

Estimating yield curves from asynchronous LIBOR and swap quotes¹

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Abstract

A class of smoothing algorithms is proposed to estimate the unobserved end-of-day six-month and one-year LIBOR rates. Empirical analysis suggests that estimates using these methods contain as little as one third of the error of more naively constructed values. These methods also allow the relationships between short-term and long-term yields to be characterized with much greater accuracy, which should be a benefit when using the data constructed using these methods for estimating dynamic term structure models.

¹I am grateful for conversations with Pierre Collin-Dufresne, Bob Goldstein, and Mike Johannes. Updated versions of this paper and bootstrapped zero coupon yield data can be found at <http://www-rcf.usc.edu/~christoj/research.html>.

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Abstract

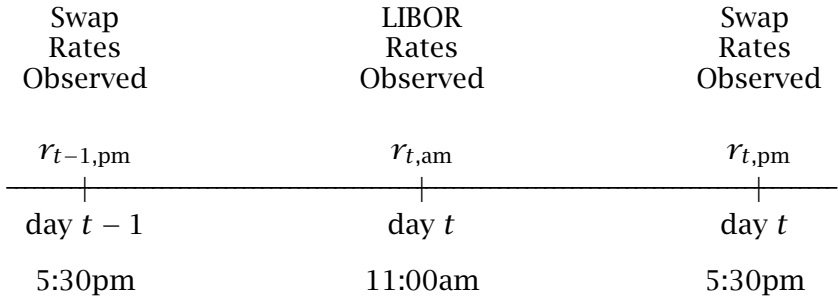
A class of smoothing algorithms is proposed to estimate the unobserved end-of-day six-month and one-year LIBOR rates. Empirical analysis suggests that estimates using these methods contain as little as one third of the error of more naively constructed values. These methods also allow the relationships between short-term and long-term yields to be characterized with much greater accuracy, which should be a benefit when using the data constructed using these methods for estimating dynamic term structure models.

1 Introduction and institutional setting

For a variety of reasons, much recent fixed income literature (e.g. Duffie and Singleton (1997), Collin-Dufresne and Solnik (2001)) has focused on the term structure of swap rates. One problem encountered when analyzing swaps is that the one-year contract is the shortest maturity available. Prior to 1997, the problem was even more severe, as the two-year contract was the shortest available.

There are at least two issues that arise from observing no shorter term yields. First, the short rate is often the primary object of interest in empirical term structure literature, and recent contributions such as Piazzesi (2003) have demonstrated that the short end of the yield curve can help uncover richer interest rate dynamics than might be suggested by longer yields alone. Second, the lack of a six-month yield prevents any reasonable attempt at bootstrapping the swap curve, effectively a par yield curve, to produce a yield curve of zero coupon rates.

In response, some authors have augmented swap rate data with short-term LIBOR rates. Dai and Singleton (2000), for example, augment a data set of two- to ten-year swap rates with the six-month LIBOR rate. A practical problem with this apparent fix is that swap rates from Datastream are recorded as of the end-of-day in London, while LIBOR rates are quoted by the British Bankers' Association at 11:00am, also in London, as depicted on the following time line:



When the use of these data is in the estimation of a dynamic term structure model, then these timing issues could be addressed, at least theoretically, by simply accounting for them in the estimation procedure. In practice, however, this additional layer of complexity would make many existing estimation methodologies, which must already contend with multiple latent factors and unknown transition densities, completely intractable. This fact necessitates a simpler if imperfect approach.

The plan for the rest of this paper is straightforward. We will propose several simple models of interest rate changes over intra-day horizons and then simultaneously estimate both the parameters of the model and the latent variables, which in this case correspond to LIBOR rates at 5:30pm and swap rates at 11:00am. We will then analyze the properties of the fitted LIBOR rates and compare them against several natural and commonly used alternatives.

2 The basic model

Both market imperfections and institutional differences between LIBOR and swap markets likely make common no-arbitrage models imperfect characterizations of high-frequency yield dynamics. In addition, expected changes in yields will likely be extremely small over half-day horizons. We therefore assume a simple martingale model of interest rates that is devoid of no-arbitrage restrictions on the covariance matrix of yield innovations.

Let r denote a vector comprised of some swap rates (r^S) observed in the evening and some LIBOR rates (r^L) observed in the late morning, and partition it as

$$r = \begin{bmatrix} r^L \\ r^S \end{bmatrix},$$

where r^L is $N_L \times 1$, r^S is $N_S \times 1$, and $N \equiv N_L + N_S$. There are T days in the sample.

Not all rates are observed at the same time. Specifically, r^S is observed only at 5:30pm, r^L is observed only at 11:00am. The timing of the latter observation corresponds to BBA's practice of quoting LIBOR rates at 11:00am London time. We assume these rates evolve according to

$$\begin{aligned} r_{t,\text{am}} - r_{t-1,\text{pm}} &\sim N(0, \Sigma) \\ r_{t,\text{pm}} - r_{t,\text{am}} &\sim N(0, \tau \Sigma). \end{aligned}$$

Besides the fact that interest rates are martingales, the specification also assumes that the covariance structure of "pm to am" changes is the same, up to a scaling constant, as the structure of "am to pm" changes. The parameter τ roughly reflects the amount of "economic time" that passes during the 11:00am to 5:30pm interval relative to the time that passes during the remaining hours of the day.

3 The estimation procedure

The goal of this analysis, extracting estimates of $r_{t,\text{pm}}^L$, could be carried out in a variety of ways. A filtering approach would use all information up to 5:00pm of day t to construct an expected value of $r_{t,\text{pm}}^L$. Because a more accurate approach would presumably use information after day t as well, we instead use a smoothing algorithm to construct expected values that are conditional on a larger information set. Following Tanner and Wong (1987), we use a Bayesian data augmentation approach that generates many draws of the unobserved $r_{t,\text{pm}}^L$, each of which is consistent with the observed time series of both r^S and r^L . The average over all these draws is a consistent estimator of the expectation we are interested in.

