Internet Appendix: Very Noisy Option Prices and Inference Regarding Volatility Risk Premium

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January 2022

Appendix A Data cleaning

In order to eliminate what appear to be obvious data errors or non-competitive quotes, we require that option prices satisfy arbitrage conditions and that option bid and ask prices are reasonable. Our goal is to preserve as much of the data as possible, excluding only the most obviously problematic observations.

First, we exclude options that violate arbitrate bounds. We are careful, however, to calculate arbitrage bounds based on the appropriate side of the bid-ask quote. This is true not only for the option price, but also for the stock price, which is more commonly proxied for using the closing price. Using the closing price is inappropriate given that it is not known to options market participants until after the option market closes, and it does not take the stock's bid-ask spread into account when computing arbitrage bounds. Given that Bogousslavsky and Muravyev (2021) find that closing prices are typically outside the end-of-day bid-ask quotes, these differences can be consequential.

We check several arbitrage conditions that relate the price of the option to the price of a stock.[1] First, we impose the lower bounds that the purchase price of an option must be no less than its immediate exercise value. For calls, this is

$$C_{\text{ask}} \geq \max\left(0, S_{\text{bid}} - X\right),$$

while for puts it is

$$P_{\text{ask}} \geq \max\left(0, X - S_{\text{ask}}\right).$$

Next, we impose upper bounds. The proceeds from selling a call must be no greater than the cost of buying a stock, or

$$C_{\text{bid}} \leq S_{\text{ask}},$$

and the proceeds of selling a put must be no greater than the strike price:

$$P_{\text{bid}} \leq X.$$

We also eliminate apparent stub (i.e., non-competitive) quotes for which the bid-ask spread is more than \$10 or more than the price of the underlying stock, which may alternatively indicate undocumented missing value codes (e.g., 999) or data errors. We further

[1] We do not check other arbitrage conditions on the relative pricing of different options on the same stock, which would be computationally difficult.

discard observations in which the bid is equal to or exceeds the ask. Finally, we eliminate large reversals in option returns (greater than 2000% followed by less than -95% or vice versa) under the assumption that they most likely represent data errors.

Appendix B A General Stochastic Volatility Model

We start with the following general stochastic volatility model:

$$\begin{aligned}\frac{dS_t}{S_t} &= \mu_t dt + \sigma_t dB_{1,t} \qquad (1)\\ d\sigma_t &= \theta_t dt + \omega_t \rho dB_{1,t} + \omega_t \sqrt{1-\rho^2} dB_{2,t} \\ \text{corr}\left(\frac{dS_t}{S_t}, d\sigma_t\right) &= \rho,\end{aligned}$$

where S_t is the price of the stock at time t, σ_t is its volatility, and ρ is the instantaneous correlation between the stock price and its volatility. $B_{1,t}$ and $B_{2,t}$ are standard Brownian motions under the physical probability measure. μ_t and θ_t are drifts of the underlying price and volatility processes under the physical measure. The price of a derivative on the underlying stock is represented by $f(S_t, \sigma_t, t)$. Ito's lemma implies:

$$\begin{aligned}\frac{df}{f} &= \mu_f dt + \frac{1}{f}\left(\frac{\partial f}{\partial S} S\sigma + \frac{\partial f}{\partial \sigma}\omega\rho\right) dB_{1,t} + \frac{1}{f}\frac{\partial f}{\partial \sigma}\omega\sqrt{1-\rho^2} dB_{2,t} \qquad (2)\\ \mu_f &= \frac{1}{f}\left(\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \theta \frac{\partial f}{\partial \sigma} + \frac{1}{2}\omega^2 \frac{\partial^2 f}{\partial \sigma^2} + \frac{\partial^2 f}{\partial \sigma \partial S}\omega\rho S\sigma\right)\end{aligned}$$

Note that in the above and following equations in this section, we drop the t-subscripts (except for dt, $dB_{1,t}$ and $dB_{2,t}$) for simplicity.

The absence of arbitrage implies that there is an equivalent martingale measure under which the dynamics of the stock price and of the volatility are

$$\begin{aligned}\frac{dS}{S} &= (r-q)\,dt + \sigma dB^*_{1,t} \qquad (3)\\ d\sigma &= \upsilon dt + \omega\rho dB^*_{1,t} + \omega\sqrt{1-\rho^2} dB^*_{2,t}\,,\end{aligned}$$

where $B^*_{1,t}$ and $B^*_{2,t}$ are Brownian motions and υ is the drift of the volatility process under the risk neutral measure, q is the instantaneous dividend yield and r is the instantaneous risk-free rate of interest. No arbitrage also implies that the price of any derivative satisfies the PDE:

$$\frac{\partial f}{\partial t} + (r-q)\,S\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \upsilon\frac{\partial f}{\partial \sigma} + \frac{1}{2}\omega^2\frac{\partial^2 f}{\partial \sigma^2} + \frac{\partial^2 f}{\partial \sigma \partial S}\omega\rho S\sigma = rf \qquad (4)$$

Substituting this PDE into the actual dynamics for f, we get

$$\begin{aligned}\frac{df}{f} &= \frac{1}{f}\left\{(\mu - r + q)\, S\frac{\partial f}{\partial S} + (\theta - \upsilon)\frac{\partial f}{\partial \sigma} + rf\right\} dt + \\ &+\frac{1}{f}\left(\frac{\partial f}{\partial S} S\sigma + \frac{\partial f}{\partial \sigma}\omega\rho\right) dB_{1,t} + \frac{1}{f}\frac{\partial f}{\partial \sigma}\omega\sqrt{1-\rho^2} dB_{2,t}\end{aligned} \tag{5}$$

This can be simplified using the definitions of $\beta_S = \frac{S}{f}\frac{\partial f}{\partial S}$ and $\beta_\sigma = \frac{1}{f}\frac{\partial f}{\partial \sigma}$ as:

$$\frac{df}{f} - \beta_S \frac{dS}{S} = \{(q - r)\, \beta_S + (\theta - \upsilon)\, \beta_\sigma + r\}\, dt + \beta_\sigma \omega\rho dB_{1,t} + \beta_\sigma \omega\sqrt{1-\rho^2} dB_{2,t} \tag{6}$$

Defining the volatility risk premium as $\lambda_\sigma = \theta - \upsilon$, we arrive at the following representation for the expected excess return of a delta-hedged derivative by taking expectations on both sides of the equation above:

$$E\left[\frac{df}{f} - \beta_S\left(\frac{dS}{S} + (q - r)\, dt\right)\right] - rdt = \beta_\sigma \lambda_\sigma dt \tag{7}$$

Appendix C Theoretical MR bias in observed delta-hedged returns

Following the notation in the main text, let $\tilde{S}_t$ be the true price of the stock and $\epsilon_{S,t}$ be its relative price error. The observed stock price $S_t = \tilde{S}_t\,(1 + \epsilon_{S,t})$. Analogously, let $\tilde{C}_t$ $\tilde{P}_t$ be the true price of the call (put) and $\epsilon_{C,t}$ ($\epsilon_{P,t}$) be the relative price error of call (put). The observed call (C_t) and put (P_t) prices can be written as $C_t = \tilde{C}_t\,(1 + \epsilon_{C,t})$ and $P_t = \tilde{P}_t\,(1 + \epsilon_{P,t})$. We make the following assumptions.

Assumption 1 (a) Price errors are independent in time-series and independent among stocks, calls and puts. (b) Price errors are also independent of true prices.

Assumption 2 Assume that $\tilde{C}_t - \tilde{C}_{t-1} \to 0$, $\tilde{P}_t - \tilde{P}_{t-1} \to 0$ and that $\tilde{S}_t - \tilde{S}_{t-1} \to 0$ as $\Delta t \to 0$. I.e., the time period is short so the change in stock and option prices is small.

Let f_1 ($\tilde{f}_1$) and f_2 ($\tilde{f}_2$) be the observed (true) price of any stock, call or put, let t and s be time periods, and let g and h be any measurable functions. Then (a) in Assumption 1 implies that

$$\text{Cov}\left(g\left(\epsilon_{f_1,t}\right), h\left(\epsilon_{f_2,s}\right)\right) = 0$$

if $f_1 \neq f_2$ or $t \neq s$. Let J be any measurable function of the time-t true prices of any combination the stock, call, and put. Then (b) in Assumption 1 implies that

$$\mathrm{Cov}\left(g\left(\epsilon_{f_1,t}\right), J\right) = 0$$

for any f_1 and g.

While Assumption 1 simplifies the derivation of the MR bias, it is possible that the data does not satisfy this assumption. For example, the errors for the call and put with the same strike and maturity might be correlated, which would violate the assumption. While this would not present a problem for portfolios of calls or puts, it might affect the bias for straddle returns. In section G, we consider the possibility that the price errors of the call and put are correlated.

Using these two assumptions, one can derive the MR bias of delta-hedged returns. Specifically, the observed return for a delta-hedged call is expressed as

$$R_t = \frac{C_t - C_{t-1}}{C_{t-1}} - \Delta^C_{S,t-1} \frac{S_{t-1}}{C_{t-1}} \frac{S_t - S_{t-1}}{S_{t-1}}, \tag{8}$$

where $\Delta^C_{S,t-1} = \Delta^C(S_{t-1}, C_{t-1})$ represents the delta of the call. The true delta-hedged call return, defined similarly except using true prices, is denoted $\tilde{R}_t$.

Following the same derivation as in Blume and Stambaugh (1983) for stock returns,

$$\mathrm{E}\left[\frac{C_t - C_{t-1}}{C_{t-1}}\right] = \mathrm{E}\left[\frac{\tilde{C}_t - \tilde{C}_{t-1}}{\tilde{C}_{t-1}}\right] + \mathrm{E}\left[\epsilon^2_{C,t-1}\right]. \tag{9}$$

Hence, the expected value of the first term of the delta-hedged return in Equation 8 is a biased estimate of the actual expected return of the call option.

