac{\rho_{us}}{\rho_{us} + \rho_{ul}}(\rho_{us} + \rho_{ul})\frac{2}{W + 1} = \rho_{us}P_{tr},$$
(5)

where ρ_{us}, ρ_{ul} denote the steady state probability that the packet under service (i.e., to be transmitted next) at the uth vehicle corresponds to first and second modes of MAC scheme, respectively. Also, $\rho_{us} + \rho_{ul}$ denotes the steady state probability that the uth vehicle (uth queueing node) is non-empty. And W denotes the maximum number of time slots at random delay mode. It is worth noting that we do not distinguish among the vehicles arriving at the tth lane (i.e., the initial lane) of the street in the steady state. Then, ρ_{us}, ρ_{ul} for vehicles with the same initial lane are the same, respectively. Therefore, in the following, we consider ρ_{us}, ρ_{ul} as $\rho_{ts}(i)$, $\rho_{tl}(i)$, respectively, such that t denotes the initial lane of the uth vehicle. Moreover, in order to differentiate between lanes of two sides of the street we include the ith subregion at the arguments of $\rho_{ts}(i)$, $\rho_{tl}(i)$.

Another effective factor in packet forwarding process reverts to the routing scheme. As we indicated previously, carry and forward technique is an inevitable feature of the routing schemes at any DTN, esp., at a street with low density of vehicles. On the other hand, since the packet forwarding through radio transmission is very faster than the packet carrying by vehicle movement, packet routing by radio means is a crucial factor in efficient routing schemes. Although we do not intend to find the optimal routing algorithm in this paper, we consider a few routing schemes and compare them. In our comparison, the average delay and the maximum stable throughput are the desired performance metrics.

It may appear that routing among faster vehicles (i.e., vehicles at the third lane) is more suitable, however, the conventional packet generation rate including the packets due to advertisements and conventional communication services is larger at faster vehicles. It is due to this fact that faster vehicles traverse more stores and malls in the roadside, then receive more new advertised packets in a time unit. Moreover, the number of slower vehicles in the steady state is more than the number of faster ones. If we transfer the packets inefficiently among the vehicles again and again, we encounter a congestion situation more probably and also, the packets at busier vehicles are compelled to bear more delay. Thus, in or-

der to design an efficient routing algorithm we should reduce the number of packet transmissions for as much as possible.

With respect to above important points, we consider three routing schemes as in the following:

Routing Scheme I: According to the first routing scheme we have a prioritization strategy as in Fig. 2, such that the farthest and the fastest vehicles are the most suitable candidates. Moreover, the packet forwarding through the vehicles at the opposite direction is not allowed.

Routing Scheme II: According to the second routing scheme we have the prioritization strategy as in Fig. 2, such that the farthest and the slowest vehicles are the most suitable candidates. Similar to routing scheme I, only the vehicles at one way forward the packets at the same direction.

Routing Scheme III: In the third routing scheme, we exploit the relaying from the vehicles at opposite direction as well. To this end, it is reasonable to consider the slower vehicles among the vehicles moving in opposite direction with higher priority. In fact, a slower vehicle has more chance to play the role of a relay because it has more time to find another vehicle in the forward direction. The prioritization strategy has been illustrated in Fig. 2. It is worth mentioning that if a packet is transmitted to a vehicle at the opposite direction as a relay but it does not find another suitable relay (i.e., in the direction of packet forwarding), no problem will happen. Because when the initial packet transmitter encounters the packet receiver (this occurs certainly because they move in different directions), the packet transfer will occur in reverse direction, i.e., the packet will be transmitted to its previous transmitter. However, more packet transmissions leads to higher collision probability.

In the above routing schemes, we assume that if the vehicle with the highest priority (i.e., the most suitable candidate) receives the packet successfully, it sends an ACK message. Clearly, all vehicles within the transmission range but with lower priorities hear the data packets as well as ACK packets. We assume that ACK packets are very small and are sent in a completely reliable manner. If a transmitted packet is received erroneously, it is due to a collision (we have neglected the effect of physical layer degradations such as noise, interference, fading, etc., as stated before). In this case, a NACK packet is returned. All vehicles

at the same column, i.e., with the same horizontal distance (see Fig. 1), are in similar collision status. However, the other vehicles nearer to the transmitter may not be in a collision status and can receive the packet and send an ACK packet according to the priority. Thus, we have the following strategy.

A typical packet is transmitted to the farthest potential vehicle within its transmission range. We have assumed that all vehicles are equipped with GPS and the position of the packet's transmitter is included in the header of the packet. Then all vehicles within the transmission range of the transmitter receive the packet. With respect to their positions (known via GPS) they know the level of their priority in receiving the packet. If the vehicle at the most suitable subregion (i.e., the highest priority) receives it successfully, a small ACK packet will be transmitted immediately and all of the other vehicles dump the corresponding data packet. Otherwise, a vehicle at that location does not exist or a collision has occurred. In both cases, the vehicles at the other locations play the role of suitable candidates according to the prioritization strategies in the routing schemes. However, in computing the routing probability the former and the latter cases are different. In fact, in the latter the vehicles at the subregions of the same column are in the collision status similarly. But in the former case, i.e., the case in which the subregion with the highest priority is empty, it is possible to consider a vehicle at the same column as the receiver without any collision. We have assumed that ACK and NACK packets are very short and when each vehicle sends it, all the other vehicles within the transmission range, receive it. Actually, we assume that the ACK and NACK packets are sent via all vehicles within the transmission range of the transmitter in distinct contiguous mini-time slots at the end of the fixed size packet duration such that they hear each other completely. Thus, they are able to make a correct decision about dumping the packet or storing it. In fact, the farthest vehicle without any collision status and with the highest priority among the vehicles at the same column is the receiver of the transmitted packet. It is worth noting that among all vehicles receiving a transmitted packet, i.e., vehicles within the transmission range of the transmitter, at most one vehicle stores the packet and the others dump it.
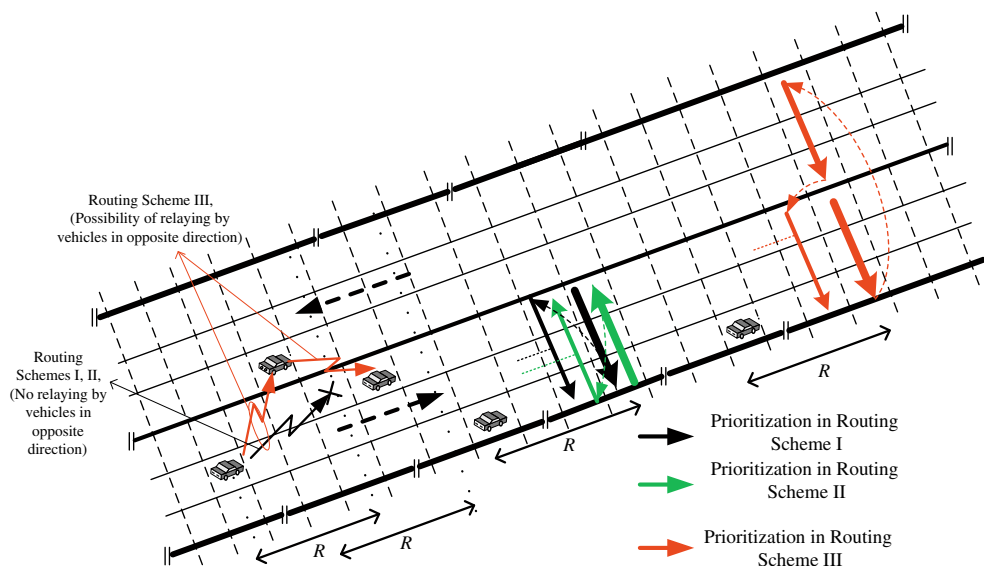

Fig. 2. Routing schemes and prioritization strategies in routing schemes (thicknesses and arrows indicate the priorities).

