Asset prices and time-varying persistence of consumption growth*

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\textbf{ABSTRACT}

Inspired by numerous results in the macro literature, we consider a model in which the correlation between shocks to consumption and to expected future consumption growth is nonzero and varies over time. Combined with a negative correlation between consumption growth and uncertainty shocks, representing a precautionary savings effect, our model embeds time variation in consumption growth persistence, which drives variation in the correlation between stock and bond returns, the volatility of stock returns, the so-called “leverage effect,” and the predictive relation between bond yields and future stock returns. Our empirical analysis finds strong support for all of these predictions.

\textit{Keywords:} Consumption persistence, long-run risk, stock/bond correlation, leverage effect

\textsuperscript{*}We thank seminar participants at Korea University and USC. All errors are our own.
I. Introduction

When consumers reduce spending, is this an indication of a higher or a lower future consumption growth rate? Whether such consumption shocks are persistent or anti-persistent is a difficult question, in part because competing theories have opposite predictions. The well-known challenge of measuring expected consumption growth (e.g., Schorfheide, Song, and Yaron (2018)) makes an empirical answer to the question elusive as well.

Nevertheless, the question is important, because a nonzero correlation has significant implications for asset prices. Using the long-run risk (LRR) framework of Bansal and Yaron (2004), we show that a positive correlation between consumption shocks and future growth rates lowers the correlation between bond and stock returns, implying that consumption growth will be more persistent when the stock/bond correlation is low. Positively correlated shocks to two main drivers of equity values – current and expected future cash flows – further lead to a higher volatility of stock returns. This correlation also affects the degree with which bond yields predict future stock returns.

We devise an empirical strategy that uses higher frequency asset price data to overcome some of the challenges in analyzing a conditional correlation involving the latent expected consumption growth process. When we take the model predictions to the data, we find broad support. In particular, we show that the stock/bond return correlation does appear to proxy for the correlation between shocks to current consumption and its future growth rate. Furthermore, the stock/bond correlation is related to stock market volatility and the predictability of stock returns in the ways predicted by our model.

In our model, consumption persistence has two sources. One is the slow variation in the expected consumption growth process, as postulated by Bansal...
and Yaron (2004). This process generates a moderately positive short-horizon autocorrelation that decays very gradually with the horizon. The other is the correlation between current consumption shocks and expected consumption growth shocks, which we refer to for brevity as consumption shock persistence or CSP. By including both sources, it is possible to induce time variation in consumption growth autocorrelation while maintaining the long-run positive autocorrelation that is critical for matching the moments of asset returns.

Macroeconomic models provide many reasons why CSP is unlikely to be zero. Transient shocks to productivity (Kaltenbrunner and Lochstoer 2010), to income (Hall and Mishkin 1982), or to uncertainty (Basu and Bundick 2017) can all drive consumption higher in the short run, while at the same time decreasing long-term consumption growth. A negative CSP is thus a natural outcome of the mean-reverting nature of these shocks.

In contrast, models that incorporate permanent shocks generally imply a positive CSP, as is the case in the production economy of Kaltenbrunner and Lochstoer (2010) when permanent shocks are specified. Separately, in the macro literature on the income/consumption relation, Campbell and Deaton (1989) find that consumption underreacts to permanent income shocks. In both cases, the positive CSP is the result of frictions that induce gradual adjustment to permanent shocks.

The empirical macro literature is abundant with evidence that both permanent and transitory shocks are necessary to explain observed patterns of persistence. Friedman (1957) observes that income and consumption likely contains both permanent and transitory components, an idea formalized by Beveridge and Nelson (1981), Watson (1986), and Clark (1987), among others. Furthermore, it is likely that the relative importance of these shocks varies over time, as there appear to be multiple sources of uncertainty that affect macroe-
conomic and financial variables to different degrees (Jurado, Ludvigson, and Ng 2015). Intuitively, then, if the most volatile shocks are transitory (e.g., pandemics, oil shocks), CSP will become negative. When permanent shocks (e.g., technology, climate change) dominate, CSP turns positive.

Our model, building on Bansal and Yaron (2004), is a stylized way to capture the net effect of these mechanisms. While preserving the core feature of long-run risk – the phenomenon that consumption growth at long horizons is positively autocorrelated – our model allows for the possibility that consumption growth may in some environments display mean reversion, which is an expected outcome when macro risks are transient. By allowing for additional flexibility in the persistence of consumption growth, our model is able to explain some conditional moments that the standard LRR model cannot.

By including this additional flexibility our model generates a number of new predictions. One is that CSP should be an important driver of the correlation between the returns on stocks and bonds. This is straightforward: Changes in expected consumption growth drive interest rates as the result of intertemporal smoothing, while changes in realized consumption growth affect cash flows. A positive CSP therefore gives rise to a higher correlation between interest rates and cash flows, resulting in a lower (and likely negative) correlation between bond and stock prices. Our model therefore predicts that consumption growth autocorrelation should be decreasing with the stock/bond correlation.

The model also implies that a higher CSP will raise stock market volatility. This is because current and expected future cash flows are two primary drivers of equity valuation. When shocks to both are positively correlated, their effects will be amplified, and market volatility will rise. The testable prediction

\footnote{Shocks to expected consumption growth have both cash flow and discount rate effects, which affect equities with opposite signs. Our calibration strongly implies that the cash flow effects dominate.}
is that market volatility is decreasing in the stock/bond return correlation.