We choose the Bayesian method only for computational convenience, and our Markov chain Monte Carlo algorithm displays good convergence properties in this application. As we use diffuse priors exclusively, our results should mirror maximum likelihood almost exactly.

The data augmentation approach also deals with the problem of simultaneously estimating the parameters τ and Σ , which are also unknown. Once our observed data ($r_{t,\text{am}}^L$ and $r_{t,\text{pm}}^S$) is

augmented with unobserved “data” ($r_{t,\text{pm}}^L$ and $r_{t,\text{am}}^S$), standard techniques can be used to draw parameter values that are consistent with the entire augmented sample.

In summary, our technique is a Gibbs sampler with four blocks: Σ , τ , the evening values of r^L (i.e. $r_{1,\text{pm}}^L, r_{2,\text{pm}}^L, \dots, r_{T,\text{pm}}^L$), and the morning values of r^S (i.e. $r_{1,\text{am}}^S, r_{2,\text{am}}^S, \dots, r_{T,\text{am}}^S$). Each block is now described in greater detail.

3.1 Block 1: drawing Σ

Conditional on the augmented data set and on a value of τ , we can rewrite the model of yield changes as

$$\begin{aligned} r_{t,\text{am}} - r_{t-1,\text{pm}} &\sim N(0, \Sigma) \\ \frac{r_{t,\text{pm}} - r_{t,\text{am}}}{\sqrt{\tau}} &\sim N(0, \Sigma) \end{aligned}$$

A natural estimator of Σ is therefore given by

$$\hat{\Sigma} = \frac{1}{2T-1} \left(\sum_{t=2}^T (r_{t,\text{am}} - r_{t-1,\text{pm}}) (r_{t,\text{am}} - r_{t-1,\text{pm}})' + \frac{1}{\tau} \sum_{t=1}^T (r_{t,\text{pm}} - r_{t,\text{am}}) (r_{t,\text{pm}} - r_{t,\text{am}})' \right).$$

While the actual value of Σ is unknown, its distribution under diffuse priors is that of an inverted Wishart (see Zellner, 1971, for example) with parameters $\hat{\Sigma}$ and $\nu = 2T - 1$ (the degrees of freedom, which is equal to the sample size since means are assumed zero). Following the algorithm of Kennedy and Gentle (1980), for example, it is simple to draw the matrix Σ from this distribution.

3.2 Block 2: drawing τ

Now given Σ and the augmented data set, we draw the parameter τ . Assuming the diffuse prior $p(\sqrt{\tau}) \propto 1/\sqrt{\tau}$, Bayes rule implies

$$p(\sqrt{\tau}|\Sigma, r) \propto \prod_{t=2}^T \phi(r_{t,\text{am}} - r_{t-1,\text{pm}}; 0, \Sigma) \times \prod_{t=1}^T \phi(r_{t,\text{pm}} - r_{t,\text{am}}; 0, \tau\Sigma) \times \frac{1}{\sqrt{\tau}},$$

where $\phi(x; \mu, \Omega)$ is the multivariate normal density of x with mean μ and covariance matrix Ω . Since the first product does not depend on τ , we have

$$p(\sqrt{\tau}|\Sigma, r) \propto \frac{1}{\sqrt{\tau}} \prod_{t=1}^T \phi(r_{t,\text{pm}} - r_{t,\text{am}}; 0, \tau\Sigma).$$

Using standard results (see Zellner, 1971, p. 225), this can be shown to be proportional to

$$\frac{1}{\sqrt{\tau}} \det(\tau\Sigma)^{-T/2} \exp\left(-\frac{1}{2\tau} \text{tr}(T\hat{\Sigma}\Sigma^{-1})\right).$$

where

$$\tilde{\Sigma} = \frac{1}{T} \sum_{t=1}^T (r_{t,\text{pm}} - r_{t,\text{am}}) (r_{t,\text{pm}} - r_{t,\text{am}})'$$

We can rewrite this density as proportional to

$$\sqrt{\tau}^{-(2\alpha+1)} \exp\left(-\frac{1}{\gamma\tau}\right),$$

where $\alpha = TN/2$ and $\gamma = 2/\text{tr}(T\tilde{\Sigma}\Sigma^{-1})$, which is the kernel of the inverted gamma density. It is therefore the case that $\sqrt{\tau} \sim \text{IG}(\gamma, \alpha)$ conditional on Σ and r . Kennedy and Gentle (1980) also describes methods for drawing from this distribution.

3.3 Block 3: drawing $r_{t,\text{pm}}^L$

Given the Markovian nature of the interest rate process, the only rates that are relevant for drawing $r_{t,\text{pm}}^L$ are those that are realized immediately prior ($r_{t,\text{am}}^S$ and $r_{t,\text{am}}^L$), immediately after ($r_{t+1,\text{am}}^S$ and $r_{t+1,\text{am}}^L$), and contemporaneously ($r_{t,\text{pm}}^S$) to $r_{t,\text{pm}}^L$. Our goal, therefore, is to draw from

$$p^*(r_{t,\text{pm}}^L) \equiv p(r_{t,\text{pm}}^L | r_{t,\text{am}}^S, r_{t,\text{am}}^L, r_{t,\text{pm}}^S, r_{t+1,\text{am}}^S, r_{t+1,\text{am}}^L, \Sigma, \tau).$$

Bayes rule and the Markov property imply that

$$p^*(r_{t,\text{pm}}^L) \propto p(r_{t,\text{pm}}^L | r_{t,\text{am}}^S, r_{t,\text{am}}^L, r_{t,\text{pm}}^S, \Sigma, \tau) p(r_{t+1,\text{am}}^S, r_{t+1,\text{am}}^L | r_{t,\text{pm}}^S, r_{t,\text{pm}}^L, \Sigma, \tau).$$