To characterize the second term on the right-hand side of equation 8, recall that $\beta^C_{S,t-1} = \beta^C(S_{t-1}, C_{t-1}) = \Delta^C(S_{t-1}, C_{t-1})\frac{S_{t-1}}{C_{t-1}}$. Define the hedge return as

$$H(S_{t-1}, C_{t-1}, S_t) = \beta^C(S_{t-1}, C_{t-1}) \frac{S_t - S_{t-1}}{S_{t-1}}. \tag{10}$$

The second order Taylor expansion in the stock and option errors is therefore

$$
\begin{aligned}
H(S_{t-1}, C_{t-1}, S_t) = H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t) + \frac{\partial H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)}{\partial S_t} \tilde{S}_t \epsilon_{S,t} \\
+ \frac{\partial H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)}{\partial S_{t-1}} \tilde{S}_{t-1} \epsilon_{S,t-1} \\
+ \frac{\partial H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)}{\partial C_{t-1}} \tilde{C}_{t-1} \epsilon_{C,t-1} \\
+ \frac{1}{2} \frac{\partial^2 H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)}{\partial S_{t-1}^2} (\tilde{S}_{t-1} \epsilon_{S,t-1})^2 \\
+ \frac{1}{2} \frac{\partial^2 H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)}{\partial C_{t-1}^2} (\tilde{C}_{t-1} \epsilon_{C,t-1})^2 \\
+ \frac{\partial^2 H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)}{\partial S_{t-1} \partial C_{t-1}} (\tilde{S}_{t-1} \epsilon_{S,t-1})(\tilde{C}_{t-1} \epsilon_{C,t-1}) \\
+ \frac{\partial^2 H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)}{\partial S_{t-1} \partial S_t} (\tilde{S}_{t-1} \epsilon_{S,t-1})(\tilde{S}_t \epsilon_{S,t}) \\
+ o(\epsilon_{S,t-1}^2) + o(\epsilon_{C,t-1}^2)
\end{aligned}
\tag{11}
$$

To take expectations, note that the errors $\epsilon_{S,t-1}$, $\epsilon_{C,t-1}$, and $\epsilon_{S,t}$ are mean zero. Together with Assumption 1 (errors are independent), we obtain

$$
\begin{aligned}
\mathrm{E}\left[H(S_{t-1}, C_{t-1}, S_t)\right] = \mathrm{E}\left[H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)\right] \\
+ \mathrm{E}\left[\frac{1}{2} \frac{\partial^2 H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)}{\partial S_{t-1}^2} (\tilde{S}_{t-1} \epsilon_{S,t-1})^2\right] \\
+ \mathrm{E}\left[\frac{1}{2} \frac{\partial^2 H(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{S}_t)}{\partial C_{t-1}^2} (\tilde{C}_{t-1} \epsilon_{C,t-1})^2\right] \\
+ o(\epsilon_{S,t-1}^2) + o(\epsilon_{C,t-1}^2) \,.
\end{aligned}
\tag{12}
$$

Substituting $H(S_{t-1}, C_{t-1}, S_t)$ for its definition (Equation 10), taking derivatives, and applying Assumption 1, we arrive at the following:

$$
\begin{aligned}
\mathrm{E}\left[H(S_{t-1}, C_{t-1}, S_t)\right] = \mathrm{E}\left[\beta^C(\tilde{S}_{t-1}, \tilde{C}_{t-1})(\frac{\tilde{S}_t - \tilde{S}_{t-1}}{\tilde{S}_{t-1}})\right] + \mathrm{E}\left[\beta^C(\tilde{S}_{t-1}, \tilde{C}_{t-1}) \frac{\tilde{S}_t}{\tilde{S}_{t-1}}\right] \mathrm{E}\left[\epsilon_{S,t-1}^2\right] \\
- \mathrm{E}\left[\frac{\partial \beta^C(\tilde{S}_{t-1}, \tilde{C}_{t-1})}{\partial S} \tilde{S}_t\right] \mathrm{E}\left[\epsilon_{S,t-1}^2\right] + \mathrm{E}\left[\frac{\tilde{S}_t - \tilde{S}_{t-1}}{2\tilde{S}_{t-1}} \frac{\partial^2 \beta^C(\tilde{S}_{t-1}, \tilde{C}_{t-1})}{\partial S^2} \tilde{S}_{t-1}^2\right] \mathrm{E}\left[\epsilon_{S,t-1}^2\right] \\
+ \mathrm{E}\left[\frac{\tilde{S}_t - \tilde{S}_{t-1}}{2\tilde{S}_{t-1}} \frac{\partial^2 \beta^C(\tilde{S}_{t-1}, \tilde{C}_{t-1})}{\partial C^2} \tilde{C}_{t-1}^2\right] \mathrm{E}\left[\epsilon_{C,t-1}^2\right] + o(\epsilon_{S,t-1}^2) + o(\epsilon_{C,t-1}^2) \,,
\end{aligned}
\tag{13}
$$

Under Assumption 2, $\frac{\tilde{S}_t-\tilde{S}_{t-1}}{\tilde{S}_{t-1}} \to 0$, $\frac{\tilde{S}_t}{\tilde{S}_{t-1}} \to 1$, and $\tilde{S}_t \to \tilde{S}_{t-1}$ as $\Delta t \to 0$. Hence, Equations 9 and 13 imply that the bias in the delta-hedged return ($\mathrm{E}[R_t] - \mathrm{E}\left[\tilde{R}_t\right]$) is approximately

$$\mathrm{E}\left[\epsilon^2_{C,t-1}\right] - \mathrm{E}\left[\beta^C(\tilde{S}_{t-1}, \tilde{C}_{t-1})\right]\mathrm{E}\left[\epsilon^2_{S,t-1}\right] + \mathrm{E}\left[\frac{\partial \beta^C(\tilde{S}_{t-1}, \tilde{C}_{t-1})}{\partial S}\tilde{S}_{t-1}\right]\mathrm{E}\left[\epsilon^2_{S,t-1}\right] . \qquad (14)$$

Similarly, define the observed delta-hedge put return as

$$\frac{P_t - P_{t-1}}{P_{t-1}} - \beta^P(S_{t-1}, P_{t-1})\frac{S_t - S_{t-1}}{S_{t-1}} , \qquad (15)$$

where $\beta^P(S_{t-1}, P_{t-1}) = \Delta^P(S_{t-1}, P_{t-1})\frac{S_{t-1}}{P_{t-1}}$. Following the same derivations above, the bias in the delta-hedged put is

$$\mathrm{E}\left[\epsilon^2_{P,t-1}\right] - \mathrm{E}\left[\beta^P(\tilde{S}_{t-1}, \tilde{P}_{t-1})\right]\mathrm{E}\left[\epsilon^2_{S,t-1}\right] + \mathrm{E}\left[\frac{\partial \beta^P(\tilde{S}_{t-1}, \tilde{P}_{t-1})}{\partial S}\tilde{S}_{t-1}\right]\mathrm{E}\left[\epsilon^2_{S,t-1}\right] \qquad (16)$$

Appendix D Bias Decomposition and Subsample results

Table A1 shows that the MR bias is large for OTM options on stocks. Table A1 aims to decompose the biases in the sorting of call options by moneyness. The results that are based on sorting and filtering on $t-1$ variables are identical to the ones in Table 2 of the paper, and they reflect all the biases in the estimation of call expected returns with equal-weighted means. Assuming that our bias adjustment procedure properly addresses the CEIV and SS biases, the results based on $t-2$ sorting and filtering eliminate the CEIV and SS biases. The differences between the $t-2$ equal-weighted and gross-return weighted averages are estimates of the MR bias. For instance, in the case of the deep OTM options, this difference is about 79.11 (117.88-38.77) bps per day, which is economically very significant.

Table A1 Panel B indicates that the MR bias is also quite large for OTM options on the index, even though the total bias in index options is small. Panel B shows the results of the empirical decomposition of the biases for index options. The results for the deep-OTM portfolio show an MR bias of about 66 bps per day (14.06+52.11), which is very large. The total bias is much smaller, at about -7 bps (-59.29+52.11), because the CEIV+SS bias for these options is negative and highly economically significant, at -73 bps (-59.29-14.06). Moreover, in Portfolio 4 the CEIV+SS bias is also economically large, at 22 bps (61.49-39.42). Naturally, the fact that these bias compensate for each other may not happen in other instances. Therefore, the large economic magnitude of the these biases indicates that, even for OTM index options, bias adjustments matter in general.

Tables A2 to A5 present the results of subsample analysis of our main empirical tables. One subsample is from January 1996 to December 2005, and the other is from January 2006 to June 2019.

Appendix E Simulated Heston Model

The process under the risk neutral measure for stock i is given by

$$d\tilde{S}_{i,t} = r\tilde{S}_{i,t}dt + \sqrt{\tilde{V}_{i,t}}\tilde{S}_{i,t}dB^*_{1,i,t}$$

$$d\tilde{V}_{i,t} = \kappa^*(\theta^* - \tilde{V}_{i,t})dt + \sigma\sqrt{\tilde{V}_{i,t}}\left(\rho dB^*_{1,i,t} + \sqrt{1-\rho^2}dB^*_{2,i,t}\right),$$

and the change of measure is determined by

$$dB^*_{1,i,t} = dB_{1,i,t} + \lambda_1\sqrt{\tilde{V}_{i,t}}dt$$

$$dB^*_{2,i,t} = dB_{2,i,t} + \lambda_2\sqrt{\tilde{V}_{i,t}}dt.$$

The process under the physical measure is therefore

$$d\tilde{S}_{i,t} = (r + \lambda_1\tilde{V}_{i,t})\tilde{S}_{i,t}dt + \sqrt{\tilde{V}_{i,t}}\tilde{S}_{i,t}dB_{1,i,t}$$

$$d\tilde{V}_{i,t} = \kappa(\theta - \tilde{V}_{i,t})dt + \sigma\sqrt{\tilde{V}_{i,t}}\left(\rho dB_{1,i,t} + \sqrt{1-\rho^2}dB_{2,i,t}\right),$$

where

$$\kappa = \kappa^* - \sigma\rho\lambda_1 - \sigma\sqrt{1-\rho^2}\lambda_2$$

$$\theta = \theta^*\kappa^*/\kappa$$

To capture correlations between stocks, we assume that

$$\text{Corr}\left(dB_{1,i,t}, dB_{1,j,t}\right) = p_1$$

$$\text{Corr}\left(dB_{2,i,t}, dB_{2,j,t}\right) = p_2$$

for all $i \neq j$.

Stocks are identical in terms of all these parameters, which are chosen to approximately match the empirical properties of the option portfolios we analyze. To match the main sample of individual equity options, we choose $\kappa^* = 3$, $\theta^* = 0.47^2$, $\sigma = 0.5$, $\rho = -0.4$, $\theta_1 = 0.554$, $\theta_2 = -3.226$, and $p_1 = p_2 = 0.5$, which imply $\theta = .38^2$ and an average equity premium $(\theta\lambda_1)$ of 0.08. For simulations that match the index option sample, $\kappa = 3$, $\theta^* = 0.225^2$, $\sigma = 0.3$, $\rho = -0.6$, $\theta_1 = 3.5556$, and $\theta_2 = -12.9583$, implying $\theta = .15^2$ and an average equity premium of 0.08. (Because there is only one underlying asset, p_1 and p_2 are undefined.) All parameters are annualized, and the risk-free rate r is zero.

Stocks differ only with respect to the starting values of the stock price and variance process and the amount of measurement error in the observed stock and option prices. Starting values of $\tilde{V}$ are chosen by simulating the joint steady-state distribution of all variance processes. We assume that each firm's initial stock price and bid-ask spread are jointly lognormal. Log stock prices have a mean of 3.75 and standard deviation 0.67 (implying a median stock price of \$42.42), which approximately match the stock prices in our dataset. Log stock relative bid-ask spreads have a mean of -7 (giving a median relative spread of about 9 bps.) and a standard deviation of 0.57. The correlation between log prices and log spreads is -0.47, indicating that firms with lower share prices tend to have larger relative spreads.