3.1. Computation of routing probabilities in the queueing network regarding MAC and routing schemes

With respect to the details of MAC and routing schemes, the routing probabilities among the queueing nodes (i.e., vehicles) in the queueing network, representing the packet forwarding process among vehicles, are computed as in the following:

$$r_{u,v} = \sum_{i'=1}^{\frac{S}{6}-\frac{R}{L}} \sum_{i''=1}^{6} P_{loc}^{t}(6(i'-1)+i'') \sum_{j'=i'+1}^{\frac{S}{6}-\frac{R}{L}+1} \sum_{j''=1}^{6} P_{loc}^{q}(6(j'-1)+j'')$$
$$\times P(NV \equiv (v,j))(1 - P_{col}(i',j'));$$
$$i' = \left\lceil \frac{i}{6} \right\rceil, \quad j' = \left\lceil \frac{j}{6} \right\rceil; \quad 1 \leqslant i,j \leqslant S, \quad i = 6(i'-1)+i'',$$
$$j = 6(j'-1)+j'', \quad NV : Next\ Vehicle; \quad t = IL(u), \quad q = IL(v),$$
(6)

where $r_{u,v}$ indicates the routing probability from the uth vehicle located at the ith subregion to the vth vehicle located at the jth subregion. In (6), we have considered both ways of the streets for the location of the vehicles in general. In this respect, we distinguish the speed category (lane) to which the vehicles belong at the arrival time in the street, i.e., the initial lane. The location probabilities are obtained with respect to (2). Clearly, the vth vehicle should be within the transmission range of the uth vehicle. The distance between uth and vth vehicles is equal to $|i'-j'|$, $i' = \left\lceil \frac{i}{6} \right\rceil$, $j' = \left\lceil \frac{j}{6} \right\rceil$, because the width of the street is very small compared to the transmission range. It is worth mentioning that uth and vth vehicles are typical vehicles and the location probabilities, obtained according to (2), are strongly dependent on the initial lanes of vehicles, i.e., $IL(u)$, $IL(v)$. Another part in (6), $P(NV \equiv (v,j))$, reverts to the probability that the vth vehicle at the jth subregion is the next vehicle depending on the routing scheme. In computing the routing probability, we distinguish two following cases.

In the first case, the jth subregion is the farthest subregion within the transmission range of the uth vehicle at the ith subregion, i.e., $|j'-i'| = \frac{R}{L}$ (it is assumed that R is divisible on L). Hence, if the vth vehicle at the jth subregion has the highest priority with respect to the routing scheme then the probability that the vth vehicle is the next vehicle equals one. Otherwise, $P(NV \equiv (v,j))$ equals the probability that there is not any vehicle in the subregions (at the same distance of the jth subregion, R/L) with higher priorities, with respect to the routing schemes.

In the second case, the distance between the jth subregion and the ith subregion is less than the transmission range, i.e., $|j'-i'| < \frac{R}{L}$. Then, the probability that the vth vehicle at the jth subregion is the next vehicle in the routing process equals the probability that the farther subregions are empty in addition to the probability that the vehicles in farther subregions (within the transmission range) are in the collision status.

The above discussion clarifies the computation of $P(NV \equiv (v,j))$ in (6). Obviously, the packet will be transmitted successfully if no collision occurs. So, we include the non-collision probability in writing the routing probability from node u to node v in (6). The non-collision probability equals the probability that no transmitter vehicles exist in the interference range (double the transmission range) of the vth vehicle. We consider the interference ranges at both front and back of the vth vehicle because we have assumed omni-directional antenna.

Therefore, with respect to above discussions in order to compute $P(NV \equiv (v,j))$ as well as the non-collision probability, we consider routing scheme I for the sake of simplicity. Thus, we have the following equation:

$$P(NV \equiv (v,j))(1 - P_{col}(i',j'))$$
$$= (1 - \rho_{qs}(j)P_{tr}) \left(\prod_{\substack{y'=j'-\frac{2R}{L} \\ y' \neq i',j'}}^{j'-1} A(y') \frac{A(i')}{\left(1 - \sum_{t=1}^{3} \rho_t^{mob}(i)\rho_{ts}(i)P_{tr}\right)} \right)$$
$$\times \left(\prod_{y'=i'+\frac{R}{L}+1}^{j'+\frac{2R}{L}} A(y') \cdot \left(\sum_{w'=j'+1}^{i'+\frac{R}{L}-1} \left((1 - B(w')) \left(\prod_{g'=j'+1}^{w'-1} B(g') \right) \right. \right. \right.$$
$$\times \left. \left. \left(\prod_{x'=w'+1}^{i'+\frac{R}{L}} A(x') \right) \left(1 - \prod_{g'=j'+\frac{2R}{L}+1}^{w'+\frac{2R}{L}} A(g') \right) \right) \right) \right);$$
$$A(x') = \prod_{x''=1}^{6} \left(1 - \sum_{t=1}^{3} \rho_t^{mob}(6(x'-1)+x'')\rho_{ts}(x)P_{tr} \right),$$
$$B(y') = \prod_{y''=1}^{6} (1 - \rho^{mob}(6(y'-1)+y'')); \quad i' = \left\lceil \frac{i}{6} \right\rceil, \quad j' = \left\lceil \frac{j}{6} \right\rceil,$$
$$i = 6(i'-1)+i'', \quad j = 6(j'-1)+j'', \quad w = 6(w'-1)+w'',$$
$$q = IL(v), \quad NV : Next\ Vehicle,$$
(7)

where $\rho_t^{mob}(i)$ denotes the probability that the ith subregion is occupied by a typical vehicle with the initial lane t. Also, with respect to discussions following (4), and due to sparse situations, we approximate Poisson distribution with a binary distribution. On the other hand, at a typical time slot, each vehicle with the initial lane t is a transmitter with probability $\rho_{ts}P_{tr}$. Since the number of vehicles at the ith subregion is a Poisson RV (see (4)), the number of transmitter vehicles at the ith subregion is also a Poisson RV. Thus, by binary approximation the probability of having a typical transmitter vehicle at the ith subregion equals $\sum_{t=1}^{3} \rho_t^{mob}(i)\rho_{ts}(i)P_{tr}$. Hence, $A(x')$ indicates the probability that there is not any transmitter vehicle at the x'th column. It is worth noting that we have considered both sides of the street because the vehicles at both sides are effective in the collision status. Moreover, $B(y')$ indicates the probability that there is not any vehicle at the y'th column.

Now, we explain (7) step by step. The first product term $(1 - \rho_{qs}(j)P_{tr})$ indicates the probability that the vth vehicle at the jth subregion does not transmit any packet at the desired time slot. As we explained in (5), ρ_{vs} only depends on the initial lane of the vth vehicle (q) and we consider j as its argument in order to distinguish the both sides of the street. Thus, ρ_{vs} in (5), equals $\rho_{qs}(j)$ in (7). The second product term denotes the probability that there is not any transmitter vehicle in the detection range preceding the vth vehicle, except the uth vehicle at the ith subregion (this vehicle has been excluded by the fraction in the second product term). The third product term equals the probability that there is not any transmitter vehicle in the interval between the transmission range of the uth vehicle and the interference range of the vth vehicle. This probability must be non-zero in order to have a non-collision status for the packet transmission from the uth vehicle to the vth vehicle. The fourth product term reverts to several situations that may occur for the vehicles within the transmission range of the uth vehicle and in front of the vth vehicle. Actually, according to the above discussion, when the vth vehicle at the jth subregion is the receiver of the transmitted packet, there is not any non-collided vehicle within the transmission range of the uth vehicle (transmitter) and in front of the vth vehicle (see Fig. 3). Otherwise, the farther non-collided vehicle would be the next vehicle in the routing process. Dependent upon the location of the nearest vehicles to the vth vehicle at its front (located at the wth subregion, see Fig. 3), we consider the fourth product term in the form of a summation, $\sum$.