The first density in $p^*(r_{t,\text{pm}}^L)$ is the conditional density of a multivariate normal distribution. If we partition the covariance matrix as

$$\tau\Sigma = \begin{bmatrix} \tau\Sigma_L & \tau\Sigma_{L,S} \\ \tau\Sigma_{L,S}' & \tau\Sigma_S \end{bmatrix},$$

then we have the standard result that

$$p(r_{t,\text{pm}}^L | r_{t,\text{am}}^S, r_{t,\text{am}}^L, r_{t,\text{pm}}^S, \Sigma, \tau) = \phi(r_{t,\text{pm}}^L; r_{t,\text{am}}^L + \Sigma_{L,S}\Sigma_S^{-1}(r_{t,\text{pm}}^S - r_{t,\text{am}}^S), \tau[\Sigma_L - \Sigma_{L,S}\Sigma_S^{-1}\Sigma_{L,S}']).$$

The second density in $p^*(r_{t,\text{pm}}^L)$ is the multivariate normal

$$p(r_{t+1,\text{am}}^S, r_{t+1,\text{am}}^L | r_{t,\text{pm}}^S, r_{t,\text{pm}}^L, \Sigma, \tau) = \phi\left(\begin{bmatrix} r_{t+1,\text{am}}^L \\ r_{t+1,\text{am}}^S \end{bmatrix}; \begin{bmatrix} r_{t,\text{pm}}^L \\ r_{t,\text{pm}}^S \end{bmatrix}, \Sigma\right).$$

(Note that τ is not relevant for these ‘‘pm to am’’ changes.) Since the normal density is symmetric in deviations from the mean, it is the case that

$$\phi\left(\begin{bmatrix} r_{t+1,\text{am}}^L \\ r_{t+1,\text{am}}^S \end{bmatrix}; \begin{bmatrix} r_{t,\text{pm}}^L \\ r_{t,\text{pm}}^S \end{bmatrix}, \Sigma\right) = \phi\left(\begin{bmatrix} r_{t,\text{pm}}^L \\ r_{t,\text{pm}}^S \end{bmatrix}; \begin{bmatrix} r_{t+1,\text{am}}^L \\ r_{t+1,\text{am}}^S \end{bmatrix}, \Sigma\right),$$

which is proportional (in $r_{t,\text{pm}}^L$) to

$$\phi\left(r_{t,\text{pm}}^L; r_{t+1,\text{am}}^L + \Sigma_{L,S} \Sigma_S^{-1} (r_{t,\text{pm}}^S - r_{t+1,\text{am}}^S), \Sigma_L - \Sigma_{L,S} (\Sigma_S)^{-1} \Sigma_{L,S}'\right)$$

We therefore have

$$p^*\left(r_{t,\text{pm}}^L\right) \propto \phi\left(r_{t,\text{pm}}^L; m_1, \tau V\right) \phi\left(r_{t,\text{pm}}^L; m_2, V\right),$$

where

$$\begin{aligned} m_1 &= r_{t,\text{am}}^L + \Sigma_{L,S} \Sigma_S^{-1} (r_{t,\text{pm}}^S - r_{t,\text{am}}^S) \\ m_2 &= r_{t+1,\text{am}}^L + \Sigma_{L,S} \Sigma_S^{-1} (r_{t,\text{pm}}^S - r_{t+1,\text{am}}^S) \\ V &= \Sigma_L - \Sigma_{L,S} \Sigma_S^{-1} \Sigma_{L,S}' \end{aligned}$$

Using the standard result for convolutions of normals, we obtain

$$p^*\left(r_{t,\text{pm}}^L\right) = \phi\left(r_{t,\text{pm}}^L; \frac{m_1}{1+\tau} + \frac{\tau m_2}{1+\tau}, \frac{\tau}{1+\tau} V\right).$$

We therefore draw the $r_{t,\text{pm}}^L$ from this distribution.

3.4 Block 4: drawing $r_{t,\text{am}}^S$

Following identical logic, we find that

$$p\left(r_{t,\text{am}}^S | r_{t-1,\text{pm}}^S, r_{t-1,\text{pm}}^L, r_{t,\text{am}}^L, r_{t,\text{pm}}^S, r_{t,\text{pm}}^L, \Sigma, \tau\right) = \phi\left(r_{t,\text{am}}^S; \frac{\tau \tilde{m}_1}{1+\tau} + \frac{\tilde{m}_2}{1+\tau}, \frac{\tau}{1+\tau} \tilde{V}\right),$$

where

$$\begin{aligned} \tilde{m}_1 &= r_{t-1,\text{pm}}^S + \Sigma_{L,S}' \Sigma_L^{-1} (r_{t,\text{am}}^L - r_{t-1,\text{pm}}^L) \\ \tilde{m}_2 &= r_{t,\text{pm}}^S + \Sigma_{L,S}' \Sigma_L^{-1} (r_{t,\text{am}}^L - r_{t,\text{pm}}^L) \\ \tilde{V} &= \Sigma_S - \Sigma_{L,S}' \Sigma_L^{-1} \Sigma_{L,S} \end{aligned}$$

3.5 Final steps: estimation

Rather than producing a single estimate for $r_{t,\text{pm}}^L$, for example, the algorithm described above produces an entire distribution of $r_{t,\text{pm}}^L$. Under a mean-squared loss function, optimal estimates are given by posterior means. Accurate results appear to be generated by running the MCMC algorithm for 11,000 iterations and then discarding the first 1,000 draws. The remaining 10,000 are averaged to obtain the estimates used in the remainder of the paper. Results are unchanged when the Gibbs chains are run ten times as long.