Appendix F A model of bid-ask spreads and simulations for measurement errors in prices

The simulation of price errors requires a model of relative bid-ask spreads. Our model is calibrated to match the spreads of the stocks and options traded on S&P 500 member firms and is based on the main regression model in Table V of De Fontnouvelle, Fishe, and Harris (2003), who examine the effects of multiple listing on stock-option bid-ask spreads. While multiple listings are outside the scope of our simulation model, their findings nevertheless guide our model specification. In particular, they find that option bid-ask spreads are significantly related to option price, delta, and gamma, as well as the effective spread and volatility of the underlying stock. Furthermore, the relation between spreads and option prices is highly nonlinear, which they capture with a set of dummy variables. After proxying for stock volatility with the option's observed implied volatility, the specification we estimate is

$$\begin{aligned}
\ln\left((\text{Ask}_{i,j,t} - \text{Bid}_{i,j,t})\,/\text{Midpoint}_{i,j,t}\right) &= \beta_0 \\
&+ \beta_1 \times \text{Option Price}_{i,j,t} \\
&+ \beta_2 \times 1(\text{Option Price}_{i,j,t} < \$2) \\
&+ \beta_3 \times 1(\$5 \leq \text{Option Price}_{i,j,t} < \$10) \\
&+ \beta_4 \times 1(\$10 \leq \text{Option Price}_{i,j,t} < \$20) \\
&+ \beta_5 \times 1(\$20 \leq \text{Option Price}_{i,j,t}) \\
&+ \beta_6 \times \text{Stock bid-ask spread}_{i,t} \\
&+ \beta_7 \times \text{Option delta}_{i,j,t} \\
&+ \beta_8 \times \text{Option gamma}_{i,j,t} \\
&+ \beta_9 \times \text{Implied volatility}_{i,j,t} \\
&+ \eta_i + \eta_t + \eta_{i,j,t}
\end{aligned} \tag{17}$$

where i, j, and t denote the stock, option, and date, respectively, and where 1() is the indicator function. We assume that the coefficients (βs) are different for puts and calls. The error components η_i and η_t are included to capture the fact that option bid-ask spreads exhibit commonality across options on the same firm and that average spreads vary significantly through time.

We estimate Equation 17 via Fama-MacBeth regression, using all options written on S&P 500 member stocks that are part of the main (bias-adjusted) sample used in the main text. Given the full-sample estimates of the regression coefficients in Equation 17, we compute the regression residuals as

$$\hat{e}_{i,j,t} = \ln\left((\text{Ask}_{i,j,t} - \text{Bid}_{i,j,t})\,/\text{Midpoint}_{i,j,t}\right) - \text{fitted value}_{i,j,t}, \tag{18}$$

which are estimates of the sum of the firm, time, and idiosyncratic error terms, $\eta_i + \eta_t + \eta_{i,j,t}$.

We assume that the firm and time components, η_i and η_t, are independent normal draws with zero means and constant variances. Furthermore, we assume that the firm and time components are identical for calls and puts. We therefore estimate the variances of these terms by computing the sample variances of $\hat{\eta}_i$ and $\hat{\eta}_t$, where $\hat{\eta}_i$ is the average of all residuals $\hat{e}_{i,j,t}$ for all options on firm i, and $\hat{\eta}_t$ is the average of all residuals for all options at time t.

Variances of the idiosyncratic errors $\eta_{i,j,t}$ are assumed to depend on the same factors that determine expected spreads. Given estimates of the firm and time component, we can compute idiosyncratic residuals as

$$\hat{\eta}_{i,j,t} = \hat{e}_{i,j,t} - \hat{\eta}_t - \hat{\eta}_i. \tag{19}$$

These residuals are used to compute the dependent variable in the following variance equation, which is also estimated via Fama-MacBeth:

$$
\begin{aligned}
\eta^2_{i,j,t} &= \beta'_0 \\
&+ \beta'_1 \times \text{Option Price}_{i,j,t} \\
&+ \beta'_2 \times 1(\text{Option Price}_{i,j,t} < \$2) \\
&+ \beta'_3 \times 1(\$5 \leq \text{Option Price}_{i,j,t} < \$10) \\
&+ \beta'_4 \times 1(\$10 \leq \text{Option Price}_{i,j,t} < \$20) \\
&+ \beta'_5 \times 1(\$20 \leq \text{Option Price}_{i,j,t}) \\
&+ \beta'_6 \times \text{Stock bid-ask spread}_{i,t} \\
&+ \beta'_7 \times \text{Option delta}_{i,j,t} \\
&+ \beta'_8 \times \text{Option gamma}_{i,j,t} \\
&+ \beta'_9 \times \text{Implied volatility}_{i,j,t} \\
&+ e'_{i,j,t}
\end{aligned} \tag{20}
$$

We use a similar model for the index option simulations. The only differences are that we omit the firm component η_i and drop two regressors from each regression. The first, the stock bid-ask spread, is unidentified, as it is the same for all options in each cross section. The second, implied volatility, is excluded because it produced counterfactual results in our simulation exercise.[2]

The results of estimating these regressions are presented in Tables A6 and A7. In general, regression coefficients are highly significant for both the mean and variance equations.

These results are used to simulate stock and option-bid ask spreads using the following steps:

1. Draw one log stock price and one log relative stock spread for each simulated firm from a bivariate normal distribution.

2. Draw each idiosyncratic error $\eta_{i,j,t}$ from a normal distribution with a zero mean and a standard deviation implied by the evaluation of the variance equation above.

3. Draw each firm error component, η_i, and each time error component, η_t, from independent normal distributions with zero means and constant standard deviations.

[2]In the data, deep OTM puts have high implied volatilities and high bid-ask spreads, and a regression that included implied volatility explained the high spreads as a consequence of high implied volatilities. The Heston (1993) model is not able to produce such high implied volatilities for deep OTM puts, and as a result a model of bid-ask spreads that included implied volatility as a regressor did not match deep OTM put spreads when the model was simulated.

4. Compute the log relative option spread as the sum of the conditional mean (Equation 17) and the error components $(\eta_i + \eta_t + \eta_{i,j,t})$.

Once we have the relative bid-ask spread for each option and stock in our simulated sample, we simulate the measurement errors in stock and option prices. Specifically, we assume that, at each time t in the simulated sample, we observe $S = \tilde{S}(1+\epsilon^S)$, $C = \tilde{C}(1+\epsilon^C)$, $P = \tilde{P}(1+\epsilon^P)$ where $\tilde{S}$, $\tilde{C}$, and $\tilde{P}$ are the true stock, call and put prices, while S, C, and P are the observed prices. All measurement errors (ϵ^S, ϵ^C, and ϵ^P) are drawn from symmetric triangular distributions, which are bounded distributions with probability density functions that are piecewise linear, increasing below the median and decreasing above it, reaching zero at either bound. This choice of density reflects a view that price errors are likely bounded by the size of the bid-ask spread. By choosing lower and upper bounds equal to $-1/2$ or $+1/2$ times the relative bid-ask spread, we ensure that the difference between observed prices and true prices is never larger than the spread, with differences closer to zero more likely than those further away.

Let ϵ^k with $k = S$, P or C be the triangular distributed relative pricing error for the underlying stock, call or put. By construction, ϵ^k is bounded between $-\text{Spread}^k/2$ and $\text{Spread}^k/2$, where Spread^k is the relative spread generated in the simulation using the steps described above. To generate ϵ^k, we generate a random number u^k from a standard uniform distribution and use the formula

$$\epsilon^k = \begin{cases} \text{Spread}^k \times \left(\sqrt{2 \times u^k} - 1\right)/2 & \text{if } u^k \leq 0.5 \\ \text{Spread}^k \times \left(1 - \sqrt{2 \times (1 - u^k)}\right)/2 & \text{otherwise.} \end{cases} \tag{21}$$

Appendix G Bias adjustment in straddles and weighted option returns

G.1 Straddles

A straddle is a combination of a call and a put on the same underlying, with the same maturity and strike price, constructed to have zero total delta. This is accomplished with the portfolio weights

$$w^C_{t-1} = \frac{-C_{t-1}\Delta^P_{S,t-1}}{P_{t-1}\Delta^C_{S,t-1} - C_{t-1}\Delta^P_{S,t-1}}$$

and

$$w_{t-1}^{P} = \frac{P_{t-1}\Delta_{S,t-1}^{C}}{P_{t-1}\Delta_{S,t-1}^{C} - C_t\Delta_{S,t-1}^{P}},$$

so that the straddle return is

$$R_t = w_{t-1}^{C}\frac{C_t - C_{t-1}}{C_{t-1}} + w_{t-1}^{P}\frac{P_t - P_{t-1}}{P_{t-1}}.$$

G.1.1 MR Biases in straddle mean returns

We derive the MR bias based on Assumptions 1 and 2 from Section C. Define

$$H\left(S_{t-1}, P_{t-1}, C_{t-1}, C_t\right) = w_{t-1}^{C}\left(S_{t-1}, P_{t-1}, C_{t-1}\right)\frac{C_t - C_{t-1}}{C_{t-1}} \tag{22}$$

Using the same procedure to derive Equation 12, we find:

$$\begin{aligned}
\mathrm{E}\left[H\left(S_{t-1}, P_{t-1}, C_{t-1}, C_t\right)\right] = {} & \mathrm{E}\left[H\left(\tilde{S}_{t-1}, \tilde{P}_{t-1}, \tilde{C}_{t-1}, \tilde{C}_t\right)\right] \\
& + \mathrm{E}\left[\frac{1}{2}\frac{\partial^2 H\left(\tilde{S}_{t-1}, \tilde{P}_{t-1}, \tilde{C}_{t-1}, \tilde{C}_t\right)}{\partial S_{t-1}^2}\left(\tilde{S}_{t-1}\epsilon_{S,t-1}\right)^2\right] \\
& + \mathrm{E}\left[\frac{1}{2}\frac{\partial^2 H\left(\tilde{S}_{t-1}, \tilde{P}_{t-1}, \tilde{C}_{t-1}, \tilde{C}_t\right)}{\partial C_{t-1}^2}\left(\tilde{C}_{t-1}\epsilon_{C,t-1}\right)^2\right] \\
& + \mathrm{E}\left[\frac{1}{2}\frac{\partial^2 H\left(\tilde{S}_{t-1}, \tilde{P}_{t-1}, \tilde{C}_{t-1}, \tilde{C}_t\right)}{\partial P_{t-1}^2}\left(\tilde{P}_{t-1}\epsilon_{P,t-1}\right)^2\right] \\
& + o\left(\epsilon_{S,t-1}^2\right) + o\left(\epsilon_{C,t-1}^2\right) + o\left(\epsilon_{P,t-1}^2\right).
\end{aligned} \tag{23}$$