At each term of $\sum$ in (7), we have several product terms. The first term determines the probability that there is at least one vehicle at the w'th column. Since it is the nearest non-empty column in
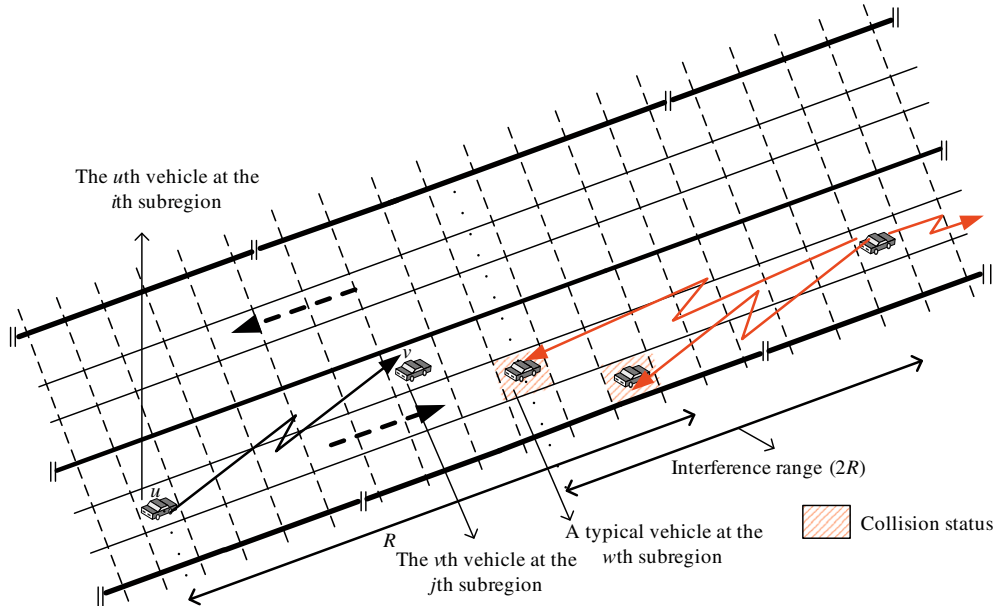
Fig. 3. Packet transfer from the uth vehicle at the ith subregion to the vth vehicle at the jth subregion.

the front of the vth vehicle, we multiply the recent probability by the probability that there is not any vehicle in the interval between the j'th and the w'th columns, denoted by the second product term. The third product term indicates the probability that there is not any transmitter vehicle in front of the wth subregion within the transmission range of the uth vehicle. In fact, this term complements the range of subregions included in the third product term discussed in the previous paragraph. The fourth product term indicates that the vehicle at the w'th column is in the collision status but the vth vehicle at the j'th column is not in the collision status. That is, there exists at least one transmitter vehicle within the interference range of the vehicle located at the w'th column and beyond the interference range of the vth vehicle.

If the jth subregion is not the highest priority according to the prioritization strategy in the routing scheme, we should also multiply (7) by the probability that there is not any vehicle with higher priority in the subregions at the j'th column, i.e., $\prod_{k=1}^{j-6(j'-1)-1}(1-\rho^{mob}(6(j'-1)+k))$. It is worth noting that if the lower index of $\sum$ or $\prod$ in (7) exceeds its higher index we consider it equal to zero. Such a case occurs for example when $j' = i' + R/L$ in the lower index of $\sum$ in (7).

On the other hand, we consider the packet delivery to destination (fixed at the intersection) as the routing to the 0th node (known as exogenous world) in the queueing network. Such a packet routing occurs when the destination is in the transmission range of the uth vehicle (i.e., the transmitter vehicle). Then, we have the following equation:

$$r_{u,0} = \sum_{i'=\frac{S-R}{L}+1}^{\frac{S}{6}} \sum_{i''=1}^{6} P_{loc}^t(6(i'-1)+i'')(1-P_{col}(i',0));$$

$$t = IL(u), \quad 1-P_{col}(i',0) = \left(\prod_{\substack{w'=\frac{S}{6}-\frac{2R}{L} \\ w'\neq i'}}^{\frac{S}{6}+\frac{2R}{L}} A(w') \frac{A(i')}{\left(1-\sum_{t=1}^{3}\rho_t^{mob}(i)\rho_{ts}(i)P_{tr}\right)} \right);$$

$$i' = \left\lceil \frac{i}{6} \right\rceil, \quad i = 6(i'-1)+i''.\tag{8}$$

In (8), we have considered the probability that there is not any transmitter vehicle in the interference range of the destination (fixed at the intersection) at each side of the street, except the de-

sired transmitter, i.e., the uth vehicle at the ith subregion. It is worth noting that in computing the non-collision probability in (7)–(8), the traffic status of the nodes has been considered separately. This is true for a product-form queueing network (e.g., a network comprised of M/M/1 or M/G/1/PS (processor sharing) nodes) and it is an approximation in general.

Then, we compute the probability of routing to itself, due to collision or not finding the next suitable vehicle, as in the following:

$$r_{u,u} = 1 - \sum_{\substack{l=1 \\ l\neq IL(u)}}^{6} N_l r_{u,v}|_{IL(v)=l} - \left(N_t - \sum_{i=1}^{S} \rho_t^{mob}(i) \right) \Gamma_{u,v}\bigg|_{IL(v)=IL(u)} - r_{u,0}; \quad t = IL(u),\tag{9}$$

where N_l denotes the average number of vehicles at the street and with the lth initial lane (the average number is considered regarding the discussions in the last paragraph in Section 2) and we exploit the symmetry existed in the routing probabilities, among the vehicles. It is worth noting that some of the routing probabilities in the above equation may be zero. For example, for routing schemes I, II, $r_{u,v}$ is zero when uth and vth vehicles are at the opposite direction (the other side of the street). In the next section, we focus on computing the maximum stable throughput and the average delay with respect to above proposed queueing network model.

4. Computation of the maximum stable throughput and the average delay

Among the QoS parameters, the maximum stable throughput and the average delay are two important ones. These parameters are of crucial importance with respect to conventional communication services and emergency services in a typical VANET. Since in this paper, we intend to compute the average delay for a typical packet along the street, i.e., from an intersection to the next one, we only focus on computing the average delay for marked packets, i.e., the packets generated at one intersection and delivered to arriving vehicles. To this end, we classify the packets into three types; marked packets at the first direction (i.e., from left to right as in Fig. 1), conventional packets at the first direction, and conventional packets at the second direction (i.e., from right to left). We distinguish between conventional packets at two directions because their

destinations are different in our scenario (located at two intersections). Moreover, we have assumed that the packets generated by vehicles at each direction are sent towards the same direction, however, in the case of routing scheme III, the vehicles at the other direction may relay them. In this case, since we distinguish the packets by their destinations, no problem occurs in packet forwarding towards their destinations (note that all vehicles have GPS, so the vehicles know their position, velocity, and direction).