An additional feature of our model is the incorporation of a negative correlation between shocks to consumption and consumption volatility. Like CSP, allowing this correlation to be nonzero is an attempt to reconcile the standard LRR model with observations from the macro literature. Specifically, this correlation is implied by the presence of a precautionary savings motive, which is found in a number of studies, including Carroll (1997) and Basu and Bundick (2017). The direct effect of this correlation is to strengthen the unconditionally negative correlation between stock returns and volatility changes, or what the literature refers to as the “leverage effect.”

A nonzero CSP magnifies the effect of the correlation between consumption shocks and volatility by linking volatility shocks to changes in expected future consumption growth. Since a higher CSP also implies a lower stock/bond return correlation, the model implies that the strength of the predictive relation between volatility and future consumption growth will vary with the stock/bond correlation. This mechanism also implies that the stock market leverage effect will be amplified (i.e. made more negative) by a higher CSP. An additional prediction of our model is therefore that the stock/bond correlation and the stock market leverage effect should be positively related.

Finally, our model has implications for the predictive relation between interest rates and future stock market returns. Empirically, the weakness of such a relation presents something of a puzzle, as it is predicted by most macrofinance models. This is the case in our model as well, except that the strength of the relationship also depends on CSP. Since bond yields are closely related to the level of expected consumption growth, they are more strongly related to consumption volatility – which drives the market risk premium – when CSP is high. When CSP is negative, more model implies that yields should have
little predictive power for future market returns, which could explain the weak unconditional relationship. The testable prediction is that yields will predict market returns more strongly when the stock/bond return correlation is low.

Our empirical analysis begins by showing that consumption growth autocorrelations are decreasing with the stock/bond return correlation. This justifies the use of the return correlation, which can be measured at high frequency using daily data, as a proxy for CSP.

We then turn to the strong link between CSP and the stock market leverage effect, which measures the relation between stock returns and volatility changes. This link implies that the stock/bond return correlation and the leverage effect should be positively related, which we confirm in the data. This also suggests that the leverage effect should be able to proxy for time-varying persistence in consumption growth. It appears to do this as well, though with lower significance.

In LRR models, expected consumption growth is closely linked to the real interest rate. Thus, a higher CSP is implied by a higher correlation between interest rates and consumption growth. Empirically, we find that lower stock/bond correlations imply a stronger relation between consumption and interest rate shocks, which is additional evidence of the relation between the stock/bond correlation and CSP.

CSP should also be positively related to market volatility. Specifically, our model implies that the stock/bond correlation should have a negative interactive effect with macroeconomic uncertainty. We confirm this relation using the macro uncertainty measure of Jurado, Ludvigson, and Ng (2015), the monetary policy uncertainty measure of Baker, Bloom, and Davis (2016), and a simple measure of consumption growth variance.
In addition to finding that consumption is more autocorrelated when the stock/bond correlation is low, we also find that volatility is more predictive of future consumption growth in the same low correlation environments. This is consistent with our prediction that CSP modulates the correlation between shocks to volatility and expected future consumption growth.

Our final set of results demonstrate that the stock/bond correlation is an important state variable for the predictive relation between yields and future stock returns. Unconditionally, this predictive relation is weakly negative, as documented in prior work. However, as the stock/bond correlation drops, the negative relation strengthens. The insignificant unconditional relation, which is at odds with the predictions of many models, is therefore a natural result of the stock/bond correlation being positive over much of our sample.

In the next section we describe and calibrate our model. Section III describes our data and strategies for measuring latent processes. Section IV presents our empirical results, while Section V concludes.

II. The model

1. Consumption growth dynamics

Our model is a generalization of the now-standard framework of Bansal and Yaron (2004). In our baseline specification, the representative agent has Epstein and Zin (1991) preferences, and consumption growth ($\Delta c_{t+1}$) has a persistent time-varying component $x_t$ and time-varying uncertainty $\sigma_t^2$:

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \epsilon_{c,t+1}$$

$$x_{t+1} = p_t x_t + \phi_x \sigma_t \epsilon_{x,t+1}$$

$$\sigma_{t+1}^2 = s_0 + s_1 \sigma_t^2 + \sigma_\sigma \sigma_t \epsilon_{v,t+1}$$
Under the assumption that the three shocks in the model are uncorrelated, Bansal and Yaron (2004) show that the wealth-consumption ratio is linear in \(x_t\) and \(\sigma_t^2\). Similarly, in our baseline model we have
\[
z_t = A_0 + A_1 x_t + A_2 \sigma_t^2,
\]
where the constants \(A_0\) and \(A_1\) are positive, while \(A_2\) is negative. As is standard, valuations are therefore raised by greater expected consumption growth and reduced, via a discount rate effect, by higher volatility.

Our generalized models deviate from Bansal and Yaron in several important dimensions. First, we allow shocks to consumption growth (\(\epsilon_{c,t+1}\)) and expected long-run consumption growth (\(\epsilon_{x,t+1}\)) to be stochastically correlated. We refer to this correlation as consumption shock persistence, or CSP, given that it determines whether a shock to current consumption is associated with higher or lower consumption growth in the future. This correlation, which we denote as \(\rho_t\), follows a stochastic process that will be specified below.