4 Model extensions

4.1 Stochastic volatility

Because financial market volatility is in general time-varying, the previous homoskedastic model may be deficient. A natural extension is to augment the previous model with a stochastic volatility process, which is assumed univariate for simplicity. We therefore assume the existence of a mean zero h , given by

$$\begin{aligned} h_{t,\text{am}} - h_{t-1,\text{pm}} &= -\kappa h_{t-1,\text{pm}} + \omega \eta_t \\ h_{t,\text{pm}} - h_{t,\text{am}} &= -\tau \kappa h_{t,\text{am}} + \sqrt{\tau} \omega \eta_t, \end{aligned}$$

where $\eta_t \sim N(0, 1)$. This process affects yield dynamics via the assumption that

$$\begin{aligned} r_{t,\text{am}} - r_{t-1,\text{pm}} &\sim N(0, \exp(h_{t,\text{am}})\Sigma) \\ r_{t,\text{pm}} - r_{t,\text{am}} &\sim N(0, \tau \exp(h_{t,\text{pm}})\Sigma). \end{aligned}$$

The parameter κ may be interpreted as the speed of mean reversion of h towards zero. ω determines the degree of heteroskedasticity implied by the model. Assuming a zero mean for h is without loss of generality and imposes a necessary identifying assumption, since mean effects may be incorporated into Σ .

4.1.1 Drawing $r_{t,\text{pm}}^L$ and $r_{t,\text{am}}^S$

Modifications to the sampling algorithm of Section 3 are fairly straightforward. The distribution of $r_{t,\text{pm}}^L$ remains Gaussian, but now has mean

$$\frac{\exp(2h_{t+1,\text{am}})}{\tau \exp(2h_{t,\text{pm}}) + \exp(2h_{t+1,\text{am}})} m_1 + \frac{\tau \exp(2h_{t,\text{pm}})}{\tau \exp(2h_{t,\text{pm}}) + \exp(2h_{t+1,\text{am}})} m_2,$$

which is still a precision-weighted average of the previously defined m_1 and m_2 . The covariance matrix of $r_{t,\text{pm}}^L$ is

$$\frac{\tau \exp(2h_{t,\text{pm}} + 2h_{t+1,\text{am}})}{\exp(2h_{t,\text{pm}}) + \tau \exp(2h_{t+1,\text{am}})} V,$$

where V is also the same as before. The mean and covariance matrix of $r_{t,\text{am}}^S$ are now

$$\frac{\tau \exp(2h_{t,\text{pm}})}{\exp(2h_{t,\text{am}}) + \tau \exp(2h_{t,\text{pm}})} \tilde{m}_1 + \frac{\exp(2h_{t,\text{am}})}{\exp(2h_{t,\text{am}}) + \tau \exp(2h_{t,\text{pm}})} \tilde{m}_2$$

and

$$\frac{\tau \exp(2h_{t,\text{am}} + 2h_{t,\text{pm}})}{\exp(2h_{t,\text{am}}) + \tau \exp(2h_{t,\text{pm}})} \tilde{V},$$

respectively, again where \tilde{m}_1 , \tilde{m}_2 , and \tilde{V} are as before.

4.1.2 Drawing the h process

As in Jacquier, Polson, and Rossi (1994), we may use Bayes rule and the Markov property to show that the density of $h_{t,\text{am}}$, conditional on all other variables and parameters, may be decomposed into three components as

$$p^*(h_{t,\text{am}}) \propto p(h_{t,\text{am}}|h_{t-1,\text{pm}}) p(h_{t,\text{pm}}|h_{t,\text{am}}) p(r_{t,\text{am}} - r_{t-1,\text{pm}}|h_{t,\text{am}}),$$

where conditioning on parameter values is implicit.

A simple albeit inefficient draw of $h_{t,\text{am}}$ may be obtained via the Metropolis-Hastings algorithm using a candidate generating density defined by the first two components of $p^*(h_{t,\text{am}})$. These together imply a normal density for $h_{t,\text{am}}$ with mean

$$\frac{\tau}{\tau + (1 - \tau\kappa)^2} h_{t-1,\text{pm}}(1 - \kappa) + \frac{(1 - \tau\kappa)^2}{\tau + (1 - \tau\kappa)^2} \frac{h_{t,\text{pm}}}{1 - \tau\kappa}$$

and variance

$$\frac{\tau(1 - \tau\kappa)^2}{\tau + (1 - \tau\kappa)^2} \omega^2.$$

After drawing a candidate, $h_{t,\text{am}}^*$, from this density, we accept it over the previous draw, $h_{t,\text{am}}$, with probability

$$\min \left\{ 1, \frac{p(r_{t,\text{am}} - r_{t-1,\text{pm}}|h_{t,\text{am}}^*)}{p(r_{t,\text{am}} - r_{t-1,\text{pm}}|h_{t,\text{am}})} \right\},$$

which is simply the truncated ratio of two multivariate normal densities.

A similar result obtains for $h_{t,\text{pm}}$.

4.1.3 Drawing κ and ω

Conditional on the observation of the entire h process, we may draw κ and ω using standard linear regression techniques. Diffuse priors are employed for both parameters.

4.2 Yield discreteness

In practice, interest rates are quoted on a discrete grid. Current swap rate quotes, for example, are typically multiples of one half a basis point. Historically, the minimum “tick” size has been considerably larger for some yields. From 1988 to 1990, the most common movement in BBA six-month LIBOR rates was +/- 6.25 basis points, and no movement was recorded about every other day during this period. The relative frequencies of other moves are shown in Table 1.

Based on these numbers it may be argued that 6.25bp represents the typical minimum tick size over this period, which is large relative to the sample standard deviation of yield changes of 8.1bp. (Furthermore, it is well known that discreteness biases estimated standard deviations upward.)

A number of approaches have been proposed to estimate standard deviations in the presence of data discreteness. Following Gottlieb and Talay (1985) and Harris (1990), we assume that the observed data are rounded versions of unobserved “true” values from some stochastic process. This approach melds easily with the Bayesian data augmentation algorithm developed previously.