In order to obtain the average delay for the marked packets, we exploit the concept of class in the queueing networks [21]. To this end, we consider class 1 for the generated marked packets and increase their classes after each transmission attempt. Thus, we know the number of transmission attempts experienced on the marked packets reaching the destination. By considering the average delay before each transmission attempt, we are able to estimate total average delay for the marked packets. It is worth noting that some of the transmission attempts may be unsuccessful due to a collision or the nonexistence of another vehicle within the transmission range. We have modeled unsuccessful packet transmission by routing to itself ($r_{u,u}$ in (9)). With respect to discussions in Section 3, if unsuccessful packet transmission is due to a collision (indicated by receiving only NACK packets) another short random wait should commence. Otherwise, no packets (ACK or NACK) are returned to the transmitter. Then, the transmitter knows that it is in the disconnection status and a long wait begins. Therefore, we need to distinguish between two modes of MAC schemes, i.e., long deterministic wait and short random wait, because the average service times corresponding to packet waits in these two modes are different. To this end, we consider the class of each packet composed of two parts, its type (short (s), long (l)) corresponding to two modes of MAC scheme, and its number (1 to C) denoting the number of transmission attempts. In this respect, we consider the following class change rules for the marked packets:

1. Each generated marked packet is assigned to class s_1 (short #1).
2. After expiring the short random wait, the packet is transmitted and its class number increases (e.g., from s_c to s_{c+1}).
3. If a packet is transmitted and only NACK packets are returned to the transmitter, then the packet transmission is collided and we increase packet's class number (e.g., from s_c to s_{c+1}). It is worth noting that by considering W in the first mode of MAC scheme sufficiently large, several consecutive collisions occur rarely.
4. If a packet is transmitted and no ACK or NACK packets are returned to the transmitter, then the packet transmission is not successful because there is not any vehicle within the transmission range of the transmitter. Then, we increase packet's class number and change its type from s to l, i.e., from s_c to l_{c+1}.
5. If the long wait is expired, the type of packet's class is changed from l to s, without any increase in class number (e.g., from l_c to s_c). Then, after a short random wait, the packet will be transmitted.

By employing the above packet class changing rules we are able to consider the status of the marked packets at each queueing node. Also, considering a maximum number for classes enables us to consider a Time-To-Live tag for each marked packet such that we dump the marked packets remained in the network more than a threshold in average.

In addition to C classes for marked packets, two extra classes are considered for conventional packets as well. We also consider the above five class changing rules for conventional packets without any increase in class number. Thus, we have totally $2C + 4$ classes, i.e., $2C$ classes for two types of marked packets (i.e., s_1 to s_C and l_1 to l_C) and two classes for conventional packets at each direction (s_{C+1}, l_{C+1} and s_{C+2}, l_{C+2}, respectively). Moreover, we need to modify the

routing probabilities, (6)–(9) moderately, in order to include the packet class concept as in the following:

$$R_{us_c, ul_{c+1}} = \prod_{w'=i'+1}^{i'+\frac{R}{L}} B(w'), \quad R_{us_c, us_{c+1}} = r_{u,u} - R_{us_c, ul_{c+1}},$$

$$R_{ul_c, us_c} = 1, \quad R_{us_c, vs_{c+1}} = r_{u,v}, \quad R_{us_c, 0} = r_{u,0}, \quad R_{us_C, 0} = 1;$$
$$1 \leqslant c \leqslant C - 1, \tag{10}$$

where $r_{u,v}$'s are computed as in (7)–(9). Moreover, we consider ρ_{us}, ρ_{ul} in (5) as $\sum_{c=1}^{C+2} \rho_{us_c}$, $\sum_{c=1}^{C+2} \rho_{ul_c}$, respectively. In (10), we consider class C as the maximum acceptable class number for the marked packets. Thus, we dump the marked packets with a class higher than C. Since at each transmission attempt we increase packet's class number, we consider $R_{us_C, 0}$ equal to one, indicating that we dump class-C packets in the network. We also exclude these packets in computing throughput. In the following, we will explain how we compute the average delay for the marked packets and the maximum stable throughput with respect to above multi-class queueing network.

In a typical queueing system, in stable conditions the number of packets departed from the system in a sufficiently large time interval equals the number of arrived packets at the system within the same time interval. However, in our queueing network model, some packet departures are due to dumping as the result of excess delay. Then, by writing the traffic equations [21], we are able to compute the throughput, i.e., the rate of packets successfully received by the destination, as in the following:

$$\alpha_{us_1} = \lambda_{us_1};$$

$$\alpha_{us_{c+1}} = \sum_v \alpha_{vs_c} R_{vs_c, us_{c+1}} + \alpha_{ul_{c+1}} R_{ul_{c+1}, us_{c+1}}; \quad 1 \leqslant c \leqslant C - 1$$

$$\alpha_{ul_{c+1}} = \alpha_{us_c} R_{us_c, ul_{c+1}}; \quad 1 \leqslant c \leqslant C - 1$$

$$\alpha_{us_{C+1}} = \lambda_{us_{C+1}} + \sum_v \alpha_{vs_{C+1}} R_{vs_{C+1}, us_{C+1}} + \alpha_{ul_{C+1}} R_{ul_{C+1}, us_{C+1}};$$

$$\alpha_{ul_{C+1}} = \alpha_{us_{C+1}} R_{us_{C+1}, ul_{C+1}};$$

$$\alpha_{us_{C+2}} = \lambda_{us_{C+2}} + \sum_v \alpha_{vs_{C+1}} R_{vs_{C+1}, us_{C+2}} + \alpha_{ul_{C+2}} R_{ul_{C+2}, us_{C+2}};$$

$$\alpha_{ul_{C+2}} = \alpha_{us_{C+2}} R_{us_{C+2}, ul_{C+2}};$$

$$Throughput = \sum_v \left(\sum_{c=1}^{C-1} \alpha_{vs_c} R_{vs_c, 0} + \alpha_{vs_{C+1}} R_{vs_{C+1}, 0} \right); \tag{11}$$

where α_{us_c}, α_{ul_c} denote the total arrival rate of the cth class packets corresponding to both types (short and long) at the uth vehicle, respectively. Moreover, class numbers $C + 1$ and $C + 2$ are related to conventional packets at first and second directions of the street. However, in computing the throughput we consider the packets successfully received by the destination with a class number at most equal to C (related to the marked packets) as well as class number $C + 1$ (related to conventional packets at first direction). It is worth noting that we increase the class number at each transmission attempt, so when a marked packet with class $C - 1$ is transmitted its class number at the receiver reaches the maximum acceptable number, C.

In order to obtain the maximum stable throughput, we compute the maximum attainable packet generation rate such that the queueing nodes are in stable conditions. Since in our scenario, the vehicles arriving at different lanes of the street are not completely similar, their traffic intensities (ρ_u) will not be equal. Among them we should find the maximum traffic intensity. In fact, these queueing nodes are bottlenecks in our queueing network. Then, we increase the packet generation rate until we reach the border of instability at the bottleneck nodes. It is worth mentioning that we have six types of queueing nodes in our queueing network, corresponding to vehicles with three initial lanes at both ways (directions) of the street. Actually, the vehicles arriving at each typical lane of the street are similar with respect to previous discus-

sions. In this case, (11) delivers the maximum stable throughput, as in the following:

$$Max\ Stable\ Throughput = \left(\sum_v \sum_{c=1}^{C-1} \alpha_{vs_c} R_{vs_c,0} + \max \sum_v \alpha_{vs_{C+1}} R_{vs_{C+1},0} \right) \Bigg|_{\max_u \rho_u = 1} ;$$

$$\rho_u = \rho_{us_{C+1}} + \rho_{ul_{C+1}} + \rho_{us_{C+2}} + \rho_{ul_{C+2}} + \sum_{c=1}^{C} \rho_{us_c} + \sum_{c=1}^{C} \rho_{ul_c},$$

$$\rho_{us_c} = \frac{\alpha_{us_c}}{\mu_s}, \quad \rho_{ul_c} = \frac{\alpha_{ul_c}}{\mu_l}. \tag{12}$$

where ρ_u, ρ_{us_c} denote the total traffic intensity and the traffic intensity of class s_c, respectively, corresponding to the uth vehicle. In (12), we consider fixed and variable rates for marked and conventional packets, respectively, and evaluate the maximum stable throughput for all packets in one direction. Nevertheless, we are able to evaluate the maximum stable throughput for only marked packets or conventional packets.