As discussed in the introduction, allowing for a stochastic correlation can be viewed as a reduced form approach to modeling time variation in the relative importance of permanent and transitory shocks. As an example, in the production economy of Kaltenbrunner and Lochstoer (2010), the assumption of permanent productivity shocks results in a positive CSP, while transitory shocks generate a negative CSP. This is the result of differences in how investment (and therefore consumption) responds to changing productivity, and also about how adjustment costs and mean reversion induce trends in future output. Given our view that both types of shocks are likely, either effect could dominate depending on which type of shock is currently more volatile. Furthermore, this phenomenon is not limited to shocks to productivity. Permanent and transitory shocks to income generate similar responses, as discussed, for example, by Hall and Mishkin (1982) and Campbell and Deaton (1989).
We also deviate from Bansal and Yaron by allowing consumption shocks to be correlated with consumption variance shocks. A negative correlation is a natural result of a precautionary savings motive, which has been confirmed empirically in a number of studies, including Carroll and Samwick (1998) and Basu and Bundick (2017). We assume this correlation, denoted $\varrho_{ps}$, is constant.

Finally, given that consumption shocks are correlated with shocks to expected growth rates ($\rho_t$) and to consumption volatility ($\varrho_{ps}$), it is natural that expected consumption growth and consumption volatility would be correlated as well. For example, an increase in precautionary savings that is induced by greater uncertainty should reduce current consumption, as households increase their savings, but should also lead to a rise in expected long-run consumption growth as uncertainty wanes and consumption returns to normal. Empirically, a nonzero correlation between $\sigma_t$ and $x_t$ is found by Nakamura, Sergeyev, and Steinsson (2017), who show that it tends to be more negative during economic contractions. In other work, Parker and Preston (2005) find significant evidence, using household survey data to measure the relative importance of precautionary savings, that the precautionary motive explains the predictable component of consumption growth. In the interest of parsimony, we avoid introducing unnecessary additional parameters by assuming that this correlation between shocks to consumption volatility and expected consumption growth is equal to the product $\rho_t \varrho_{ps}$.

Closing the model requires a specification of the dynamics of consumption shock persistence, or $\rho_t$. To obtain closed form solutions, we parameterize the covariance, rather than correlation, between $\epsilon_{c,t}$ and $\epsilon_{x,t}$ as an autoregressive process. The covariance, $p_t = \text{Cov}_t (\epsilon_{c,t+1}, \epsilon_{x,t+1})$, follows

$$p_{t+1} = \omega_0 + \omega_1 p_t + \sigma_p \sigma_t \epsilon_{p,t+1},$$

(3)

where, for simplicity, we assume that $\epsilon_{p,t+1}$ is uncorrelated with other shocks.
Given the same preference assumptions as Bansal and Yaron, the price-to-consumption ratio $z_t$ can be represented as a linear function of long-run expected consumption growth ($x_t$), the variance of consumption growth ($\sigma_t^2$), and the leverage covariance ($p_t$). That is,

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 + A_3 p_t.$$  

(4)

In the appendix, we derive the values for $A_0$, $A_1$, $A_2$, and $A_3$. Under conventional parameter assumptions ($\gamma > 1$ and $\psi > 1$), we find that $A_1 > 0$ and $A_2 < 0$, which is consistent with the model of Bansal and Yaron and with our own baseline specification. In addition, we find that $A_3 < 0$, implying that the price-consumption ratio is lower when CSP is higher.

We refer to the above specification as the “consumption-only” model given that is solely describes the dynamics of consumption. As is standard, we extend this model further by adding a dividend process. In this model, which we label as the “full” model, dividend growth is specified as

$$\Delta d_{t+1} = \mu_d + \phi_d x_t + \sigma_i \varphi_{cd} \epsilon_{c,t+1} + \sigma_d \varphi_{d} \epsilon_{d,t+1},$$

(5)

where $\epsilon_{d,t+1}$ is assumed to be uncorrelated with other shocks. Dividend growth therefore shares similarities with consumption because of its dependence on the long-run growth process $x_t$, and also because of its sensitivity to the consumption shock $\epsilon_{c,t}$. The strength of this commonality is determined by the values of $\phi_d$ and $\varphi_{cd}$ relative to the volatility of dividend-specific shocks, which is determined by $\varphi_d$.

We define the return on the market portfolio as

$$R_{m,t+1} = \kappa_0 + \kappa_1 z_{m,t+1} - z_{m,t} + \Delta d_{t+1},$$

(6)

where $z_m$ is the price-dividend ratio of that portfolio and where $\kappa_0$ and $\kappa_1$ are constants of the log linearization. Similar to the wealth-consumption ratio, we can verify the conjecture that the price-dividend ratio is a linear function of
the three state variables. We show in the appendix that the signs of the four coefficients match those of equation (4).

2. Calibration

We perform a calibration of the model in order to examine its quantitative implications. In doing so, our priority is match the parameters of Bansal and Yaron (2004) as closely as possible. In our baseline model, in which correlations are all zero, most parameters are equal to their counterparts in that paper. The only exception is the volatility-of-variance parameter $\sigma_v$, which is different because of a difference in how we specify the volatility of the $\sigma_t^2$ process. We set this parameter to the value that equates the unconditional volatility-of-variance with the constant value assumed by Bansal and Yaron.