Specifically, the unrounded interest rates represented by $r_{t,\text{am}}^L$ and $r_{t,\text{pm}}^S$, before assumed observable, are now assumed latent. Instead we observe the rounded versions of these two series, $\hat{r}_{t,\text{am}}^L$ and $\hat{r}_{t,\text{pm}}^S$. If we initially suppose, for ease of exposition, that all yields have the same constant tick size of 2δ , then our data now imply only the approximate location of the true process, or

$$\begin{aligned}\hat{r}_{t,\text{am}}^L - \delta &< r_{t,\text{am}}^L < \hat{r}_{t,\text{am}}^L + \delta \\ \hat{r}_{t,\text{pm}}^S - \delta &< r_{t,\text{pm}}^S < \hat{r}_{t,\text{pm}}^S + \delta\end{aligned}$$

We draw the unknown $r_{t,\text{am}}^L$ and $r_{t,\text{pm}}^S$ in two additional steps of the data augmentation algorithm. Draws of the parameters are unchanged given the augmented data set.

4.2.1 Drawing $r_{t,\text{am}}^L$

Similarly to before, we require a draw from

$$p^* \left(r_{t,\text{am}}^L \right) \equiv p \left(r_{t,\text{am}}^L | \hat{r}_{t,\text{am}}^L, r_{t-1,\text{pm}}^S, r_{t-1,\text{pm}}^L, r_{t,\text{am}}^S, r_{t,\text{pm}}^S, r_{t,\text{pm}}^L, \Sigma, \tau \right).$$

Conditioning arguments include the *true* rates observed before and after $r_{t,\text{am}}^L$, the contemporaneous *true* value of $r_{t,\text{am}}^S$, and the contemporaneously observed *rounded* rate $\hat{r}_{t,\text{am}}^L$.

Application of Bayes rule and the Markov property imply that

$$p^* \left(r_{t,\text{am}}^L \right) \propto p \left(r_{t,\text{am}}^L | r_{t-1,\text{pm}}^S, r_{t-1,\text{pm}}^L, r_{t,\text{am}}^S, r_{t,\text{pm}}^S, r_{t,\text{pm}}^L, \Sigma, \tau \right) p \left(\hat{r}_{t,\text{am}}^L | r_{t,\text{am}}^L \right),$$

where the first density on the right hand side is similar to those derived in sections 3.4 and 3.5. Following that logic we find that

$$p^* \left(r_{t,\text{am}}^L \right) \propto \phi \left(r_{t,\text{am}}^L; \frac{\tau m_1^*}{1 + \tau} + \frac{m_2^*}{1 + \tau}, \frac{\tau}{1 + \tau} V^* \right) \mathbf{1}(\max |r_{t,\text{am}}^L - \hat{r}_{t,\text{am}}^L| < \delta),$$

where

$$\begin{aligned}m_1^* &= r_{t-1,\text{pm}}^L + \Sigma_{L,S} \Sigma_S^{-1} \left(r_{t,\text{am}}^S - r_{t-1,\text{pm}}^S \right) \\ m_2^* &= r_{t,\text{pm}}^L + \Sigma_{L,S} \Sigma_S^{-1} \left(r_{t,\text{am}}^S - r_{t,\text{pm}}^S \right),\end{aligned}$$

V is the same as before, and $\mathbf{1}(\max |r_{t,\text{am}}^L - \hat{r}_{t,\text{am}}^L| < \delta)$ is an indicator function that takes the value of one if the maximum deviation between the corresponding elements of $r_{t,\text{am}}^L$ and $\hat{r}_{t,\text{am}}^L$ is less than δ .

The form of this density implies that an accept/reject algorithm may be used to draw a candidate vector of $r_{t,\text{am}}^L$ from the given normal density. If all elements of that vector are within half a tick of their corresponding values in $\hat{r}_{t,\text{am}}^L$, then the draw is accepted. Otherwise, the draw is rejected and the previous value of $r_{t,\text{am}}^L$ is repeated.

4.2.2 Drawing $r_{t,\text{pm}}^S$

Following the same steps, we find

$$\begin{aligned} & p\left(r_{t,\text{pm}}^S | r_{t,\text{am}}^S, r_{t,\text{am}}^L, \hat{r}_{t,\text{pm}}^S, r_{t,\text{pm}}^L, r_{t+1,\text{am}}^S, r_{t+1,\text{am}}^L, \Sigma, \tau\right) \\ & \propto \phi\left(r_{t,\text{pm}}^S; \frac{\tilde{m}_1^*}{1+\tau} + \frac{\tau\tilde{m}_2^*}{1+\tau}, \frac{\tau}{1+\tau}\tilde{V}\right) \mathbf{1}(\max|r_{t,\text{pm}}^S - \hat{r}_{t,\text{pm}}^S| < \delta), \end{aligned}$$

where

$$\begin{aligned} \tilde{m}_1^* &= r_{t,\text{am}}^S + \Sigma'_{L,S} \Sigma_L^{-1} (r_{t,\text{pm}}^L - r_{t,\text{am}}^L) \\ \tilde{m}_2^* &= r_{t+1,\text{am}}^S + \Sigma'_{L,S} \Sigma_L^{-1} (r_{t,\text{pm}}^L - r_{t+1,\text{am}}^L) \end{aligned}$$

and \tilde{V} is the same as before. An accept/reject algorithm is also used to draw $r_{t,\text{pm}}^S$.

4.2.3 Estimating δ

To implement the discreteness correction, we need an estimate of δ , half of the effective minimum tick size. For added realism, we allow δ to vary across yields and through time. Because the tick size has declined over time, a rolling procedure is used to estimate a time series of δ_t . After some experimentation, we settle on estimating δ_t using a moving window beginning three months before t and ending three months after. Within that interval, δ_t is estimated as the 10th percentile of the distribution of all nonzero values of $|r_{t,\text{pm}}^S - r_{t-1,\text{pm}}^S|$ for swap rates or $|r_{t,\text{am}}^L - r_{t-1,\text{am}}^L|$ for LIBOR rates.

5 Empirical results

Daily data on six-month and one-year LIBOR yields are available beginning in January 1988 from Datastream. As described above, these yields are quoted at 11:00am London time by the British Banker's Association. These two yields constitute the entire vector r^L .