Substituting $H\left(S_{t-1}, P_{t-1}, C_{t-1}, C_t\right)$ for its definition (Equation 22), taking derivatives, and applying Assumption 1, we arrive at the following:

$$
\begin{aligned}
\mathrm{E}\left[H\left(S_{t-1},P_{t-1},C_{t-1},C_t\right)\right] = {} & \mathrm{E}\left[w_{t-1}^{C}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\frac{\tilde{C}_t-\tilde{C}_{t-1}}{\tilde{C}_{t-1}}\right] \\
& +\mathrm{E}\left[w_{t-1}^{C}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\frac{\tilde{C}_t}{\tilde{C}_{t-1}}\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right] \\
& -\mathrm{E}\left[\frac{\partial w_{t-1}^{C}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial C}\tilde{C}_t\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right] \\
& +\mathrm{E}\left[\frac{\tilde{C}_t-\tilde{C}_{t-1}}{2\tilde{C}_{t-1}}\frac{\partial^2 w_{t-1}^{C}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial C^2}\tilde{C}_{t-1}^2\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right] \\
& +\mathrm{E}\left[\frac{\tilde{C}_t-\tilde{C}_{t-1}}{2\tilde{C}_{t-1}}\frac{\partial^2 w_{t-1}^{C}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial S^2}\tilde{S}_{t-1}^2\right]\mathrm{E}\left[\epsilon_{S,t-1}^2\right] \\
& +\mathrm{E}\left[\frac{\tilde{C}_t-\tilde{C}_{t-1}}{2\tilde{C}_{t-1}}\frac{\partial^2 w_{t-1}^{C}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial P^2}\tilde{P}_{t-1}^2\right]\mathrm{E}\left[\epsilon_{P,t-1}^2\right] \\
& +o\left(\epsilon_{S,t-1}^2\right)+o\left(\epsilon_{C,t-1}^2\right)+o\left(\epsilon_{P,t-1}^2\right)\ ,
\end{aligned} \tag{24}
$$

Under Assumption 2, $\frac{\tilde{C}_t-\tilde{C}_{t-1}}{\tilde{C}_{t-1}} \to 0$, $\tilde{C}_t \to \tilde{C}_{t-1}$ and $\frac{\tilde{C}_t}{\tilde{C}_{t-1}} \to 1$ as $\Delta t \to 0$. Then $\mathrm{E}\left[H\left(S_{t-1},P_{t-1},C_{t-1},C_t\right)\right]-\mathrm{E}\left[H\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1},\tilde{C}_t\right)\right]$ converges (as $\Delta t \to 0$) to

$$
\begin{aligned}
& \mathrm{E}\left[w_{t-1}^{C}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right]-\mathrm{E}\left[\tilde{C}_{t-1}\frac{\partial w_{t-1}^{C}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial C}\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right] \\
& \qquad +o\left(\mathrm{E}\left[\epsilon_{S,t-1}^2\right]\right)+o\left(\mathrm{E}\left[\epsilon_{C,t-1}^2\right]\right)+o\left(\mathrm{E}\left[\epsilon_{P,t-1}^2\right]\right)\ .
\end{aligned} \tag{25}
$$

Similarly, we can show that the bias in put side is

$$
\begin{aligned}
& \mathrm{E}\left[w_{t-1}^{P}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon_{P,t-1}^2\right]-\mathrm{E}\left[\tilde{P}_{t-1}\frac{\partial w_{t-1}^{P}\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial P}\right]\mathrm{E}\left[\epsilon_{P,t-1}^2\right] \\
& \qquad +o\left(\mathrm{E}\left[\epsilon_{S,t-1}^2\right]\right)+o\left(\mathrm{E}\left[\epsilon_{C,t-1}^2\right]\right)+o\left(\mathrm{E}\left[\epsilon_{P,t-1}^2\right]\right)\ .
\end{aligned} \tag{26}
$$

Combining the call and put parts together, we obtain

$$\begin{aligned}
\mathrm{E}\left[R_t\right]-\mathrm{E}\left[\tilde{R}_t\right] &= \mathrm{E}\left[w_{t-1}^C\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right] \\
&\quad -\mathrm{E}\left[\tilde{C}_{t-1}\frac{\partial w_{t-1}^C\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial C}\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right] \\
&\quad +\mathrm{E}\left[w_{t-1}^P\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon_{P,t-1}^2\right] \\
&\quad -\mathrm{E}\left[\tilde{P}_{t-1}\frac{\partial w_{t-1}^P\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial P}\right]\mathrm{E}\left[\epsilon_{P,t-1}^2\right] \\
&\quad +o\left(\mathrm{E}\left[\epsilon_{S,t-1}^2\right]\right)+o\left(\mathrm{E}\left[\epsilon_{C,t-1}^2\right]\right)+o\left(\mathrm{E}\left[\epsilon_{P,t-1}^2\right]\right)\;.
\end{aligned} \tag{27}$$

This implies that the MR bias in straddle mean return is approximately

$$\begin{aligned}
&\mathrm{E}\left[w_{t-1}^C\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right]-\mathrm{E}\left[\tilde{C}_{t-1}\frac{\partial w_{t-1}^C\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial C}\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right]+ \\
&\mathrm{E}\left[w_{t-1}^P\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon_{P,t-1}^2\right]-\mathrm{E}\left[\tilde{P}_{t-1}\frac{\partial w_{t-1}^P\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial P}\right]\mathrm{E}\left[\epsilon_{P,t-1}^2\right]\;.
\end{aligned} \tag{28}$$

In this expression, $\mathrm{E}\left[w_{t-1}^C\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right]$ and $\mathrm{E}\left[w_{t-1}^P\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon_{P,t-1}^2\right]$ represent the DMR bias, or the bias that would result from errors in option returns were the two weights perfectly observed. In addition, an IMR biase arises as the result of errors in prices simultaneously affecting returns and weights. This bias is represented by the terms $\mathrm{E}\left[\tilde{C}_{t-1}\frac{\partial w_{t-1}^C\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial C}\right]\mathrm{E}\left[\epsilon_{C,t-1}^2\right]$ and $\mathrm{E}\left[\tilde{P}_{t-1}\frac{\partial w_{t-1}^P\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial P}\right]\mathrm{E}\left[\epsilon_{P,t-1}^2\right]$.

The source of the IMR bias is different, however, from that of delta-hedged options. With the latter, IMR bias results from errors in the stock price simultaneously affecting stock returns and the weight on the stock in the delta-hedged call strategy. Since stock errors tend to be small, the IMR bias for delta-hedged options can be safely ignored, as demonstrated by our simulation results. For straddles, however, the spurious covariance between weights and returns, which drives the IMR bias, arises as the result of errors in option prices, which are likely much larger than errors in stock prices. Thus, the IMR bias is a greater concern for straddles.

This far, we have followed Assumption 1 by assuming that the contemporaneous price errors for the put and call are uncorrelated. If we instead allow this correlation to be nonzero, the MR bias will change. Specifically, equation 23 becomes

$$
\begin{aligned}
\mathrm{E}\left[H\left(S_{t-1},P_{t-1},C_{t-1},C_t\right)\right] = {} & \mathrm{E}\left[H\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1},\tilde{C}_t\right)\right] \\
& + \mathrm{E}\left[\frac{1}{2}\frac{\partial^2 H\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1},\tilde{C}_t\right)}{\partial S_{t-1}^2}\left(\tilde{S}_{t-1}\epsilon_{S,t-1}\right)^2\right] \\
& + \mathrm{E}\left[\frac{1}{2}\frac{\partial^2 H\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1},\tilde{C}_t\right)}{\partial C_{t-1}^2}\left(\tilde{C}_{t-1}\epsilon_{C,t-1}\right)^2\right] \\
& + \mathrm{E}\left[\frac{\partial^2 H\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1},\tilde{C}_t\right)}{\partial C_{t-1}\partial P_{t-1}}\tilde{C}_{t-1}\tilde{P}_{t-1}\epsilon_{C,t-1}\epsilon_{P,t-1}\right] \\
& + \mathrm{E}\left[\frac{1}{2}\frac{\partial^2 H\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1},\tilde{C}_t\right)}{\partial P_{t-1}^2}\left(\tilde{P}_{t-1}\epsilon_{P,t-1}\right)^2\right] \\
& + o\left(\epsilon_{S,t-1}^2\right) + o\left(\epsilon_{C,t-1}^2\right) + o\left(\epsilon_{P,t-1}^2\right) .
\end{aligned}
\tag{29}
$$

This equation has one additional term,

$$
\mathrm{E}\left[\frac{\partial^2 H\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1},\tilde{C}_t\right)}{\partial C_{t-1}\partial P_{t-1}}\tilde{C}_{t-1}\tilde{P}_{t-1}\epsilon_{C,t-1}\epsilon_{P,t-1}\right] .
$$

Substituting $H\left(S_{t-1},P_{t-1},C_{t-1},C_t\right)$ for its definition (Equation 22), this term can be written as:

$$
\begin{aligned}
& -\mathrm{E}\left[\frac{\partial w_{t-1}^C\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial P}\frac{\tilde{C}_t\tilde{P}_{t-1}}{\tilde{C}_{t-1}}\right]\mathrm{E}\left[\epsilon_{C,t-1}\epsilon_{P,t-1}\right] \\
& + \mathrm{E}\left[\frac{\tilde{C}_t-\tilde{C}_{t-1}}{\tilde{C}_{t-1}}\frac{\partial^2 w_{t-1}^C\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial C\partial P}\tilde{C}_{t-1}\tilde{P}_{t-1}\right]\mathrm{E}\left[\epsilon_{C,t-1}\epsilon_{P,t-1}\right] .
\end{aligned}
$$

Apply Assumption 2, $\tilde{C}_t \to \tilde{C}_{t-1}$, and the second term in above equation converges to zero. Hence, the additional bias in the call side is

$$
-\mathrm{E}\left[\frac{\partial w_{t-1}^C\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial P}\tilde{P}_{t-1}\right]\mathrm{E}\left[\epsilon_{C,t-1}\epsilon_{P,t-1}\right] .
$$

Similarly, one can show that the additional bias in the put side is

$$
-\mathrm{E}\left[\frac{\partial w_{t-1}^P\left(\tilde{S}_{t-1},\tilde{P}_{t-1},\tilde{C}_{t-1}\right)}{\partial C}\tilde{C}_{t-1}\right]\mathrm{E}\left[\epsilon_{C,t-1}\epsilon_{P,t-1}\right] .
$$

Note that these additional terms will be zero if the price errors of call and put are uncorrelated.