The generalized specifications require, in addition, several correlation parameters. These include the three parameters of the consumption shock persistence ($p_t$) process in equation (3), the constant correlation parameter $\psi_{ps}$, as well as two parameters ($\varphi_c$ and $\varphi_{cd}$) that determine the correlation between consumption and dividend shocks. Because of the difficulty in estimating unconditional means, we assume that the mean of the $p_t$ process is zero, which implies that $\omega_0 = 0$. The slope ($\omega_1$) and volatility ($\sigma_p$) parameters are chosen to match the volatility and first-order serial correlation of the covariances between stock returns and bond yields. This is justified by the strong relation between these covariances and the $p_t$ process, as shown in the following section.

For our primary specification, we set the precautionary savings parameter $\psi_{ps}$ equal to $-0.3$ following the results of Basu and Bundick (2017), but we also show results for two alternative specifications by setting $\psi_{ps}$ equal to $-0.2$ and $-0.5$. We match the relative values of $\varphi_c$ and $\varphi_{cd}$ to the empirical correlation between dividend growth and stock market variance under the constraint
that the combination of the total volatility of dividend shocks matches that of Bansal and Yaron.\footnote{One constraint of our model, in the interest of parsimony, is the zero correlation assumption between $\epsilon_{d,t+1}$ and $\epsilon_{x,t+1}$. This assumption may be somewhat counterfactual, as it results in excess correlation between consumption and dividend growth.} The assumed parameters are summarized in Panel A of Table I.

Panel B compares the asset moments generated by our three specifications. These include the baseline model, in which correlations are set to zero, the consumption-only model, in which the total wealth portfolio is assumed to be the stock market portfolio, and the full model, which incorporates a dividend process. For each specification, we generate 1 million observations and evaluate the first two moments of stock and bond returns, as well as several other relevant asset pricing moments. The moments generated by the simulations are generally comparable to that of other standard long-run risk models.

3. Stock/bond return correlations

In this section we show that consumption shock persistence has strong implications for the contemporaneous relationship between stock and bond returns (the SB correlation), which suggests a new explanation for why the SB correlation may vary over time. For our purposes, it is also important because its close relation with consumption shock persistence implies that it is a good proxy for $\rho_t$, particularly since it can be computed as long as bond and stock returns are available.

To understand how the SB correlation is related to each shock, it helps to first understand how bond yields, stock returns, and stock variances are affected. Here, we explain these relationships, which are summarized in Table I.

We first consider the two channels that drive SB correlations under the baseline model, in which $\rho_t$ is assumed to be zero. The first is through shocks
to the expected consumption growth ($\epsilon_{x,t+1}$). If this shock is positive, then higher expected future cash flows will lead to higher stock returns. Bond yields will also increase, as the demand for money rises through the intertemporal consumption smoothing motive. Stock and bond returns will therefore have opposite responses, implying a negative SB correlation.

The second channel is from shocks to consumption growth uncertainty ($\epsilon_{v,t+1}$). When there is a positive uncertainty shock, the stock market return variance will rise, which raises risk premia and lowers valuations. At the same time, bond yields will drop as higher higher consumption risk induces households to reduce their holdings of risky assets and replace them with riskless bonds. This increase in precautionary savings leads to a decrease in bond yields or an increase in bond prices.

Panel C of Table I shows that the first channel is much stronger than the second. The correlation between yield changes and shocks to expected consumption growth is around 0.98. In contrast, the correlation between yield changes and volatility shocks is just -0.18. Importantly, both channels both imply a negative correlation between bond and stock returns, since lower yields imply higher bond returns.

While our generalized models exhibit the same negative SB correlation on average, consumption shock persistence causes this correlation to vary over time. For example, suppose there is a positive expected consumption shock ($\epsilon_{x,t+1} > 0$) during a period in which $\rho_t$ is negative. This positive shock is therefore likely to coincide with a decline in current consumption growth. In this case, bond yields will increase as the economy expects higher growth, while the negative shock to current consumption will push equity values lower. While the net effect may nevertheless be that equity values rise, due to higher expected long run growth, the rise will be moderated by the negative shock to
current consumption. Negative $\rho_t$ will therefore lead to a SB correlation that is less negative than usual, perhaps even slightly positive.

The stock/bond return correlation is also affected by the negative relationship between uncertainty shocks ($\epsilon_{v,t+1}$) and consumption shocks ($\epsilon_{c,t+1}$), which captures a precautionary savings motive. With a positive uncertainty shock, this negative relation will cause current consumption to decrease as investors save more in response to the greater uncertainty. Because positive uncertainty shocks ($\epsilon_{v,t+1} > 0$) and negative current consumption shocks ($\epsilon_{c,t+1} < 0$) both reduce equity values, the negative stock return is amplified relative to the model without a precautionary savings motive. This is observed in Panel C of Table I, which shows a much more negative correlation between uncertainty shocks and market returns for the two generalized models. Because bond yields are also negatively affected by higher uncertainty, precautionary savings effect makes the SB correlation lower on average.