On the other hand, in order to compute the average delay for the marked packets, in the first stage we compute the average number of delay units included in the number of classes as in the following:

$$P(D = k - 1) = \frac{\sum_u \alpha_{us_{k-1}} R_{us_{k-1},0}}{\sum_u \sum_{c=2}^{C} \alpha_{us_{c-1}} R_{us_{c-1},0}}; \quad 2 \leqslant k \leqslant C \tag{13}$$

$$\overline{D} = \sum_{k=1}^{C-1} k P(D = k). \tag{14}$$

where D is the random variable denoting the delay units for marked packets received by the destination.

In the second stage, we need to compute the average delay corresponding to each unit (class number). Thus, we need to compute the average delay for a typical packet with respect to the arrival rates of different classes of packets derived by solving the traffic equations. To this end, we consider FIFO policy that is commonly used in practice. Although we assume packets generation as a Poisson process, the packets arrival at each vehicle is not a Poisson. This is due to non-memoryless distribution for packet service times and possible feedback in the queueing network (e.g., due to collision). However, for the sake of simplicity and analytical tractability we assume Poisson approximation for the packet arrival rates at each vehicle. It is worth mentioning that the departure rate from each queueing node is the same as its arrival rate in a conservative queueing node irrespective of the service policy. Nevertheless, the distribution of inter-departure time is strongly dependent upon the service policy. In computing the throughput, we only need the rates, so there is not any approximation in this respect. But in computing the average delay our approach is only an approximation and we confirm the accuracy of our results via simulation.

By applying the above approximation, each node is considered as an M/G/1 queueing node with two classes of packets. Actually, we have multiple classes but in view of service time they are classified into only two types corresponding to two modes of MAC scheme. Then, we have the following relations for the average delay time for the marked packets at each queueing node (e.g., uth vehicle) [24]:

Delay Time = Service Time(waiting time due to MAC)
+ Queueing Time;

$$T_u = \frac{\alpha_{us}}{\alpha_{us} + \alpha_{ul}} \frac{1}{\mu_s} + \frac{\alpha_{ul}}{\alpha_{us} + \alpha_{ul}} \frac{1}{\mu_l} + \frac{\alpha_{us}\overline{x_s^2} + \alpha_{ul}\overline{x_l^2}}{2(1 - \rho_{us} - \rho_{ul})}; \quad \overline{x_l^2} = x_l^2;$$

$$\overline{x_s^2} = \frac{1}{W}\left(\frac{W(W+1)(2W+1)}{6} \right), \quad \alpha_{us} = \sum_{c=1}^{C+2} \alpha_{us_c},$$

$$\alpha_{ul} = \sum_{c=1}^{C+2} \alpha_{ul_c}, \quad \rho_{us} = \sum_{c=1}^{C+2} \rho_{us_c}, \quad \rho_{ul} = \sum_{c=1}^{C+2} \rho_{ul_c}, \tag{15}$$

where x_s, x_l denote the service time corresponding to short random wait and long deterministic wait. In (15), we have added the service time (comprised of two first additive terms in (15)) to the queueing time (i.e., the third additive term in (15) obtained by Pollaczek–Khintchine theorem [24]) in order to obtain the average delay (T_u) equivalent to each delay unit specified by the class number. Since on one hand, we have heterogeneous and non-similar queueing nodes and on the other hand, class numbers are due to packet traversing among different queueing nodes, we obtain the lower and the upper bounds for the average delay with respect to different types of queueing nodes (i.e., vehicles with three initial lanes at two sides of the street). If we multiply the average delay units (14) by the maximum and the minimum delay time per unit, we obtain the upper and lower bounds for the average delay. Although we apply an approximation in computing the average delay at each node in (15) (i.e., considering Poisson arrival process at each node), simulation results in the next section confirm the validity of our approximate lower and upper bounds.

5. Numerical and simulation results

It is assumed that there is a source of marked packets at the beginning of the street (i.e., at the intersection) at only one way (direction) and we consider a fixed and small marked packet generation rate. Then, we obtain the average delay for the marked packets as well as the maximum stable throughput for all packets along the street. In this respect, we consider the conventional packet generation rate at each vehicle proportional to the vehicle's velocity. This is in accordance with the discussion preceding the routing schemes in Section 3. The typical parameters corresponding to the packet generation rate, mobility patterns, transmission range, MAC and routing schemes have been illustrated in Table 1.

In order to confirm our analytical results we have done several simulations. With respect to the new mobility model, MAC, and routing schemes we have implemented a simulation program in MATLAB environment. The simulation program is an event-based driven one consisting of the following events; packet generations, vehicle movements, and packet class changes (that is, ending instants of waiting times). We consider the mobility patterns of the vehicles according to the mobility model in Section 2, so, we spatially quantize the movement of the vehicles. Moreover, we do not consider any restriction for overtaking of the vehicles, because we keep the arrival rate sufficiently small in order to preserve sparse situations. With respect to the simulation setup and the lane transition probabilities shown in Table 1, the vehicles are interested in keeping their lanes and speeds. If a vehicle changes

Table 1
The typical values for the parameters in numerical analyses and simulation setup.

Parameter	Value	Unit
Speed distribution at the first lane	Uniform [3,14]	m/s
Speed distribution at the second lane	Uniform [14,22]	m/s
Speed distribution at the third lane	Uniform [22,33]	m/s
Length of the street	2000	m
Length of each subregion (L)	10	m
Transmission range (R)	100	m
Maximum number of slot times in the first mode of MAC (W)	50	NA
Typical deterministic long wait (second mode of MAC)	1	s
Number of classes for marked packets (C)	50	NA
Time slot	200	μs
Total simulation time	200,000	Time slot
Transient time at simulation	100,000	Time slot
Lane transition probability for the 2nd lane	0.2	NA
Lane transition probability for 1st and 3rd lanes	0.1	NA
Marked packet generation rate at each vehicle	10	packets/s

its lane after traversing a subregion, it selects a new speed according to the corresponding distribution of the new lane. We also consider independent Poisson distributions for the arriving vehicles and the packet generations.

In Figs. 4 and 5 we have focused on routing schemes I–III. In these figures, we have changed the arrival rate of vehicles at both ways of the street symmetrically and evaluated the maximum stable throughput for the total packets at only one way as well as the average delay for the marked packets. As we observe in Fig. 4 by increasing the arrival rate of vehicles the maximum stable throughput increases as well until a point beyond which the maximum stable throughput decreases. Clearly, for higher arrival rate of vehicles the number of packet generators and then packet forwarding opportunities will be increased. However, the collision probability increases too. The trend of the maximum stable throughput shows the result of these two contradicting factors. It is worth noting that the maximum stable throughput for routing scheme III is lower than the other two routing schemes. In fact, the packets are generated at the vehicles at both sides. Moreover, in routing scheme III, the packets are exchanged among the vehicles at both sides as well. This leads to higher number of transmissions due to more opportunities and thus a larger probability of collision. So, the nodes will be more congested and go to saturation (the border of instability) earlier.