G.1.2 Bias-adjustment in straddle mean returns

Our approach to bias adjustment for straddles is to remove some of the spurious correlation between weights and option returns. Specifically, we construct an alternative set of portfolio weights by replacing several terms with their own lags:

$$\hat{w}^C_{t-1} = \frac{-C_{t-1}\Delta^P_{S,t-1}}{P_{t-1}\Delta^C_{S,t-2} - C_{t-2}\Delta^P_{S,t-1}}$$

and

$$\hat{w}^P_{t-1} = \frac{P_{t-1}\Delta^C_{S,t-1}}{P_{t-2}\Delta^C_{S,t-1} - C_{t-1}\Delta^P_{S,t-2}}$$

Using these weights to recompute an alternative straddle return results in

$$\hat{R}_t = \frac{-\Delta^P_{S,t-1}}{P_{t-1}\Delta^C_{S,t-2} - C_{t-2}\Delta^P_{S,t-1}}\left(C_t - C_{t-1}\right) + \frac{\Delta^C_{S,t-1}}{P_{t-2}\Delta^C_{S,t-1} - C_{t-1}\Delta^P_{S,t-2}}\left(P_t - P_{t-1}\right).^{3} \quad (30)$$

To understand why this formulation avoids MR/RC biases, consider the effect of an error in C_{t-1}, first on the first term and then on the second. Because C_{t-1} is not used in the construction of $\frac{-\Delta^P_{S,t-1}}{P_{t-1}\Delta^C_{S,t-2}-C_{t-2}\Delta^P_{S,t-1}}$, the relation between C_{t-1} and the first term is linear. While errors will induce noise, linearity prevents them from causing bias. Noise in C_{t-1} will have some effect on the second term through the fraction $\frac{\Delta^C_{S,t-1}}{P_{t-2}\Delta^C_{S,t-1}-C_{t-1}\Delta^P_{S,t-2}}$, but this noise will be uncorrelated with $P_t - P_{t-1}$, implying no bias due to measurement error there as well. A similar logic applies to errors in P_{t-1}.

Because each straddle constructed in this way avoids MR/RC biases, there is no need to use value weighting or gross return weighting. In fact, either of those weighting schemes would disrupt the unbiasedness of this formulation, though it would be straightforward to derive an alternative straddle construction that is appropriate in those cases. It remains necessary to use the same procedure we proposed earlier to avoid CEIV and SS biases.

[3] When the errors in call and put are correlated, we can use alternative straddle return below to adjust for the MR bias:

$$\hat{R}_t = \frac{-\Delta^P_{S,t-2}}{P_{t-2}\Delta^C_{S,t-2} - C_{t-2}\Delta^P_{S,t-2}}\left(C_t - C_{t-1}\right) + \frac{\Delta^C_{S,t-2}}{P_{t-2}\Delta^C_{S,t-2} - C_{t-2}\Delta^P_{S,t-2}}\left(P_t - P_{t-1}\right).$$

G.2 β_S-adjusted returns

Constantinides, Jackwerth, and Savov (2013) examine option returns adjusted by their elasticity with respect to the underlying stock price, which we denote

$$\beta_{S,t-1}^{f} = \frac{S_{t-1}\Delta_{S,t-1}^{f}}{f_{t-1}},$$

where f is the price of a call or put and Δ is its delta. These β_S-adjusted returns are therefore given by

$$R_t = \frac{f_t - f_{t-1}}{f_{t-1}} \frac{1}{\beta_{S,t-1}^{f}} = \frac{1}{S_{t-1}\Delta_{S,t-1}^{f}} \left(f_t - f_{t-1}\right) . \tag{31}$$

We follow the same method as before to derive the MR bias. Starting with call options ($f = C$), Equation 31 becomes

$$R\left(S_{t-1}, C_{t-1}, C_t\right) = \frac{1}{\beta_{S,t-1}^{C}} \frac{\left(C_t - C_{t-1}\right)}{C_{t-1}} = w_{t-1}^{C}\left(S_{t-1}, C_{t-1}\right) \frac{\left(C_t - C_{t-1}\right)}{C_{t-1}}, \tag{32}$$

where we define $w_{t-1}^{C}\left(S_{t-1}, C_{t-1}\right) = 1/\beta_{S,t-1}^{C}$ to simplify the derivation.

Using the same procedure to derive Equation 12, the expected β_S-adjusted return can be written as

$$\begin{aligned} \mathrm{E}\left[R\left(S_{t-1}, C_{t-1}, C_t\right)\right] = \mathrm{E}\left[R\left(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{C}_t\right)\right] + \mathrm{E}\left[\frac{1}{2}\frac{\partial^2 R\left(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{C}_t\right)}{\partial S_{t-1}^2}\left(\tilde{S}_{t-1}\epsilon_{S,t-1}\right)^2\right] \\ +\mathrm{E}\left[\frac{1}{2}\frac{\partial^2 R\left(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{C}_t\right)}{\partial C_{t-1}^2}\left(\tilde{C}_{t-1}\epsilon_{C,t-1}\right)^2\right] + o\left(\epsilon_{S,t-1}^2\right) + o\left(\epsilon_{C,t-1}^2\right) . \end{aligned} \tag{33}$$

Substituting for $R\left(\tilde{S}_{t-1}, \tilde{C}_{t-1}, \tilde{C}_t\right)$ using Equation 33, taking derivatives, and applying Assumption 1 results in the following:

$$
\begin{aligned}
&\mathrm{E}\left[R\left(S_{t-1},C_{t-1},C_t\right)\right]-\mathrm{E}\left[R\left(\tilde{S}_{t-1},\tilde{C}_{t-1},\tilde{C}_t\right)\right]\\
&=\mathrm{E}\left[\frac{\left(\tilde{C}_t-\tilde{C}_{t-1}\right)}{2\tilde{C}_{t-1}}\frac{\partial^2 w^C_{t-1}\left(\tilde{S}_{t-1},\tilde{C}_{t-1}\right)}{\partial S^2}\tilde{S}^2_{t-1}\right]\mathrm{E}\left[\epsilon^2_{S,t-1}\right]\\
&+\mathrm{E}\left[\frac{\left(\tilde{C}_t-\tilde{C}_{t-1}\right)}{2\tilde{C}_{t-1}}\frac{\partial^2 w^C_{t-1}\left(\tilde{S}_{t-1},\tilde{C}_{t-1}\right)}{\partial C^2}\tilde{C}^2_{t-1}\right]\mathrm{E}\left[\epsilon^2_{C,t-1}\right]\\
&+\mathrm{E}\left[\frac{\tilde{C}_t}{\tilde{C}_{t-1}}w^C_{t-1}\left(\tilde{S}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon^2_{C,t-1}\right]\\
&-\mathrm{E}\left[\tilde{C}_t\frac{\partial w^C_{t-1}\left(\tilde{S}_{t-1},\tilde{C}_{t-1}\right)}{\partial C}\right]\mathrm{E}\left[\epsilon^2_{C,t-1}\right]\\
&+o\left(\epsilon^2_{S,t-1}\right)+o\left(\epsilon^2_{C,t-1}\right)+o\left(\epsilon^2_{C,t}\right)\ .
\end{aligned}
\tag{34}
$$

Under Assumption 2, $\tilde{C}_t \to \tilde{C}_{t-1}$, and $\frac{\tilde{C}_t-\tilde{C}_{t-1}}{\tilde{C}_{t-1}} \to 0$. Then $\mathrm{E}\left[R\left(S_{t-1},C_{t-1},C_t\right)\right]-\mathrm{E}\left[R\left(\tilde{S}_{t-1},\tilde{C}_{t-1},\tilde{C}_t\right)\right]$ converges (as $\Delta t \to 0$) to

$$
\begin{aligned}
&\mathrm{E}\left[w^C_{t-1}\left(\tilde{S}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon^2_{C,t-1}\right]-\mathrm{E}\left[\tilde{C}_{t-1}\frac{\partial w^C_{t-1}\left(\tilde{S}_{t-1},\tilde{C}_{t-1}\right)}{\partial C}\right]\mathrm{E}\left[\epsilon^2_{C,t-1}\right]+\\
&o\left(\epsilon^2_{S,t-1}\right)+o\left(\epsilon^2_{C,t-1}\right)+o\left(\epsilon^2_{C,t}\right)\ .
\end{aligned}
\tag{35}
$$

Hence, the MR bias of the β_S-adjusted call is approximately

$$
\mathrm{E}\left[w^C_{t-1}\left(\tilde{S}_{t-1},\tilde{C}_{t-1}\right)\right]\mathrm{E}\left[\epsilon^2_{C,t-1}\right]-\mathrm{E}\left[\tilde{C}_{t-1}\frac{\partial w^C_{t-1}\left(\tilde{S}_{t-1},\tilde{C}_{t-1}\right)}{\partial C}\right]\mathrm{E}\left[\epsilon^2_{C,t-1}\right]\ . \tag{36}
$$

Similarly, one can show that the MR bias of the β_S-adjusted put is

$$
\mathrm{E}\left[w^P_{t-1}\left(\tilde{S}_{t-1},\tilde{P}_{t-1}\right)\right]\mathrm{E}\left[\epsilon^2_{P,t-1}\right]-\mathrm{E}\left[\tilde{P}_{t-1}\frac{\partial w^P_{t-1}\left(\tilde{S}_{t-1},\tilde{P}_{t-1}\right)}{\partial P}\right]\mathrm{E}\left[\epsilon^2_{P,t-1}\right]\ . \tag{37}
$$

As with straddles, these expressions reveal both DMR and IMR biases. The first terms represent the DMR bias, which would arise even the option's $\beta^f_{S,t-1}$ was perfectly observed. The IMR bias, seen in the second terms, arises through the implicit dependence of $\Delta^f_{S,t-1}$ on option prices. For example, if an OTM call option were observed with a positive pricing error at time $t-1$, then the implied volatility of this option would be artificially raised.

For OTM options, a higher implied volatility will lower $\Delta^f_{S,t-1}$, inducing a spurious positive covariance between $1/\left(S_{t-1}\Delta^f_{S,t-1}\right)$ and $f_t - f_{t-1}$ that results in a positive IMR bias. In contrast, a positive pricing error for an ITM call would lower $\Delta^f_{S,t-1}$, causing the IMR bias to become negative.

Similarly to straddles, we eliminate the MR bias by replacing time $\Delta^f_{S,t-1}$ with $\Delta^f_{S,t-2}$ in the weight on the option. The alternative β_S-adjusted return is therefore

$$\hat{R}_t = \frac{1}{S_{t-1}\Delta^f_{S,t-2}} \left(f_t - f_{t-1}\right) . \tag{38}$$

This breaks the false covariance between $\Delta^f_{S,t-2}$ and $f_t - f_{t-1}$, eliminating MR bias. We again repeat the same methods for eliminating the CEIV and SS biases.

A delta-hedged version of either return can be obtained simply by subtracting the contemporaneous return on the underlying stock.

G.3 Leverage-adjusted returns

More recently, Fournier, Jacobs, and Orlowski (2021) examine deleveraged returns, which are defined as the return on the derivative multiplied by the ratio of option to stock prices, or

$$R_t = \frac{f_t - f_{t-1}}{f_{t-1}} \frac{f_{t-1}}{S_{t-1}} = \frac{f_t - f_{t-1}}{S_{t-1}}.$$

The sensitivity of this return with respect to underlying stock returns is therefore Δ_{t-1}.

The advantage of this approach is that the MR bias should be small due to the linearity in f_{t-1} and the small measurement errors in stock prices, although when the position is hedged a small IMR bias may arise as the result of errors in the stock price. A reasonable bias adjustment of these returns may therefore require no changes other than those taken to avoid the CEIV and SS biases.