Furthermore, consumption shock persistence can magnify the precautionary savings effect. When $\rho_t$ is positive, for instance, then a positive uncertainty shock and a negative shock to current consumption will also be associated with lower expected future consumption growth. The latter effect amplifies the decline in equity values implied by the first two shocks, implying an even more strongly negative stock return. Meanwhile, the effect of the positive uncertainty shock on bond yields is also amplified by the reduction in expected future consumption growth, as both effects cause bond yields to fall. The implication is that a precautionary savings effect will make the SB correlation even more negatively related to $\rho_t$, though below we show that the magnitude of this effect is likely small.

The first two panels of Figure 1 shows how the SB correlation varies as a function of the model state variables. Panel (a) shows the relationship be-
between the SB correlation and the $\rho_t$ process for the consumption-only model, while Panel (b) presents corresponding results for the full model. In both, we show correlations for several different fixed levels of the precautionary savings parameter $\varrho_{ps}$. For comparison, each panel includes a flat line indicating the constant correlation that is obtained under the baseline model, in which current and expected consumption growth shocks are uncorrelated. It is worth noting that the baseline does not match the empirical observation that the SB correlation is time varying.

We derive these relationships using closed form solutions for the variances of stock returns and bond yields as well as for the covariance between the two. We show in the appendix that all three may be expressed as linear combinations of $\sigma^2_t$ and $p_t$. Furthermore, both the covariance and the two variances are increasing in consumption growth volatility and in $p_t$. The stock/bond return correlation, however, is a univariate function of just $\rho_t$. Whether this function is increasing or decreasing in $\rho_t$, however, is not straightforward and must be addressed using our calibrated model.

The figure confirms the negative relation between $\rho_t$ and the SB correlation. In both panels, one for the consumption-only model and one for the full model, the SB correlation is slightly convex in $\rho_t$, and in both cases the value of the precautionary savings parameter $\varrho_{ps}$ has relatively little effect. Lastly, while low values of consumption shock persistence are associated with positive SB correlations in both models, positive SB correlations are rarer in the full model.

This relationship is also shown in the simulation result in Panel D of Table I, which examines the “correlation of correlations.” While the figure shows that

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3For example, Connolly, Stivers, and Sun (2005) and Baele, Bekaert, and Inghelbrecht (2010) report a negative relationship between SB correlations and stock market uncertainty. Campbell, Pflueger, and Viceira (2020), among others, report a decreasing trend in SB correlations.
the relation between $\rho_t$ and the SB correlation is slightly nonlinear, the table is useful in that it assesses the goodness of fit of a linear projection of the SB correlation onto $\rho_t$. For both the consumption-only model and the full model, the relationship between $\rho_t$ and SB correlation is extremely negative, with correlations below $-0.98$. Thus, our model suggests that the SB correlation is a very good proxy for the less easily observed $\rho_t$ process.

4. Stock market volatility and the leverage effect

The negative relationship between stock returns and changes in stock volatility, referred to in the literature simply as the “leverage effect,” is found in virtually all equity markets and sample periods. While robust, the relationship is nevertheless time-varying, as demonstrated by Pyun (2019). In this section, we examine the relationship between consumption shock persistence and the leverage effect, which we measure as the slope coefficient of the regression of stock returns on shocks to stock return variance.

The relation between consumption persistence and stock market leverage is intuitive in our model. When there is a shock to consumption variance, the direct effect is to raise market volatility, which increases discount rates. Due to the precautionary savings effect, this rise in volatility is also associated with a decline in current consumption. Market prices therefore decrease due both to higher discount rates and lower cash flows.

If, in addition, CSP is positive, then positive volatility shocks are also likely to be associated with an decrease in expected future consumption growth. This decline in the growth rate of future cash flows depresses stock values further, which has two implications.

One implication is that it will raise stock return variances. Stock returns are both positively related to current and expected future consumption growth
shocks. When the two shocks are positively correlated, then their risks are magnified and stocks become riskier. When the two shocks are negatively correlated, then market risk falls due to a hedging effect. Thus, the model implies that the variance of stock returns will increase with CSP. This is formalized with an analytical result, which we prove in the appendix, which is that

$$\text{Var}_t(R_{m,t+1}) = V_2 \sigma_t^2 + V_3 \rho_t,$$

(7)

where $V_2, V_3 > 0$.

The second implication is that a positive CSP will strengthen the negative relationship between stock market returns and volatility shocks. This is because volatility shocks affect cash flows both through their relation with current consumption, and also because of their relation with future consumption growth. On the other hand, when CSP is negative, then a positive volatility shock is more likely to be associated with higher expected future consumption growth. During these periods, stock returns will react less to the same volatility shock, decreasing the magnitude of the leverage effect.

The last two panels of Figure 1 shows the relationship between consumption shock persistence and stock market leverage, which is defined as the time-varying slope coefficient of a regression of market returns on volatility shocks. For this figure, we use exact formulas that are provided in the appendix. Results for the consumption-only model and our full specification are provided in Panels (c) and (d), respectively. For comparison, we again include flat lines indicating the values obtained under the baseline model, which produces a leverage effect that is negative and constant.

The figure shows that stock market leverage is negatively related to CSP. This relationship is essentially linear, and it implies a perfect correlation between the leverage effect and $\rho_t$, as we report in Panel D of Table 1. The
figure also shows that the leverage effect is sensitive to the value chosen for
the precautionary savings parameter $\varrho_{ps}$, as the intuition above suggests.