Rates on a variety of swap contracts are also available on Datastream starting from this date. Notably, until January 1997 the shortest maturity swap had a tenor of two years. Even after this point, there is still no six-month swap, as such a contract would be degenerate. Since swap rates can be interpreted (ignoring default) as par yields on bond paying semi-annual coupons, the lack of a six-month rate prohibits us, even today, from extracting zero coupon yields entirely from the swap curve. We take the two-year swap rate as one element of r^S , deliberately ignoring the one-year swap rate during the period it is available.

Since we have no data on the actual LIBOR rates that might have been quoted at the close, it is impossible to evaluate our "synchronization" procedures by asking how well they predict the true values. We therefore gauge effectiveness by asking how well changes in the time-synchronized estimates predict changes in the in the one-year swap rate. Because the one-year swap rate is not used in the estimation, this analysis represents a kind of out-of-sample assessment.

To make comparisons between LIBOR and swap rates, we convert the fitted LIBOR rates, which are zero coupon yields, to par yields so that they will be comparable to swap rates. Since one-year swap rates are only available starting in January 1997, our evaluations cover the period from January 6, 1997, to December 31, 2002, the last day of our sample.

We estimate three versions of the model. The first is the basic specification of section 2. The second specification adds a stochastic volatility factor, and the third implements the rounding correction described in Section 4.2. We also consider three different estimation procedures. In the first, we simply estimate the models using the entire sample, from January 4, 1988, to December 31, 2002. Next, we limit the sample to the period starting in January 1997, at which time the one-year swap rate becomes available. Finally, to account for the possibility of parameter drift, we re-estimate the model yearly. In this case, values of $r_{t,pm}^L$ for a given year are estimated using fourteen months of data beginning in December of the previous year and ending in January of the following year. The one month of overlap in each direction is intended to increase the estimation accuracy for rates at the very beginning and very end of the year.

In addition to the model-based methods, we also consider three naive proxies for each LIBOR rate at the end of the day. The first is simply the LIBOR rate's quote from earlier on the same day, while the second is the quote from the morning of the following day. Last is the average of the two.

Predicted changes in the one-year swap rate are evaluated in terms of bias, root mean squared error (RMSE), and mean absolute error (MAE). The correlation between predicted and actual changes is given as a less formal diagnostic. Finally, we report the frequency with which errors exceed the minimum tick size. These results are displayed in Table 2.

The table shows that the gains from modeling rate changes rather than using previous and/or future rates naively yields significant improvements in all statistics. Among the model-free estimates, averaging the same and next day's mornings produces the best forecasts of one-year swap changes, but the RMSE is still approximately 3.4 basis points and the MAE is 2.4 basis points. In comparison, all three models produce RMSEs of around 1.5 basis points and MAEs around 1 basis point.

Although the differences in Table 2 are small, estimates based on the entire sample period (1988-2002) appear to do slightly worse than those estimated over 1997-2002 or re-estimated yearly. Stochastic volatility tends to improve performance slightly, while attempting to correct for discreteness results in a slight degradation, although the effects are again extremely minor. It is possible that yield discreteness was more of a problem earlier in the sample.

Although they are a less formal diagnostic, correlations are particularly informative for understanding the magnitudes of the improvements offered by the models. The best "model-free" predictions are clearly offered by the estimates that average the same day's morning LIBOR rate with the next day's morning value. Even in this case, however, the correlation of the implied predictions with the true changes in the one-year swap rate is just .724. In contrast, most model-based predictions yield correlations above .95.

The final column of Table 2 shows that roughly 60% of the model-implied rate changes are within a single tick of the actual values, while the corresponding amounts for the model-free estimates are quite low, between 20 and 30%.

Figure 1 displays relations between true and predicted levels, first differences, and fifth differences (corresponding to weekly changes) for the third and tenth methods listed in Table 2. While the table examined changes in yields, the left column shows that one-year swap rate *levels* are also well-predicted by synchronized LIBOR rates. This is notable since Collin-Dufresne and Solnik (2001) show that the LIBOR and swap curves need not agree, especially for longer maturities. Apparently, a one-year maturity is sufficiently short to be able to ignore the swap/LIBOR spread.

The middle column of the figure shows graphically some of the same results described in Table 2. It is clear that the model-based method dominates in the accuracy of its predictions, displaying a correlation of .95 between predicted and actual values, compared with just .72 for the model-free predictions. Fifth differences (usually representing weekly yield changes) are less affected by small timing differences, so the model-free predictions are fairly accurate. Nevertheless, model-based predictions continue to be superior.

Tables 3 and 4 report pairwise tests comparing the predictive accuracy of different methods. Following Diebold and Mariano (1995), if we define $\hat{e}_{i,t}$ as the time- t error of method i in predicting the one-day change in the one-year swap rate, then method i is superior to method j (in the sense of RMSE) if the mean of

$$\hat{e}_{i,t}^2 - \hat{e}_{j,t}^2,$$

is significantly negative. Table 2 reports the corresponding t-statistics for these means using standard errors calculated according to White (1980). Table 3 reports t-statistics of

$$|\hat{e}_{i,t}| - |\hat{e}_{j,t}|,$$

corresponding to an MAE-based forecast evaluation.

The results show that many of the differences in RMSE and MAE in Table 2 were highly statistically significant. In particular, all model-based estimates are superior to all non-model estimates without exception. Among the four models, the basic swap/LIBOR model and the model with stochastic volatility generally outperform the others, though the performance between these two models is closely matched. If anything, the stochastic model displays slightly favorable performance.

Finally, to provide at least suggestive evidence on how asynchronous data could affect the estimation of dynamic term structure models, Table 5 shows the correlations and covariances between changes in the 1-year swap rate, both actual and predicted, and actual changes in the 10-year swap rate. The top row in the table shows that actual 1-year and 10-year swap rate changes had a correlation coefficient of about .73 over the 1997-2002 period. The corresponding covariance (in terms of basis points squared) was about 23.