G.4 Simulation results

We examine the performance of the unadjusted and adjusted methods using the same simulations analyzed in Section 4 of the paper. As in that section, we report estimates using "true" prices that are free of measurement errors, estimates that use noisy prices without adjustment, and estimates that follow the bias adjustment methods we have proposed. We also decompose the bias of the unadjusted estimates into DMR/DRC, IMR/IRC, CEIV, and SS components. We show results for quintile sorts and Fama-MacBeth regressions, both based on moneyness.

We present simulation results only for stock options. We find no relevant biases for index options using any of the three types of returns considered here, though we cannot rule out the possibility that biases in index options would be important given other sorting variables or sample selection procedures, or that biases in actual data would be stronger. In addition, we do not report results for β_S-adjusted or leverage-adjusted put returns, which are similar to the call returns we discuss below.

Panel A of Table A8 shows simulation results for straddles. Using true prices, we see no relation between moneyness and average returns, which is due to the fact that all straddles considered have no exposure to the underlying stock price and very similar exposure to volatility risk. When noisy prices are used, however, a noisy U shape emerges among the five quintile means, and the Fama-MacBeth slope coefficient becomes significantly negative on average. Bias adjustment performs extremely well, however, leaving only a small bias on the low moneyness quintile mean.

The table further shows that the CEIV and SS biases are generally more important than the DMR or IMR biases. The different biases cancel each other out to some extent, suggesting that an attempt to control one bias without addressing another (e.g., using doubly lagged variables for sample selection but not for sorting) may only worsen the problem.

Panel B shows result on β_S-adjusted and delta-hedged call returns. As opposed to straddles, we now see a clear negative relation between moneyness and true average returns. When prices are measured with error, however, the relationship weakens substantially, and the Fama-MacBeth slope coefficient drops in magnitude by almost one half. Bias adjustment restores the true relation almost exactly, on average.

The DMR, IMR, CEIV, and SS biases are all important for at least one of the portfolios of β_S-adjusted calls. As with straddles, IMR bias is significant due to errors in option prices having a simultaneous impact on the option return and the weight with which the option is held. In some cases, the different biases mostly cancel each other out. In other cases, they do not. In general, however, all four biases are individually large enough to affect inferences.

Panel C shows results for leverage-adjusted and delta-hedged calls. Similarly to straddles, the relation between true average returns and moneyness is weak. When option prices are measured with error, however, individual portfolio means become erratic but highly significant, and the Fama-MacBeth slope coefficient turns strongly positive.

While all four biases have some impact, the CEIV bias is the most important, and the SS bias is very small. While the DMR bias and IMR bias are both nonzero, they cancel each other out almost perfectly, consistent with the earlier claim that the total MR bias would be zero for this type of return. The cancellation is imperfect only because of errors in stock prices, which are relatively small.

Bias adjustment, which in this case only involves procedures to eliminate the CEIV and SS biases, results in approximately unbiased estimates.

Table A1: **Mean returns of unhedged call options on S&P 500 stocks and on the S&P 500 Index.** Panels A and B present the average return of stock and index call options sorted by moneyness. The moneyness of an option at time t is $\ln(e^{-r_t(T-t)}K/S_t)/(\sigma_t\sqrt{T-t})$, where S_t is the observed closing stock price on day t; K is the option strike price; r_t is the risk-free rate between t and the option expiration date T from IvyDB; and σ_t is the volatility implied by the closing option price from IvyDB. Moneyness groups are defined as follows. Low: Moneyness < -1.25. Group 2: $-1.25 \leq$ Moneyness < -0.5. Group 3: $-0.5 \leq$ Moneyness < 0.5. Group 4: $0.5 \leq$ Moneyness < 1.25. High: Moneyness ≥ 1.25. The return of call option i is $(C_{i,t} - C_{i,t-1})/C_{i,t-1}$. Two scenarios are considered. First, options are based on the *baseline* sample, which is constructed with selection criteria based on $t-1$ variables. Moreover, calls are sorted by their moneyness at the beginning of the holding period ($t-1$). Second, options are based on the a sample constructed with selection criteria based on $t-2$ variables, and calls are sorted by their moneyness one day before the beginning of the holding period ($t-2$). In each scenario, four methods are applied: (1) returns are equally weighted, (2) returns are weighted by the dollar value of open interest ($\$OI_{i,t-2}$), (3) returns are weighted by one-day-lagged call option gross returns ($C_{i,t-1}/C_{i,t-2}$), (4) returns are weighted by the gross return times dollar value of open interest ($(C_{i,t-1}/C_{i,t-2}) \times \$OI_{i,t-2}$). The results that are based on sorting and filtering on t-1 variables are identical to the ones in Table 2 of the paper and they reflect all the biases in the estimation of the mean call returns. Conditional on our bias methodology properly addressing the CEIV and SS biases, the results based on $t-2$ sorting and filtering eliminate the CEIV and SS biases. Mean returns are displayed in basis points. The p-values are for a Wolak (1989) test of the null hypothesis that call option mean returns increase monotonically with moneyness. T-statistics are shown in parentheses.

	Low	2	3	4	High	H-L	p-value
A: Stock							
	Sorting and filtering on $t-1$ variables						
Equal weighted	1.39	0.03	51.59	83.50	-14.70	-15.93	0.00
	(0.21)	(0.00)	(3.09)	(3.75)	(-0.55)	(-0.70)	
$\$OI$ weighted	8.50	10.33	25.87	-8.26	-161.40	-169.75	0.00
	(1.17)	(0.91)	(1.53)	(-0.36)	(-5.52)	(-6.76)	
	Sorting and filtering on $t-2$ variables						
Equal weighted	14.43	24.84	55.04	106.33	117.88	103.49	0.82
	(2.18)	(2.30)	(3.29)	(4.81)	(4.42)	(4.59)	
$\$OI$ weighted	17.11	18.96	26.40	19.79	-41.84	-58.86	0.00
	(2.33)	(1.67)	(1.56)	(0.86)	(-1.43)	(-2.34)	
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}}$	12.96	22.05	47.99	73.64	38.77	25.85	0.03
	(1.97)	(2.07)	(2.92)	(3.37)	(1.42)	(1.12)	
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}} \times \$OI$	15.99	19.56	32.99	24.84	-73.48	-89.39	0.00
	(2.22)	(1.75)	(1.98)	(1.09)	(-2.46)	(-3.46)	
B: Index							
	Low	2	3	4	High	H-L	p-value
	Sorting and filtering on $t-1$ variables						
Equal weighted	20.81	29.76	41.86	61.49	-59.29	-103.69	0.00
	(2.02)	(1.77)	(1.43)	(1.29)	(-1.05)	(-2.16)	
$\$OI$ weighted	21.60	27.67	39.01	58.61	-12.74	-59.60	0.00
	(2.07)	(1.60)	(1.37)	(1.27)	(-0.22)	(-1.21)	
	Sorting and filtering on $t-2$ variables						
Equal weighted	20.21	29.00	41.17	39.42	14.06	-29.24	0.11
	(1.97)	(1.72)	(1.40)	(0.83)	(0.25)	(-0.60)	
$\$OI$ weighted	19.11	26.25	38.87	34.65	0.44	-45.29	0.04
	(1.86)	(1.51)	(1.35)	(0.76)	(0.01)	(-0.93)	
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}}$	20.07	28.96	40.47	29.93	-52.11	-94.88	0.00
	(1.96)	(1.73)	(1.38)	(0.63)	(-0.95)	(-1.98)	
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}} \times \$OI$	19.09	26.29	38.17	27.70	-41.03	-86.53	0.00
	(1.86)	(1.52)	(1.34)	(0.61)	(-0.74)	(-1.80)	

Table A2: **Mean returns of unhedged call options on S&P 500 stocks and on the S&P 500 Index from 1996-2005.** Panels A and B present the average return of stock and index call options sorted by moneyness. The moneyness of an option at time t is $\ln(e^{-r_t(T-t)}K/S_t)/(\sigma_t\sqrt{T-t})$, where S_t is the observed closing stock price on day t; K is the option strike price; r_t is the risk-free rate between t and the option expiration date T from IvyDB; and σ_t is the volatility implied by the closing option price from IvyDB. Moneyness groups are defined as follows. Low: Moneyness < -1.25. Group 2: $-1.25 \leq$ Moneyness < -0.5. Group 3: $-0.5 \leq$ Moneyness < 0.5. Group 4: $0.5 \leq$ Moneyness < 1.25. High: Moneyness ≥ 1.25. The return of call option i is $(C_{i,t} - C_{i,t-1})/C_{i,t-1}$. The bias-unadjusted results are calculated with the *baseline* sample, which is constructed with selection criteria based on $t-1$ variables. Moreover, in the bias-unadjusted method, calls are sorted by their moneyness at the beginning of the holding period ($t-1$). In contrast, the bias-adjusted results are calculated with a sample constructed with selection criteria based on $t-2$ variables, calls are sorted by their moneyness one day before the beginning of the holding period ($t-2$), and returns are weighted by one-day-lagged call option gross returns ($C_{i,t-1}/C_{i,t-2}$). Section 1 describes the sample selection criteria. β_S and β_σ are the option sensitivities with respect to the underlying price and volatility. The median β_S, β_σ, relative spread, and moneyness are the mean values across time of the median $t-2$ values of these variables. Mean returns are displayed in basis points. Relative spreads are displayed in percentages. We also present results weighted by the dollar value of open interest ($\$OI_{i,t-2}$). The p-values are for a Wolak (1989) test of the null hypothesis that call option mean returns increase monotonically with moneyness. T-statistics are shown in parentheses.