The three models display large differences in the average level of the leverage
effect. Panel B of Table I shows estimates of the leverage effect that are
obtained from simulating each model. These are estimates of the unconditional
leverage effect, but they are close to the average of the conditional values. The
table also shows the value estimated in the data using a procedure we describe
in Section I. In comparing the model-implied values with an estimate from the
data, we can see that the baseline and consumption-only models drastically
overstate the size of the leverage effect. The full model, which introduces a
dividend process with its own error term, reduces the average leverage effect
so that it is only moderately larger than the one estimated from the data.

5. The conditional moments of consumption growth

The key assumption of our generalized model is that consumption growth
shocks have time-varying persistence. In this section we address how this
assumption affects the conditional distribution of consumption growth for dif-
ferent values of $\rho_t$, with the aim of formulating empirical predictions.

While greater consumption shock persistence will clearly increase the serial
correlation in consumption growth, it is difficult to assess the strength of this
and other relations analytically. We therefore simulate 10 million months of
data from our full model and compute approximate conditional moments by
separating the simulated sample into narrow bins (e.g., [-0.05, 0), [0, 0.05),
[0.05, 0.1), etc.) according to the value of $\rho_t$. We then compute the variable
of interest (e.g., first-order autocorrelation) using all the observations in each
bin. Our model simulation is monthly, but to facilitate comparison with later
empirical results we aggregate to the quarterly level by taking the sum of three
consecutive realizations of the consumption growth process.

Panel (a) of Figure 2 shows the relationship between consumption growth and contemporaneous shocks to expected future consumption growth. As assumed in our model, there is a positive relationship between the two, and the plot serves only to quantify the effect. For example, our figure suggests that when \( \rho_t \) is at the first quartile (−0.14), one standard deviation shock to \( x_t \) implies a −0.1% consumption growth shock.

Panel (b) shows how the first-order serial correlation of consumption growth relates to consumption shock persistence. Similar to Panel (a), the consumption growth process is aggregated to the quarterly level, and we examine serial correlation in quarterly growth rates. As expected, serial correlation is positive on average, which is due to the presence of the long-run risk process, and rises with the level of \( \rho_t \). It is notable that even very negative values of \( \rho_t \) nevertheless imply conditionally positive serial correlation.

Because we have assumed that the correlation between expected consumption growth and volatility shocks is equal to \( \rho_t \varphi_{ps} \), our model implies that this correlation will be more negative when CSP is high. Panel (c) of Figure 2 shows that the same relation also holds in levels. That is, the level of expected future consumption growth is more negatively related to the level of consumption variance when CSP is high, where we measure the relation by the slope coefficient of the regression of \( x_t \) on \( \sigma_t^2 \). We examine levels here in order to be consistent with our empirical analysis, where first differences of observable proxies for \( x_t \) on \( \sigma_t^2 \) are likely to be dominated by measurement error.

6. Stock return predictability of bond yields

Most consumption-based asset pricing models imply that bond yields should negatively predict future stock returns. This is a result of stock risk premia
being increasing in consumption volatility, which will also lower bond yields
due to precautionary savings. These relationships imply that when bond yields
are lower, the equity risk premium should be higher.

However, there is at best weak empirical evidence for such a relationship.
While a number of studies starting with Fama and Schwert (1977) find a neg-
ative relation between stock returns on lagged bond yields, the negative rela-
tionship appears sample-dependent. Also, as evidenced by Welch and Goyal
(2008), for example, the statistical significance is well below that of other
predictors such as the aggregate dividend yield.

The final implication of the model is that the strength of this form of
stock market return predictability depends on the relationship between current
and expected future consumption growth. Bond yields are the inverse of the
expected marginal utility of the investors, which is closely related to the level
of the expected consumption growth. Meanwhile, the stock risk premium
is higher when volatility is higher. Therefore, when expected consumption
growth is more negatively related to volatility, the relationship between bond
yields and stock risk premia should become more strongly negative.

Given that the SB correlation and the stock market leverage betas are
each increasing in consumption shock persistence, we expect a more negative
predictive relationship between future stock returns and bond yields when
either of those correlations is low.

Using the simulations described earlier, we first examine the regression of
stock risk premia on lagged bond yields. As with other results, we examine how
the slope coefficient of this regression depends on the lagged value of \( \rho_t \). In this
analysis we make use of an exact formula for the market risk premium, which
is an increasing function of uncertainty and the current/expected consumption
growth covariance (δ), as shown in the appendix. Results based on realized excess returns would be identical save for some increase in simulation error.

Panel (d) of Figure 2 shows that for values of ρ, that are greater than −0.5, we see a negative relationship between bond yields and the market risk premium, as implied by many other asset pricing models. But whereas other models imply that the decree of predictability is constant, here it is highly time-varying. This figure suggests that bond yields should be better predictors of future stock returns when consumption shock persistence is high. Given that CSP can be proxied by either the SB correlation or the stock market leverage beta, where each relation is negative, the degree of conditional predictability should be inversely related to either of these two measures.

A more intuitive explanation of this result starts with the idea highly correlated assets are likely exposed to the same systematic risk factors. If the compensation for factor risk increases, then both assets should see higher expected returns. For bonds, the yield to maturity is the return that the investor would obtain if the bond is held until the maturity, albeit with specific assumptions on the returns to reinvestment. For stocks, no single variable encapsulates future returns in the same way, but if bond and stock returns are highly correlated, then we should be able to infer something about the expected return on bonds by looking at expected returns on bonds, as proxied by yields.