In Fig. 5, we evaluate the average delay for the marked packets. For each routing scheme, the higher number of vehicles causes that packets traverse by radio transmission more probably, leading to less delay. On the other hand, in comparison between routing scheme III and routing schemes I and II, we have more opportunities for packet forwarding via radio transmission in the former, however, we have more collision probability due to higher number of transmissions as well. With respect to the results in Fig. 5, routing scheme III has the least delay among the three routing schemes. In fact, more packet forwarding opportunities in routing scheme III, causes that the packets experience the long waits less, leading to less total delay. As we discussed in the following of (15) in Section 4, we have plotted two curves in Fig. 5 indicating lower and upper bounds for the average delay. Simulation results in Fig. 5, confirm completely our analytical results. Since the packet generation rate affects the average delay, we have considered a packet generation rate below the maximum stable throughput (obtained in Fig. 4) in order to be certain that the average delay is finite. In this figure, we have considered total packet generation rate (marked and conventional) at each way equal to 200 that is smaller
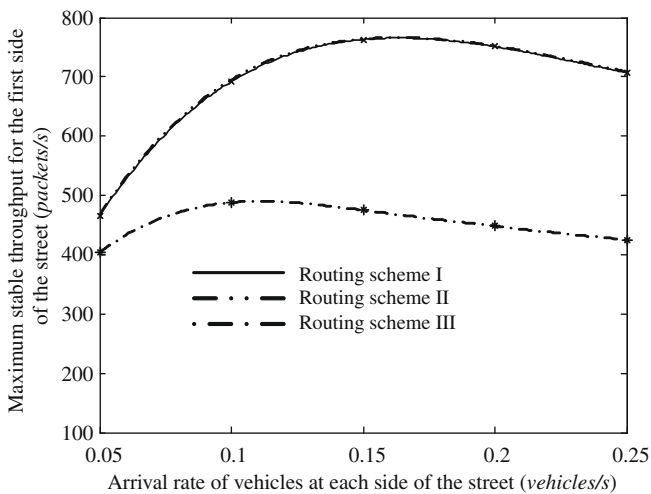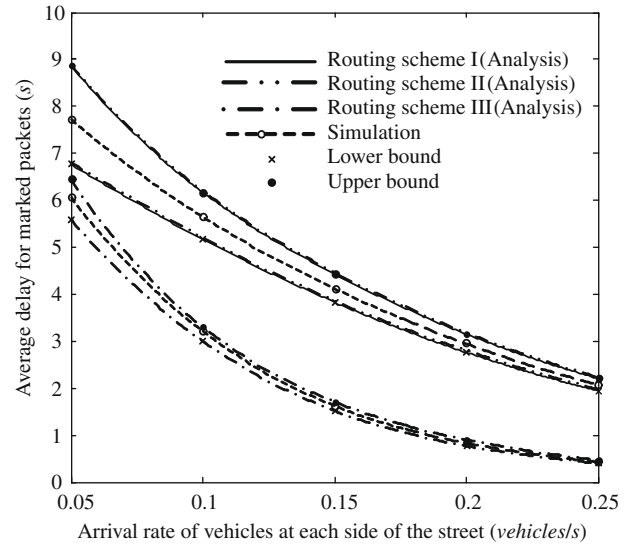

Fig. 5. The average end-to-end delay of marked packets for routing schemes I–III (total packet generation rate = 200 packets/s).

than the maximum stable throughput (see Fig. 4), because at the maximum stable throughput we will have a very large queueing delay.

In order to obtain an estimate for the maximum stable throughput by simulation, we compare the ratio of the number of successfully delivered packets to destination in a sufficiently large time interval to the number of generated packets at the same time interval, with unity. It is worth noting that we are not able to reach the maximum stable throughput exactly, because at the generation rate corresponding to the maximum stable throughput some of the nodes are in the border of instability such that the transient time for reaching the steady state goes to infinity. Therefore, we have considered the above ratio corresponding to packet generation rates a few lower and higher than the maximum rate derived analytically and illustrated the results. In Fig. 6, we have plotted the simulation results for three values of the arrival rate of vehicles for routing scheme I. By considering the knees of the curves in Fig. 6 we observe a good match between the simulation results and the analytical results in Fig. 4.

It is worth noting that routing schemes I and II, depend only on the vehicles at one direction and the vehicles at the other side of the street affect the collision status. Thus, increasing the vehicle arrival rate at both directions has a similar effect on routing schemes I and II. With respect to Figs. 4 and 5 we do not observe any meaningful difference between routing schemes I and II. Actually, since we have considered sparse situations it is rarely probable that we encounter a situation indicating more than one vehicle in the same column (i.e., at different lanes) at one side. So, we have focused on routing scheme I between schemes I and II, in the following analyses. On the other hand, routing scheme III that forwards the packets through the vehicles at the other side as well, has more complexity compared to routing schemes I and II. In fact, although increasing the arrival rate of vehicles at the other side of the street increases the collision probability, there are more opportunities for packet forwarding as well.

In Figs. 7 and 8 we have focused on the effect of the opportunistic relaying by the vehicles on the other side of the street in routing scheme III when compared to routing scheme I. Obviously, we expect that the opportunistic relaying leads to higher opportunities for packet forwarding. Such opportunistic relaying leads to significant lower average delays compared to previous routing schemes (see Fig. 8). However, since the packets from each side of the street
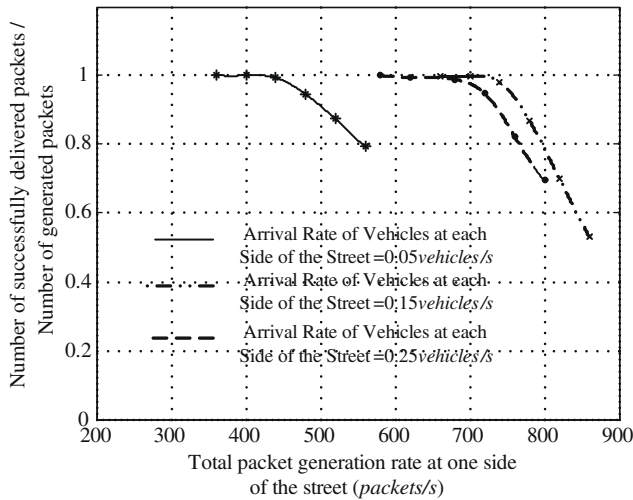

Fig. 4. The analytical maximum stable throughput for routing schemes I–III.