In comparison, the 1-year swap rates implied by model-free LIBOR-based estimates have correlations between .38 and .5. A researcher using one of these approaches might therefore conclude that short and long-term rates are relatively disconnected, when in fact they are highly correlated. In comparison, most of the model-implied estimates of 1-year swap rate changes produce correlations between .78 and .8, consistent with the tight link between actual short and long rates.

In fact, the model-implied correlations appear to be slightly too high. One possible explanation is that the smoothing procedure simply eliminates a small amount of yield-specific idiosyncratic variation. Whether this is desirable or not depends on one's interpretation of these errors as arising from mismeasurement, yield discreteness, or temporary market pressures. In any case, by eliminating these sorts of i.i.d. errors, correlations should rise but covariances should not. The final column of Table 5 indicates this is the case, as covariances using most of the model-implied 1-year swap rates are much closer to the actual covariances than were the corresponding correlations.

6 Conclusion

The methods presented in this paper represent a simple attempt to create an improved data set that will be of use to future researchers looking at the swap and LIBOR markets. Relative to common practices, these methods appear to offer a large reduction in the errors in estimated swap/LIBOR term structures. These reductions appear to offer significant advantages in being able to correctly characterize the relations between short-term and long-term yields, and we believe this will be a substantial benefit in estimating dynamic term structure models.

Bootstrapped zero coupon yield curves constructed using these estimates can be downloaded from <http://www-rcf.usc.edu/~christoj/research.html>.

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Table 1
Size of $|\Delta r|$ conditional on $|\Delta r| \neq 0$
1988-1990

Move size	Frequency
$ \Delta r < 6.25\text{bp}$.025
$ \Delta r = 6.25\text{bp}$.660
$6.25\text{bp} < \Delta r < 12.5\text{bp}$.009
$ \Delta r = 12.5\text{bp}$.214
$12.5\text{bp} < \Delta r < 18.75\text{bp}$.002
$ \Delta r = 18.75\text{bp}$.047
$18.75\text{bp} < \Delta r $.043

Table 2
Performance in predicting actual changes in the one-year swap rate, 1997-2002

#	Method	Bias*	RMSE*	MAE*	Correlation	$p(\hat{\epsilon} > 2\delta)$
Model-free estimates						
1	Same day's morning value	0.00	4.45	3.23	0.567	0.789
2	Next day's morning value	0.00	4.67	3.03	0.523	0.741
3	Average of same and next day mornings	0.00	3.39	2.40	0.724	0.712
Models estimated from 1988-2002						
4	Basic swap/LIBOR model	0.00	1.53	1.05	0.952	0.380
5	Swap/LIBOR model with stochastic volatility	0.00	1.46	1.05	0.954	0.410
6	Swap/LIBOR model with discreteness correction	0.00	1.55	1.07	0.950	0.401
Models estimated from 1997-2002						
7	Basic swap/LIBOR model	0.00	1.48	1.03	0.953	0.394
8	Swap/LIBOR model with stochastic volatility	0.00	1.44	1.01	0.956	0.391
9	Swap/LIBOR model with discreteness correction	0.00	1.50	1.05	0.952	0.399
Models re-estimated annually						
10	Basic swap/LIBOR model	0.00	1.47	1.02	0.954	0.381
11	Swap/LIBOR model with stochastic volatility	0.00	1.48	1.05	0.954	0.418
12	Swap/LIBOR model with discreteness correction	0.00	1.52	1.06	0.950	0.398

* in basis points

Table 3
Pairwise t-statistics for RMSE-based forecast superiority

Negative t-statistics indicate the superiority of the method given on the left over alternatives that are identified by the numbers at the top of each column. Tests follow the Diebold and Mariano (1995) procedure by testing for significance of the mean of

$$\hat{e}_{i,t}^2 - \hat{e}_{j,t}^2,$$

where i refers to the row number and j the column number. Test statistics use heteroskedasticity-adjusted standard errors calculated according to White (1980).

Model-free estimates							
1	Same day's morning value						
2	Next day's morning value						
3	Average of same and next day mornings						
Models estimated from 1988-2002		1	2	3	4	5	6
4	Basic swap/LIBOR model	-14.41	-10.35	-10.80		1.69	-1.53
5	Swap/LIBOR model with stochastic volatility	-14.87	-10.42	-11.18	-1.69		-2.39
6	Swap/LIBOR model with discreteness correction	-14.43	-10.36	-10.86	1.53	2.39	
Models estimated from 1997-2002		1	2	3	7	8	9
7	Basic swap/LIBOR model	-14.69	-10.54	-11.36		2.82	-2.59
8	Swap/LIBOR model with stochastic volatility	-14.83	-10.52	-11.36	-2.82		-3.66
9	Swap/LIBOR model with discreteness correction	-14.66	-10.52	-11.32	2.59	3.66	
Models re-estimated annually		1	2	3	10	11	12
10	Basic swap/LIBOR model	-14.74	-10.47	-11.25		-0.88	-5.10
11	Swap/LIBOR model with stochastic volatility	-14.79	-10.42	-11.19	0.88		-2.63
12	Swap/LIBOR model with discreteness correction	-14.59	-10.39	-11.05	5.10	2.63	

Table 4
Pairwise t-statistics for MAE-based forecast superiority

Negative t-statistics indicate the superiority of the method given on the left over alternatives that are identified by the numbers at the top of each column. Tests follow the Diebold and Mariano (1995) procedure by testing for significance of the mean of