III. Empirical Results

1. CSP proxies

Direct measurement of time variation in the relation between current and expected consumption growth shocks is hampered by the difficulty in measuring the latent expected consumption growth process (e.g., Schorfheide, Song,
and Yaron (2018)) and the relatively low frequency of consumption growth rates. When consumption shock persistence is varying through time, then direct measurement is unlikely to be successful.

We therefore examine consumption shock persistence using an indirect approach that makes use of high frequency asset price data. The stock/bond return correlation is estimated as the negative of the correlation between the first-order difference in bond yields and stock returns. This estimate approximates the true stock/bond return correlation, as it ignores the effect of convexity, but it is extremely accurate. As a baseline, the correlations are estimated using a rolling basis using daily observations of the past 365 calendar days. Since the stock/bond correlations can be measured for bond maturity, we can compute several such correlation series. In this paper, we report the results of the ten-year constant maturity bonds, though using other maturities produces very similar results.

As an alternative measure, we calculate the negative of the correlation between stock returns and the first-order difference of real yields using the past 60 monthly observations. While we expect larger measurement errors in this procedure, both due to the estimation of real yields and from using fewer observations, this measure is less likely to be contaminated by any correlations between stock returns and inflation rates (e.g., Boons, Duarte, de Roon, and Szymanowska (2017)).

We estimate the stock market leverage effect from the monthly regression

\[
R_{m,t} = \beta_0 + \beta_v (\hat{\sigma}_t - \hat{\sigma}_{t-1}) + \epsilon_t, \tag{8}
\]

where \(R_{m,t}\) is the excess market return in month \(t\). \(\hat{\sigma}_t\) is the long-run volatility forecast of stock returns from the two-factor EGARCH model of Brandt and Jones (2006), measured at the end of month \(t\) and scaled to a monthly value.
The regression is estimated using a 60-month rolling window. We call the beta estimate of this regression the stock market leverage effect and denote it by $\text{Lev}_t$.

As an additional measure, we estimate the daily regression using the daily changes in the VXO Index as the independent variable. We estimate this regression using a rolling-window of 365 calendar days.

There are benefits and drawbacks of using this alternative volatility measure. One benefit is that by using daily measures of volatility, we are either able to reduce the standard errors of the estimates or use a shorter sample period. This is especially useful when the beta estimates are time-varying. One drawback is that VXO is a measure of risk-neutral volatility, which means that it contains a component driven by the volatility risk premium. A second drawback is that such measures can only be constructed starting in 1986.

### 2. The serial correlation of consumption growth

The first implication of the model is that the persistence of consumption growth shocks should be reflected in the level of the stock/bond correlation or the stock market leverage beta. This relationship is critical, because it justifies the use of the SB correlation (or the leverage beta) as an empirical proxy for the latent consumption shock persistence process.

In interpreting these results, it is important to note that short-run autocorrelations from consumption growth data are likely high due to time-aggregation effects that are absent from our theoretical model. As shown both by Breeden, Gibbons, and Litzenberger (1989) and Heaton (1993), if investors make consumption decisions more frequently than the interval over which consumption is measured, then autocorrelation in growth rates will be high, perhaps 0.25 in quarterly data. What our model suggests is that serial correlation will be
larger during periods when the SB correlation or the stock market leverage beta is more negative.

To test the hypothesis, we first estimate a predictive regression of quarterly consumption growth on its own lag. We test whether this relationship is stronger or weaker during high or low SB correlation or stock market leverage periods by adding an interactive term. The regression we estimate is

$$\Delta c_{t+1} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 R_t \times \Delta c_t + \alpha_3 R_t + \epsilon_{t+1},$$

where $\Delta c_t$ is quarterly consumption growth and where $R_t$ is either the SB correlation or the stock market leverage beta. If, as implied by our model, the serial correlation is stronger during periods when the SB correlation or the stock market leverage is negative, then we expect to see $\alpha_2 < 0$.

Panel A of Table II summarizes the results. The simple regression onto lagged consumption growth in the first column shows that past consumption growth predicts future consumption growth. As mentioned above, this is likely due to time aggregation, at least in part. Long-run risk in consumption growth also naturally leads to a positive autocorrelation in consumption growth. The $R^2$ of 0.233 is comparable to numbers reported by previous studies, for example, Savov (2011).

Our primary interest is to test the sign and the significance of the interactive coefficient, $\alpha_2$. The panel shows that the interactive coefficient is negative across all four measures considered and statistically significant in three of them. This is consistent with the prediction of the model.

We also vary the forecast horizon of the regression, replacing the one quarter-ahead dependent variable with one that is between two and ten quarters ahead. These dependent variables are non-cumulative and therefore non-overlapping. Figure 3 shows, for various horizons, the regression slope coef-
ficients on the interactive regressor $R_t \times \Delta c_t$. Across all four measures, the figures show that our findings are not just restricted to the one quarter-ahead forecast. Using the daily SB correlation as our interactive variable, predictability is observed as far as eight quarters ahead.