Fig. 6. The simulation results of the maximum stable throughput for routing scheme I and three different arrival rates of vehicles.

will be also sent to the vehicles at the other side, we will encounter an increase in the number of transmissions leading to a higher collision probability. This leads to a reduction in the maximum stable throughput (see Fig. 7). In other words, the nodes will go to saturation earlier than the case in routing scheme I. This is a very important result that indicates different (contradicting) effects of opportunistic relaying onto the maximum stable throughput and the average delay. On the other hand, for routing scheme I, the vehicles at the other side increase the collision probability, leading to higher delay (see Fig. 8). But when the arrival rate of vehicles at the first side increases we observe similar results to Figs. 4 and 5. Since in routing scheme III the vehicles at both sides of the street are exploited for packet forwarding at each direction, increasing the arrival rate of vehicles only at one direction leads to similar results, irrespective of the desired direction. We also confirm the analytical results by simulation in Figs. 9 and 10. The discussions of the results and matching between simulation and analytical results are similar to previous discussions corresponding to Figs. 4–6.
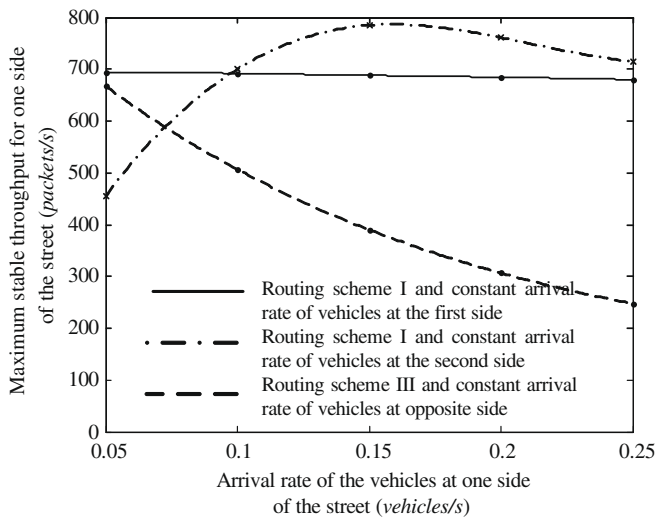

Fig. 7. Analytical comparison of the maximum stable throughput for different values of vehicles' arrival rate at one side and a fixed arrival rate at opposite side, in routing schemes I and III (the fixed arrival rate of vehicles = 0.1 vehicles/s).
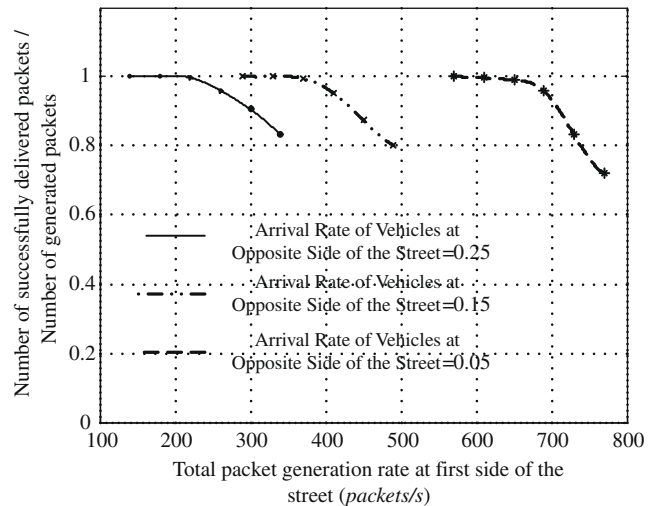

Fig. 9. The simulation results of the maximum stable throughput for routing scheme III and three different arrival rates of vehicles at opposite side of the street (the arrival rate of vehicles at first side of the street = 0.1 vehicles/s).
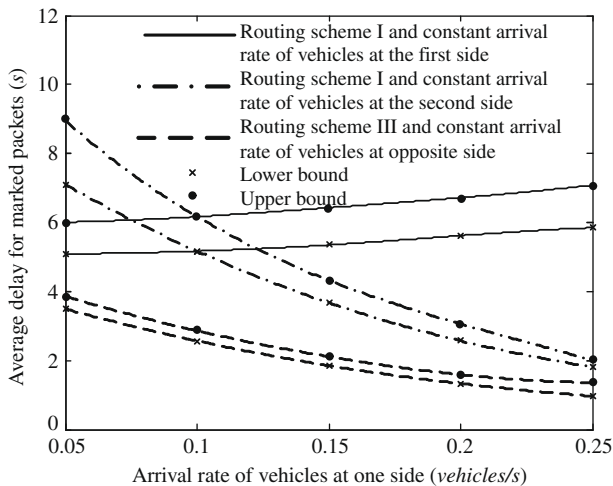

Fig. 8. Analytical comparison of the average end-to-end delay for different values of vehicles' arrival rate at one side and a fixed arrival rate at opposite side, in routing schemes I and III (the fixed arrival rate of vehicles = 0.1 vehicles/s, total packet generation rate = 200 packets/s).
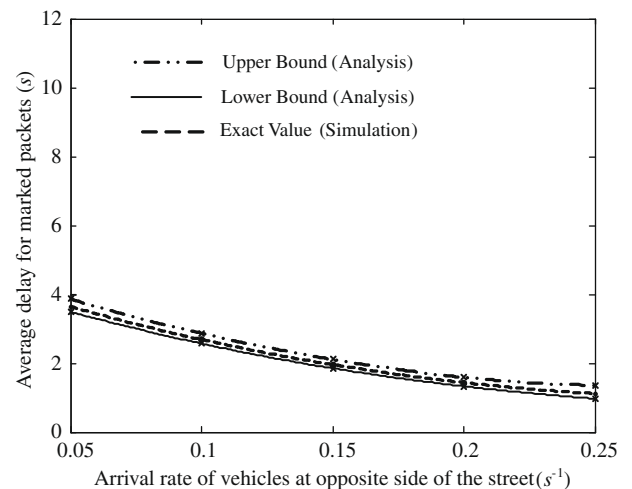

Fig. 10. The average end-to-end delay of marked packets for routing scheme III (the arrival rate of vehicles at one side of the street = 0.1 vehicles/s, total packet generation rate = 200 packets/s).

1779

In Figs. 11 and 12 we have focused on the efficiency of the second mode of the proposed MAC scheme, i.e., long deterministic wait. As we observe decreasing long wait increases inefficient transmissions that cause unnecessary collisions leading to the spoil of the rare relaying opportunities and decrease in the maximum stable throughput. On the other hand, increasing the long wait increases the average delay as we expect. From the results in Figs. 11 and 12, we can select a suitable value for the long wait in our MAC scheme.

6. Conclusions

We focused on a DTN scenario consisting of a part of a VANET, i.e., a two-way street, in sparse situations. We mapped mobility pattern of the vehicles as well as opportunistic multi-hop packet forwarding process onto several queueing networks in order to evaluate the average delay for the packet forwarding and the maximum stable thr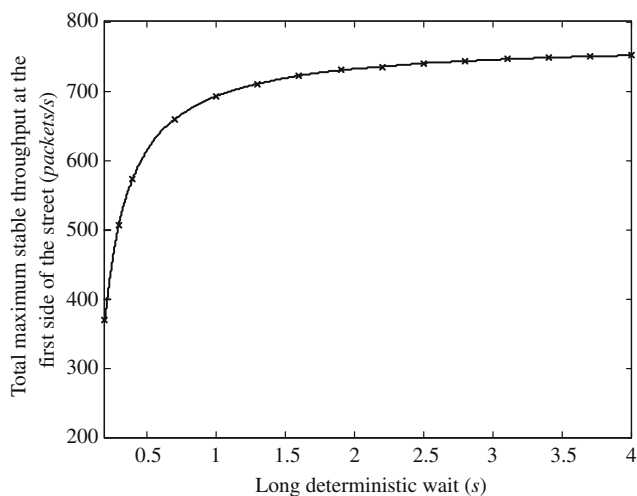oughput, along the street. In our scenarios, the packets' destination was fixed at the intersections, conventional packet sources were distributed along the street, and a packet generator was considered at the intersection. The packets generated at the intersection were marked. We evaluated the average delay corresponding to marked packets and the maximum stable throughput for all packets at one direction.

In the packet forwarding process, we proposed a new MAC scheme comprised of two modes, short random wait and long deterministic wait. The latter was suitable with respect to disconnection status occurred frequently in DTN scenarios. Moreover, we evaluated three routing schemes. At two schemes we only considered the relaying vehicles in the direction of packet forwarding but with different velocity-dependent priorities and in third routing scheme we exploited the capability of relaying of the vehicles at the other side of the street.