A: Stocks							
	Low	2	3	4	High	H-L	
Median β_σ	0.15	0.80	3.24	8.31	13.17	13.03	
Median β_S	4.97	8.50	13.70	19.59	22.01	17.04	
Median relative spread	4.02	5.78	8.22	18.89	31.35	27.33	
Median moneyness	-1.58	-0.86	0.00	0.81	1.43	3.01	
	Average daily call option returns						Wolak p-value
Unadjusted							
Equal weighted	5.18	20.64	44.36	63.52	-83.83	-88.64	0.00
	(0.51)	(1.29)	(1.81)	(1.91)	(-2.04)	(-2.56)	
$\$OI$ weighted	7.12	10.93	18.40	-3.11	-156.11	-162.87	0.00
	(0.64)	(0.62)	(0.69)	(-0.09)	(-3.36)	(-4.01)	
Adjusted							
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}}$	10.37	21.80	41.17	67.03	9.59	-0.68	0.02
	(1.04)	(1.39)	(1.71)	(2.05)	(0.23)	(-0.02)	
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}} \times \$OI$	13.65	13.26	25.34	15.48	-106.88	-120.33	0.00
	(1.24)	(0.76)	(0.97)	(0.43)	(-2.29)	(-2.93)	
B: Index							
	Low	2	3	4	High	H-L	
Median β_σ	0.26	1.34	5.63	16.88	31.43	31.17	
Median β_S	7.63	12.98	22.92	38.77	52.52	44.89	
Median relative spread	1.36	2.68	6.02	12.91	28.46	27.10	
Median moneyness	-1.59	-0.86	-0.03	0.84	1.54	3.12	
	Average daily call option returns						Wolak p-value
Unadjusted							
Equal weighted	13.87	15.93	13.27	3.30	-110.26	-131.92	0.00
	(0.87)	(0.60)	(0.29)	(0.04)	(-1.19)	(-1.62)	
$\$OI$ weighted	13.16	18.54	16.33	3.59	-60.54	-87.22	0.07
	(0.79)	(0.67)	(0.37)	(0.05)	(-0.65)	(-1.07)	
Adjusted							
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}}$	8.98	9.47	14.17	-23.25	-67.31	-73.70	0.22
	(0.57)	(0.36)	(0.31)	(-0.32)	(-0.76)	(-0.93)	
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}} \times \$OI$	5.56	11.15	17.02	-21.68	-79.48	-87.10	0.13
	(0.34)	(0.40)	(0.38)	(-0.31)	(-0.90)	(-1.11)	

Table A3: **Mean returns of unhedged call options on S&P 500 stocks and on the S&P 500 Index from 2006-2019.** Panels A and B present the average return of stock and index call options sorted by moneyness. The moneyness of an option at time t is $\ln(e^{-r_t(T-t)}K/S_t)/(\sigma_t\sqrt{T-t})$, where S_t is the observed closing stock price on day t; K is the option strike price; r_t is the risk-free rate between t and the option expiration date T from IvyDB; and σ_t is the volatility implied by the closing option price from IvyDB. Moneyness groups are defined as follows. Low: Moneyness < -1.25. Group 2: $-1.25 \leq$ Moneyness < -0.5. Group 3: $-0.5 \leq$ Moneyness < 0.5. Group 4: $0.5 \leq$ Moneyness < 1.25. High: Moneyness ≥ 1.25. The return of call option i is $(C_{i,t} - C_{i,t-1})/C_{i,t-1}$. The bias-unadjusted results are calculated with the *baseline* sample, which is constructed with selection criteria based on $t-1$ variables. Moreover, in the bias-unadjusted method, calls are sorted by their moneyness at the beginning of the holding period ($t-1$). In contrast, the bias-adjusted results are calculated with a sample constructed with selection criteria based on $t-2$ variables, calls are sorted by their moneyness one day before the beginning of the holding period ($t-2$), and returns are weighted by one-day-lagged call option gross returns ($C_{i,t-1}/C_{i,t-2}$). Section 1 describes the sample selection criteria. β_S and β_σ are the option sensitivities with respect to the underlying price and volatility. The median β_S, β_σ, relative spread, and moneyness are the mean values across time of the median $t-2$ values of these variables. Mean returns are displayed in basis points. Relative spreads are displayed in percentages. We also present results weighted by the dollar value of open interest ($\$OI_{i,t-2}$). The p-values are for a Wolak (1989) test of the null hypothesis that call option mean returns increase monotonically with moneyness. T-statistics are shown in parentheses.

A: Stocks							
	Low	2	3	4	High	H-L	
Median β_σ	0.15	0.80	3.24	8.31	13.17	13.03	
Median β_S	4.97	8.50	13.70	19.59	22.01	17.04	
Median relative spread	4.02	5.78	8.22	18.89	31.35	27.33	
Median moneyness	-1.58	-0.86	0.00	0.81	1.43	3.01	
	Average daily call option returns						Wolak p-value
Unadjusted							
Equal weighted	-1.42	-15.24	56.95	98.31	36.49	37.90	0.00
	(-0.17)	(-1.04)	(2.51)	(3.28)	(1.03)	(1.27)	
$\$OI$ weighted	9.52	9.88	31.40	-12.07	-165.32	-174.85	0.00
	(0.99)	(0.67)	(1.43)	(-0.41)	(-4.39)	(-5.52)	
Adjusted							
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}}$	14.88	22.24	53.04	78.53	60.36	45.49	0.36
	(1.71)	(1.54)	(2.38)	(2.67)	(1.68)	(1.51)	
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}} \times \$OI$	17.73	24.23	38.65	31.78	-48.77	-66.50	0.00
	(1.85)	(1.65)	(1.79)	(1.08)	(-1.26)	(-2.01)	
B: Index							
	Low	2	3	4	High	H-L	
Median β_σ	0.36	1.58	7.09	23.45	45.06	44.70	
Median β_S	9.46	15.64	29.14	51.06	72.25	62.80	
Median relative spread	1.88	2.74	5.22	13.14	28.44	26.56	
Median moneyness	-1.51	-0.86	-0.03	0.86	1.59	3.10	
	Average daily call option returns						Wolak p-value
Unadjusted							
Equal weighted	25.90	40.02	63.07	104.57	-22.53	-83.58	0.00
	(1.91)	(1.84)	(1.65)	(1.68)	(-0.32)	(-1.42)	
$\$OI$ weighted	27.76	34.44	55.82	99.26	21.57	-40.11	0.00
	(2.07)	(1.55)	(1.51)	(1.63)	(0.30)	(-0.66)	
Adjusted							
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}}$	28.19	43.39	59.97	69.27	-41.17	-109.95	0.00
	(2.09)	(2.01)	(1.57)	(1.11)	(-0.59)	(-1.84)	
Weighted by $\frac{C_{i,t-1}}{C_{i,t-2}} \times \$OI$	29.00	37.50	53.85	64.23	-13.34	-86.13	0.00
	(2.17)	(1.69)	(1.45)	(1.06)	(-0.19)	(-1.41)	

Table A4: **FM regressions of returns of delta-hedged options on S&P 500 stocks and on the S&P 500 Index from 1996 to 2005.** This table displays the results of FM regressions: $R_{f,i,t} - \beta^f_{S,i,t-1} R_{S,t} = \lambda_0 + \lambda_\sigma \times \beta^f_{\sigma,i,t-1} + \eta_{i,t}$. The dependent variable is delta-hedged option excess return, where $R_{f,i,t}$ is the excess return of call or put i between $t-1$ and t; $\beta^f_{S,i,t-1} = S_{t-1}/f_{t-1} \times \Delta^f_{S,i,t-1}$ is the β of the call or put with respect to the underlying asset; and $R_{S,t}$ is the excess return of the underlying stock or index between $t-1$ and t. In the bias-unadjusted method, the independent variable is the β of the option with respect to the volatility of the underlying at time $t-1$ ($\beta^f_{\sigma,i,t-1}$). Moreover, the sample used in the bias-unadjusted method is constructed with selection criteria based on $t-1$ variables, including the requirement that the option's delta ($\Delta^f_{S,i,t-1}$) is non-missing. In contrast, the sample used in the bias-adjusted method is constructed with selection criteria based on $t-2$ variables, with missing deltas replaced by their lagged values ($\Delta^f_{S,i,t-2}$) when calculating the delta-hedged return. In addition, in the bias-adjusted method, the independent variable is $\beta^f_{\sigma,i,t-2}$, and the regressions are estimated with WLS using gross returns as weights (either $C_{i,t-1}/C_{i,t-2}$ or $P_{i,t-1}/P_{i,t-2}$). Section 1 describes the sample selection criteria. We also present results of regressions estimated with WLS using the dollar value of open interest ($\$OI_{i,t-2}$) as weights. The options are written on the S&P 500 Index and individual stocks in the S&P 500 Index. T-statistics are shown in parentheses.

	A: Bias-unadjusted			
	Stocks		Index	
	λ_0	λ_σ	λ_0	λ_σ
Calls				
OLS	-19.21	4.07	15.12	-7.64
	(-11.14)	(2.06)	(2.32)	(-3.52)
WLS, $\$OI$ weighted	-10.06	0.53	5.67	-6.48
	(-7.25)	(0.20)	(1.20)	(-3.04)
Puts				
OLS	-21.37	2.37	75.98	-21.68
	(-9.85)	(1.22)	(9.05)	(-9.71)
WLS, $\$OI$ weighted	-34.55	7.85	59.34	-17.36
	(-10.56)	(3.14)	(8.12)	(-8.27)
	B: Bias-adjusted			
	Stocks		Index	
	λ_0	λ_σ	λ_0	λ_σ
Calls				
WLS, weighted by $\frac{C_{i,t-1}}{C_{i,t-2}}$	-6.01	2.62	3.37	-6.17
	(-3.24)	(1.43)	(0.52)	(-2.77)
WLS, weighted by $\frac{C_{i,t-1}}{C_{i,t-2}} \times \$OI$	0.40	-3.22	0.43	-6.22
	(0.27)	(-1.35)	(0.10)	(-2.90)
Puts				
WLS, weighted by $\frac{P_{i,t-1}}{P_{i,t-2}}$	-25.74	3.56	77.06	-23.30
	(-7.74)	(1.40)	(8.94)	(-11.36)
WLS, weighted by $\frac{P_{i,t-1}}{P_{i,t-2}} \times \$OI$	-10.21	-2.56	68.55	-19.93
	(-4.64)	(-1.12)	(9.09)	(-9.91)

Table A5: **FM regressions of returns of delta-hedged options on S&P 500 stocks and on the S&P 500 Index from 2006 to 2019.** This table displays the results of FM regressions: $R_{f,i,t} - \beta^f_{S,i,t-1} R_{S,t} = \lambda_0 + \lambda_\sigma \times \beta^f_{\sigma,i,t-1} + \eta_{i,t}$. The dependent variable is delta-hedged option excess return, where $R_{f,i,t}$ is the excess return of call or put i between $t-1$ and t; $\beta^f_{S,i,t-1} = S_{t-1}/f_{t-1} \times \Delta^f_{S,i,t-1}$ is the β of the call or put with respect to the underlying asset; and $R_{S,t}$ is the excess return of the underlying stock or index between $t-1$ and t. In the bias-unadjusted method, the independent variable is the β of the option with respect to the volatility of the underlying at time $t-1$ ($\beta^f_{\sigma,i,t-1}$). Moreover, the sample used in the bias-unadjusted method is constructed with selection criteria based on $t-1$ variables, including the requirement that the option's delta ($\Delta^f_{S,i,t-1}$) is non-missing. In contrast, the sample used in the bias-adjusted method is constructed with selection criteria based on $t-2$ variables, with missing deltas replaced by their lagged values ($\Delta^f_{S,i,t-2}$) when calculating the delta-hedged return. In addition, in the bias-adjusted method, the independent variable is $\beta^f_{\sigma,i,t-2}$, and the regressions are estimated with WLS using gross returns as weights (either $C_{i,t-1}/C_{i,t-2}$ or $P_{i,t-1}/P_{i,t-2}$). Section 1 describes the sample selection criteria. We also present results of regressions estimated with WLS using the dollar value of open interest ($\$OI_{i,t-2}$) as weights. The options are written on the S&P 500 Index and individual stocks in the S&P 500 Index. T-statistics are shown in parentheses.