$$|\hat{e}_{i,t}| - |\hat{e}_{j,t}|,$$

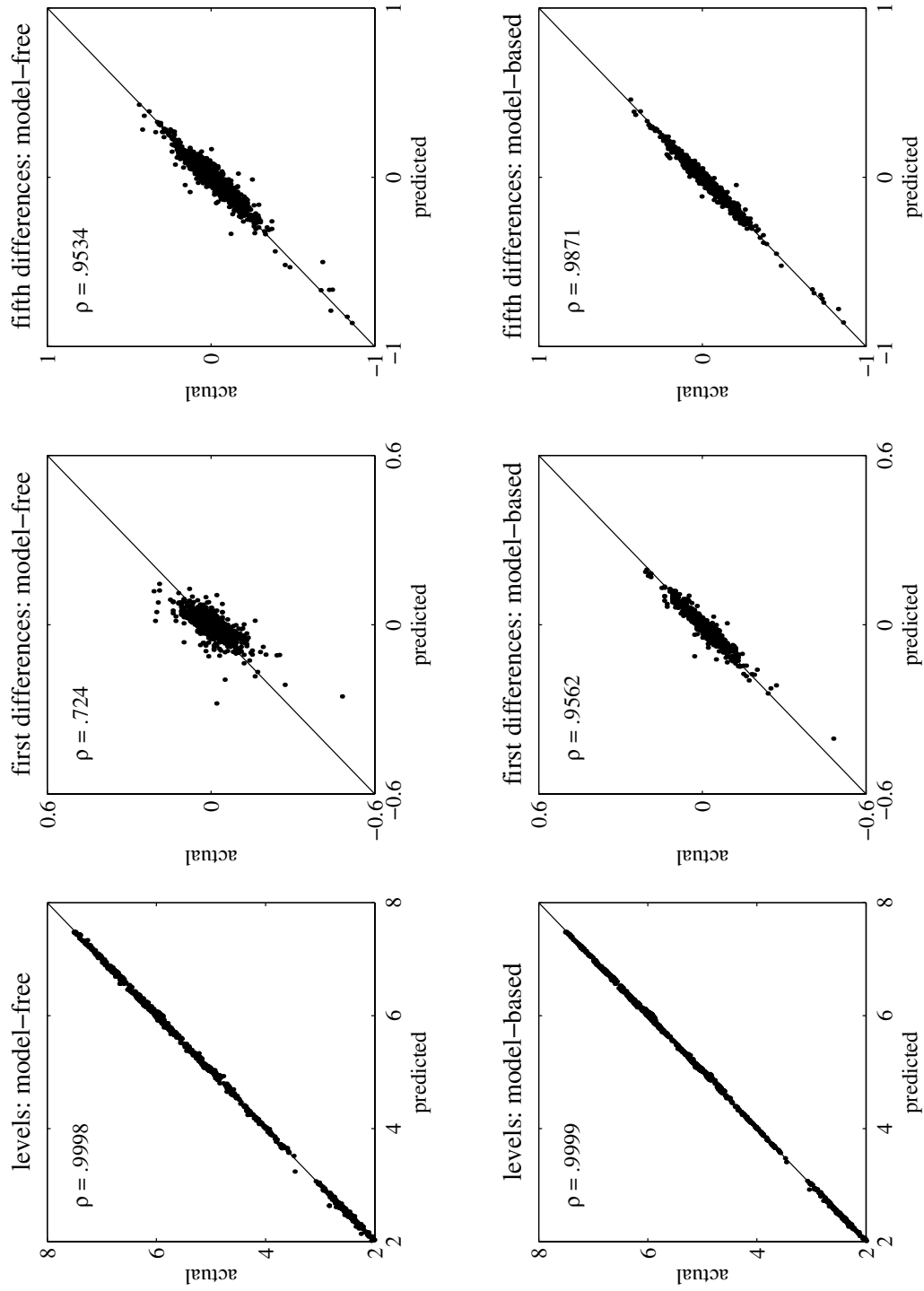
where i refers to the row number and j the column number. Test statistics use heteroskedasticity-adjusted standard errors calculated according to White (1980).

Model-free estimates							
1	Same day's morning value						
2	Next day's morning value						
3	Average of same and next day mornings						
Models estimated from 1988-2002		1	2	3	4	5	6
4	Basic swap/LIBOR model	-26.65	-22.12	-21.87		-0.07	-2.71
5	Swap/LIBOR model with stochastic volatility	-27.97	-22.35	-23.04	0.07		-1.45
6	Swap/LIBOR model with discreteness correction	-26.59	-21.86	-21.58	2.71	1.45	
Models estimated from 1997-2002		1	2	3	7	8	9
7	Basic swap/LIBOR model	-27.65	-22.61	-23.40		3.66	-2.02
8	Swap/LIBOR model with stochastic volatility	-28.12	-22.85	-23.64	-3.66		-3.96
9	Swap/LIBOR model with discreteness correction	-27.54	-22.42	-23.11	2.02	3.96	
Models re-estimated annually		1	2	3	10	11	12
10	Basic swap/LIBOR model	-27.71	-22.73	-23.50		-3.55	-5.40
11	Swap/LIBOR model with stochastic volatility	-27.76	-22.17	-23.07	3.55		-0.82
12	Swap/LIBOR model with discreteness correction	-27.14	-22.24	-22.74	5.40	0.82	

Table 5
Correlations and covariances of actual and predicted changes in 1-year
swap rates with actual changes in the 10-year swap rate, 1997-2002

	Correlation	Covariance
Actual 1-year swap rate	0.732	23.03
Model-free estimates		
1 Same day's morning value	0.377	11.21
2 Next day's morning value	0.383	11.39
3 Average of same and next day mornings	0.504	11.30
Models estimated from 1988-2002		
4 Basic swap/LIBOR model	0.802	25.57
5 Swap/LIBOR model with stochastic volatility	0.770	23.33
6 Swap/LIBOR model with discreteness correction	0.792	24.77
Models estimated from 1997-2002		
7 Basic swap/LIBOR model	0.791	23.44
8 Swap/LIBOR model with stochastic volatility	0.786	23.31
9 Swap/LIBOR model with discreteness correction	0.788	23.29
Models re-estimated annually		
10 Basic swap/LIBOR model	0.786	23.43
11 Swap/LIBOR model with stochastic volatility	0.778	22.92
12 Swap/LIBOR model with discreteness correction	0.779	23.18

Figure 1



Model-free predictions are averages of the same day's and the next day's morning LIBOR quotes. Model-based predictions use the 1997-2002 estimates of the basic swap/LIBOR model.