In our analyses, we exploited the concept of class in the queueing networks in order to distinguish between two modes of MAC scheme and specifying the average number of transmission attempts. Each transmission attempt was equivalent to an average delay, so we were able to derive the average delay incurred by a typical marked packet.

In the last part of the paper, we carried out several numerical analyses in order to obtain the average delay and the maximum stable throughput in different conditions, e.g., different vehicle arrival rates, different routing schemes, and different values for the long wait in the second mode of the proposed MAC scheme. We also confirmed our analytical results by simulation in several settings. We believe that the proposed analytical approach in this paper, will be helpful for estimating some of the decision metrics employed in the routing algorithms in VANETs.

The analytical approach in the paper can be extended to a city configuration comprised of several intersections and streets. Actually in such a configuration, we are able to compute the arrival rate of vehicles at each side of the street that is strongly dependent upon the mobility patterns of the vehicles and turning probabilities at the intersections. In this respect, [12] contains more details. Then, we are able to apply the analytical approach in this paper in order to evaluate the average delay and the maximum stable throughput along each of the streets.

Fig. 11. The analytical maximum stable throughput for different values of long wait in routing scheme I (the arrival rate of vehicles at each side of the street = 0.1 vehicles/s).
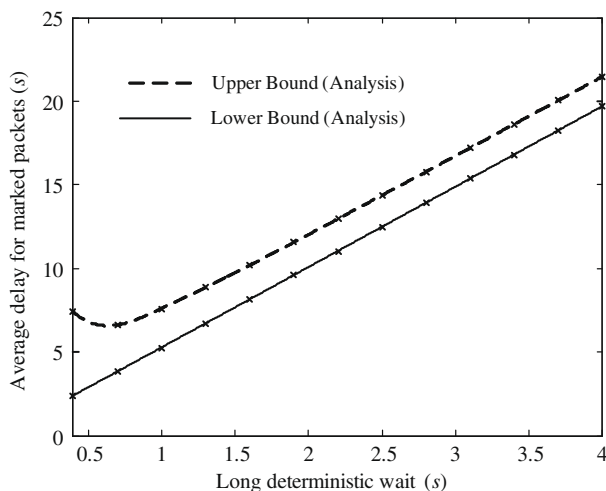

Fig. 12. The analytical average end-to-end delay for different values of long wait in routing scheme I (the arrival rate of vehicles at each side of the street = 0.1 vehicles/s, total packet generation rate = 450 packets/s).

References

[1] Evan P.C. Jones, Lily Li, Jakub K. Schmidtke, Paul A.S. Ward, Practical routing in delay – tolerant networks, IEEE Trans. Mobile Comput. 6 (8) (2007) 943–959.
[2] Padma Mundur, Sookyoung Lee, Matthew Seligman, Routing for data delivery in dynamic networks, in: Proceedings of the IEEE MILCOM'06, October 2006, pp. 1–7.
[3] A. Panagakis, A. Vaios, I. Stavrakakis, On the effects of cooperation in DTNs, in: Proceedings of the Second International Conference COMSWARE'07, 2007, pp. 1–6.
[4] Seung-Keun Yoon, Zygmunt J. Haas, Efficient tradeoff of restricted epidemic routing in mobile ad-hoc networks, in: Proceedings of the IEEE MILCOM'07, October 2007, pp. 1–7.
[5] Fazl-e-Hadi, N. Shah, A. Hussain Syed, M. Yasin, Adaptive anycast: a new anycast protocol for performance improvement in delay tolerant networks, in: Proceedings of the International Conference on Integration Technology, China, March 2007, pp. 185–189.
[6] Anders Lindgren, Kaustubh S. Phanse, Evaluation of queueing policies and forwarding strategies for routing in intermittently connected networks, in: Proceedings of the First International Conference COMSWARE'06, January 2006, pp. 1–10.
[7] Darshak Thakore, Subir Biswas, Routing with persistent link modeling in intermittently connected wireless networks, in: Proceedings of the IEEE MILCOM'05, October 2005, pp. 461–467.
[8] W. Zhao, Y. Chen, M. Ammar, M. Corner, B. Levine, E. Zegura, Capacity enhancement using throwboxes in DTNs, in: Proceedings of the IEEE MASS'06, October 2006, pp. 31–40.
[9] F.K. Karnadi, Z.H. Mo, K.-C. Lan, Rapid generation of realistic mobility models for VANET. Available from: <http://www.cse.unsw.edu.au/~klan/paper/sumo.pdf>.
[10] M. Fiore, Mobility models in inter-vehicle communications literature. Available from: <http://www.tlc-networks.polito.it/fiore/papers/mobilityModels.pdf>.

[11] M.M. Artimy, W.J. Phillips, W. Robertson, Connectivity with static transmission range in vehicular ad hoc networks, in: Proceedings of the IEEE CNSR'05, 2005.

[12] G.H. Mohimani, F. Ashtiani, A. Javanmard, M. Hamdi, Mobility modeling, spatial traffic distribution, and probability of connectivity for sparse and dense vehicular ad hoc networks, IEEE Trans. Veh. Technol., doi:10.1109/TVT.2008.2004266.

[13] M.K.M. Ali, M. Khabazian, A performance modeling of connectivity in ad hoc networks, in: Proceedings of the IEEE WCNC'06, 2006, NET17-1.

[14] T. Nadeem, P. Shankar, L. Iftode, A comparative study of data dissemination models for VANETs, in: Proceedings of the IEEE International Conference on Mobile and Ubiquitous Systems, 2006, pp. 1–10.

[15] J. Zhao, Y. Zhang, G. Cao, Data pouring and buffering on the road: a new data dissemination paradigm for vehicular ad hoc networks, IEEE Trans. Veh. Technol. 56 (6) (2007) 3266–3277.

[16] M. Nekovee, B. Bjarni Bogason, Reliable and efficient information dissemination in intermittently connected vehicular ad hoc networks, in: Proceedings of the IEEE VTC'07, April 2007, pp. 2486–2490.

[17] Thomas D.C. Little, Ashish Agarwal, An information propagation scheme for VANETs, in: Proceedings of the IEEE Conference on Intelligent Transportation Systems, Vienna, September 2005, pp. 155–160.

[18] J. LeBrun, C.N. Chuah, D. Ghosal, M. Zhang, Knowledge-based opportunistic forwarding in vehicular wireless ad hoc networks, in: Proceedings of the IEEE VTC'05, June 2005, pp. 2289–2293.

[19] J. Zhao, G. Cao, VADD: vehicle-assisted data delivery in vehicular ad hoc networks, in: IEEE Proceedings of INFOCOM'06, 2006.

[20] Y. Ding, C. Wang, L. Xiao, A static-node assisted adaptive routing protocol in vehicular networks, in: Proceedings of VANET'07, September 2007, pp. 59–67.

[21] X. Chao, M. Miyazawa, M. Pinedo, Queueing Networks, John Wiley & Sons, 1999.

[22] F. Ashtiani, J.A. Salehi, M.R. Aref, Mobility modeling and analytical solution for spatial traffic distribution in wireless multimedia networks, IEEE J. Sel. Areas Commun. 21 (10) (2003) 1699–1709.

[23] P. Gupta, P.R. Kumar, The capacity of wireless networks, IEEE Trans. Inform. Theory 46 (2) (2000) 388–404.

[24] L. Kleinrock, Queueing Systems, vols. 1 and 2, John Wiley & Sons, 1975.