	A: Bias-unadjusted			
	Stocks		Index	
	λ_0	λ_σ	λ_0	λ_σ
Calls				
OLS	-16.80	0.01	19.04	-6.00
	(-6.94)	(0.01)	(3.17)	(-3.21)
WLS, $\$OI$ weighted	-7.89	-6.05	4.14	-4.33
	(-4.40)	(-2.98)	(1.02)	(-2.36)
Puts				
OLS	-26.41	1.00	-0.18	-6.38
	(-9.57)	(0.47)	(-0.01)	(-1.66)
WLS, $\$OI$ weighted	-23.69	1.79	13.29	-7.68
	(-12.28)	(0.79)	(1.04)	(-2.60)
	B: Bias-adjusted			
	Stocks		Index	
	λ_0	λ_σ	λ_0	λ_σ
Calls				
WLS, weighted by $\frac{C_{i,t-1}}{C_{i,t-2}}$	-1.75	-2.50	11.52	-6.51
	(-0.81)	(-1.31)	(1.96)	(-3.24)
WLS, weighted by $\frac{C_{i,t-1}}{C_{i,t-2}} \times \$OI$	-1.46	-7.02	-3.80	-5.47
	(-0.85)	(-3.59)	(-0.99)	(-2.78)
Puts				
WLS, weighted by $\frac{P_{i,t-1}}{P_{i,t-2}}$	2.65	-8.83	-0.96	-8.28
	(1.02)	(-4.62)	(-0.03)	(-2.41)
WLS, weighted by $\frac{P_{i,t-1}}{P_{i,t-2}} \times \$OI$	-4.60	-6.32	10.48	-8.05
	(-2.71)	(-2.97)	(0.62)	(-2.79)

Table A6: **Option bid-ask spread model estimation results for stock options.** This table reports estimates of Fama-MacBeth regressions estimating the mean and the variance of stock option bid-ask spreads as function of the price, delta, gamma, and implied volatility of the option as well as the effective spread of the underlying stock. Newey-West t-statistics, using ten lags, are shown in parentheses.

	Mean Equations		Variance Equations	
Intercept	-1.5918	-1.63460	0.68862	0.57739
	(-73.6819)	(-72.2228)	(78.0014)	(54.9760)
Option Price$_{i,j,t}$	-0.03424	-0.04411	0.01370	0.01834
	(-23.3607)	(-20.8518)	(26.9011)	(27.8097)
1(Option Price$_{i,j,t}$ < \$2)	0.21301	0.26549	-0.00000	0.02556
	(30.2483)	(33.2277)	(-0.0003)	(4.5662)
1(\$5 $\leq$ Option Price$_{i,j,t}$ < \$10)	0.08138	0.11991	0.06302	0.07486
	(16.2799)	(18.7976)	(19.6499)	(20.5359)
1(\$10 $\leq$ Option Price$_{i,j,t}$ < \$20)	0.14566	0.24220	0.10108	0.09999
	(12.0059)	(14.2268)	(15.8112)	(12.3343)
1(\$20 $\leq$ Option Price$_{i,j,t}$)	0.25699	0.39351	-0.09957	-0.10067
	(9.3354)	(10.9060)	(-8.1271)	(-6.3520)
Stock bid-ask spread$_{i,t}$	-0.44066	-0.45837	0.44052	0.39363
	(-12.9878)	(-13.3473)	(15.3661)	(14.6447)
Option delta$_{i,j,t}$	-1.36109	1.23748	-0.56042	0.45008
	(-63.6658)	(46.9999)	(-44.5330)	(31.5000)
Option gamma$_{i,j,t}$	0.50579	0.01704	0.06013	0.22383
	(5.8541)	(0.1832)	(1.7485)	(6.8392)
Implied volatility$_{i,j,t}$	-0.03319	-0.08338	0.22336	0.24150
	(-1.6155)	(-4.7403)	(16.3359)	(16.4997)
Standard deviation of ϵ_t	0.3060			
Standard deviation of ϵ_i	0.4203			

Table A7: **Option bid-ask spread model estimation results for index options.** This table reports estimates of Fama-MacBeth regressions estimating the mean and the variance of index option bid-ask spreads as function of the price, delta, gamma of the option. Newey-West t-statistics, using ten lags, are shown in parentheses.

	Mean Equations		Variance Equations	
Intercept	-1.3606	-1.5352	0.3067	0.2237
	(-55.6081)	(-71.2277)	(12.4403)	(16.1355)
Option Price$_{i,j,t}$	-0.0087	-0.1008	0.0041	0.0614
	(-11.8520)	(-10.0167)	(4.5427)	(8.7617)
1(Option Price$_{i,j,t}$ $<$ \$2)	0.3943	0.3297	0.1714	0.2141
	(34.1439)	(42.0641)	(11.8717)	(22.3780)
1(\$5 $\leq$ Option Price$_{i,j,t}$ $<$ \$10)	-0.1671	-0.2100	-0.0359	-0.1804
	(-18.6859)	(-25.3000)	(-2.8312)	(-16.4640)
1(\$10 $\leq$ Option Price$_{i,j,t}$ $<$ \$20)	-0.1997	-0.2469	-0.0048	-0.5505
	(-11.6751)	(-14.0946)	(-0.2565)	(-22.1218)
1(\$20 $\leq$ Option Price$_{i,j,t}$)	-0.2178	-0.2143	-0.0224	-1.3191
	(-8.0829)	(-7.9077)	(-0.9002)	(-24.4526)
Option delta$_{i,t}$	-1.5586	-4.8280	-0.6318	-0.2621
	(-31.5386)	(-10.4528)	(-7.6817)	(-0.4882)
Option gamma$_{i,j,t}$	-69.1376	-231.7026	47.9531	-149.1644
	(-18.9766)	(-18.1546)	(7.1453)	(-5.5429)
Standard deviation of ϵ_t	0.6833			

Table A8: **Mean returns and Fama-MacBeth regression coefficients of simulated straddle, delta-hedged β_S-adjusted calls, and delta-hedged leverage-adjusted calls sorted or regressed on moneyness.** This table reports simulated average portfolio returns and Fama-MacBeth regression coefficients. All values reported are averages across 100 simulations. The sort/regression variable is option moneyness, defined as $\ln(e^{-r_t(T-t)}K/S_t)/(\sigma_t\sqrt{T-t})$, where S_t is the stock price; T is the maturity date; K is the option strike price; r_t is the risk-free, which is set equal to zero; and σ_t is the implied volatility. Moneyness groups are defined as follows. Low: moneyness < -1.25. Group 2: $-1.25 \leq$ moneyness < -0.5. Group 3: $-0.5 \leq$ moneyness < 0.5. Group 4: $0.5 \leq$ moneyness < 1.25. High: moneyness ≥ 1.25. "True prices" refer to results calculated from simulated prices without measurement errors. The bias-unadjusted results are calculated with a simulated sample constructed with selection criteria based on $t-1$ variables that contain measurement errors. In contrast, the bias-adjusted results are calculated with a sample constructed with selection criteria based on $t-2$ variables, also containing measurement errors. Moreover, in the bias-unadjusted sorting, returns are sorted by or regressed on moneyness at the beginning of the holding period ($t-1$), while the bias-adjusted results use moneyness one day before the beginning of the holding period ($t-2$). In addition, the alternative returns presented in equations 30 and 38 are used to calculate the bias-adjusted returns for straddle and delta-hedged β_S-adjusted calls. The total bias is decomposed into its different parts: the direct mean return bias (DMR), indirect mean return bias (IMR), CEIV bias, and sample-selection bias (SS). Returns and biases are in basis-points per day. Average t-statistics are shown in parentheses.

	A: Straddles						
	Average daily returns					FM Regression	
	Low	2	3	4	High	λ_0	λ_M
True prices	-15.63	-15.79	-15.88	-15.75	-15.19	-15.72	0.13
	(-9.57)	(-9.38)	(-9.28)	(-9.23)	(-9.38)	(-9.39)	(0.78)
Bias-unadjusted	-11.11	-18.46	-15.96	-21.30	-15.47	-17.15	-1.23
	(-6.72)	(-10.91)	(-9.30)	(-12.45)	(-9.50)	(-10.23)	(-6.92)
Bias-adjusted	-13.79	-15.86	-16.04	-15.85	-14.77	-15.57	-0.10
	(-7.70)	(-8.03)	(-8.01)	(-8.11)	(-8.09)	(-8.22)	(-0.28)
Biases							
DMR	7.28	9.73	11.12	9.06	6.20	9.35	-0.59
IMR	-3.81	-4.31	-5.52	-4.15	-3.99	-4.58	0.06
CEIV	20.17	-5.04	-5.68	-6.64	17.91	-0.10	0.67
SS	-19.12	-3.05	-0.01	-3.82	-20.40	-6.10	-1.50
Total	4.52	-2.67	-0.08	-5.55	-0.28	-1.43	-1.36

B: Delta-hedged β_S-adjusted calls							
	Average daily returns					FM Regression	
	Low	2	3	4	High	λ_0	λ_M
True prices	-0.26	-0.65	-1.62	-2.98	-4.41	-1.86	-1.40
	(-9.76)	(-9.65)	(-9.62)	(-9.61)	(-9.78)	(-9.71)	(-9.72)
Bias-unadjusted	3.61	-4.53	-2.68	-2.42	-3.51	-2.50	-0.78
	(81.23)	(-61.89)	(-15.78)	(-7.77)	(-7.76)	(-13.04)	(-5.40)
Bias-adjusted	-0.28	-0.64	-1.61	-2.95	-4.35	-1.84	-1.37
	(-6.11)	(-8.33)	(-8.73)	(-8.77)	(-8.98)	(-8.91)	(-8.89)
Biases							
DMR	0.50	0.65	1.09	1.37	1.28	1.00	0.31
IMR	-1.39	-1.15	-1.03	-0.70	-0.42	-0.95	0.30
CEIV	8.60	-2.65	-1.13	-0.05	0.92	0.07	-0.68
SS	-3.84	-0.72	0.00	-0.06	-0.88	-0.75	0.69
Total	3.87	-3.88	-1.06	0.56	0.90	-0.64	0.62

C: Delta-hedged leverage-adjusted calls							
	Average daily returns					FM Regression	
	Low	2	3	4	High	λ_0	λ_M
True prices	-0.24	-0.52	-0.81	-0.62	-0.31	-0.58	-0.01
	(-9.76)	(-9.66)	(-9.64)	(-9.64)	(-9.75)	(-9.69)	(-1.94)
Bias-unadjusted	3.49	-3.77	-1.47	-0.53	-0.25	-1.15	0.18
	(83.45)	(-63.89)	(-17.38)	(-8.12)	(-7.79)	(-19.02)	(23.68)
Bias-adjusted	-0.24	-0.51	-0.80	-0.61	-0.30	-0.57	0.00
	(-6.03)	(-9.04)	(-9.52)	(-9.52)	(-9.63)	(-9.59)	(-0.45)
Biases							
DMR	0.47	0.52	0.55	0.30	0.09	0.43	-0.14
IMR	-0.88	-0.53	-0.55	-0.30	-0.09	-0.48	0.21
CEIV	4.51	-3.24	-0.66	0.10	0.11	-0.46	0.01
SS	-0.37	0.00	0.00	-0.01	-0.05	-0.06	0.10
Total	3.73	-3.25	-0.66	0.10	0.06	-0.57	0.18

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