An alternative test exploits the high correlation implied by our model between bond yields ($y_t$) and expected consumption growth ($x_t$). This directly implies that higher consumption shock persistence will be reflected in a higher contemporaneous correlation between consumption growth and changes in yields. Given the negative relation between CSP and the SB correlation, the model therefore predicts that the $\alpha_2$ coefficient in the regression

$$\Delta c_t = \alpha_0 + \alpha_1 \Delta y_t + \alpha_2 R_t \times \Delta y_t + \alpha_3 R_t + \epsilon_{2,t}$$

will be negative. The results of this analysis are reported in Panel B.

These regressions are much different from those in Panel A. The 0.043 $R^2$ of the simple regression, consumption growth regressed on the contemporaneous first-order difference in bond yields, is much smaller as a result of these regressions being unaffected by time aggregation. Nevertheless, we find that bond yield changes are unconditionally positively related to consumption growth. The implication of our model is that the relation should be stronger when SB correlations or the stock market leverage betas are negative. This is indeed what we find by the examining the interactive coefficient $\alpha_2$. All specifications show a negative coefficient, with two that are highly significant.

A potential concern is that a significant fraction of the variation in nominal yields may be driven by inflation, which is outside our model. Using the methodology outlined in Appendix B we attempt to remove the expected inflation component from nominal yields. We then repeat the previous analysis using real yield changes instead of nominal. The results are shown in Panel C,
where we observe patterns that are similar to those based on nominal yields, though with lower statistical significance.

3. Stock/bond correlation and stock market leverage

Given the results in Table II, a natural question to ask is whether the two proxies of CSP, namely the SB correlation and the stock market leverage beta, are themselves related. Testing the relationship between the two series is challenging, as both of them must be estimated from rolling samples. If these samples are too short, estimation errors will dominate the observed variation. If the samples are too long, we will induce artificial persistence that could lead to the spurious regression problem of Granger and Newbold (1974).

We strike a balance between these concerns by using a rolling window length that is shorter than that used in Table II, in which the spurious regression problem was not a concern. For measures based on daily data, we either use one month or 12 months of data. For measures based on monthly data, we either use one or five years.

We evaluate the relationship between the two series using a time-series regression. For measures estimated with just a single month of data, the regression is monthly. For measures estimated with 12 months of data, we use annual end-of-year values to eliminate issues arising from the use of overlapping data. For measures estimated with 60 months of data, we use the just a single pair of values every five years (i.e., December of 1965, 1970, etc.) for the same reason. We further calculate the standard errors using Newey-West adjustment.

Table III summarizes the results of the regression where SB correlations are

4Pyun (2019) shows that stock market leverage betas can be estimated with reasonable accuracy using just one month of daily data. For a more accurate measure we also consider 12-month estimates.
regressed on stock market leverage betas. We choose to control for the level of stock market volatility because several studies (e.g., Baele, Bekaert, and Inghelbrecht (2010)) show that SB correlation is empirically negatively related to the level of volatility, possibly due to the ‘flight-to-quality’ phenomenon. Using daily estimates, we find that SB correlations are positively related to stock market leverage betas. This remains so even after controlling for the level of market volatility and is consistent for different measures of correlations.

4. Market variance

As shown in (7), our model implies that the variance of market returns should be related both to macroeconomic uncertainty and to the covariance between shocks to current and future expected consumption growth. The latter effect, which relates to the stock/bond return correlation, suggests a link to the “flight-to-quality” hypothesis. In this view, investors shift their portfolio from more risky stocks to safer bonds in periods of heightened uncertainty. The negative correlation between stock and bond returns induced by these flows results in a negative relation between uncertainty and the SB correlation.

In this section, we test whether the SB correlation or the stock market leverage beta are negatively related to stock market variance in the manner that model predicts. In particular, since both the SB correlation and the stock market leverage beta are proxies for the correlation between shocks to current and expected future consumption growth, not the covariance, the SB correlation should have an effect that is interactive with macroeconomic uncertainty. This suggests the predictive regression

\[ RV_{t+1} = \beta_0 + \beta_1 UNC_t + \beta_2 UNC_t \times R_t + \epsilon_{t+1}, \]  

(9)

where \( RV_t \) is the realized variance, \( UNC_t \) is a measure of macroeconomic uncertainty as defined as in the data appendix, and where \( R_t \) is either the SB
correlation or the stock market leverage beta. Our primary interest is in the interactive coefficient $\beta_2$, which we expect to be negative.

The results of these regressions are summarized in Table IV. Panels A, B, and C show the results using macroeconomic uncertainty measure of Jurado, Ludvigson, and Ng (2015), the monetary policy uncertainty of Baker, Bloom, and Davis (2016), and the consumption volatility process from Schorfheide, Song, and Yaron (2018), respectively. We also show the results after controlling for past realized variance. Overall, the results are consistent with our model. Across three measures of uncertainty, we find strong statistical significance of the $\beta_2$ coefficient if we use the daily SB correlation as our CSP proxy. The results for monthly measures are weaker, likely in part due to estimation errors in expected inflation. For stock market leverage, in contrast, we find stronger results for the monthly measures.

There are two reasons why we may find weaker results for the daily leverage measure. One is that first differences in option-implied volatility may not correctly represent volatility innovations, as they are also affected by risk premia. Second, the sample period, which starts in 1986, is shorter for the daily estimates. Overall, the result suggests that economic uncertainty predicts stock market variance with a higher slope when SB correlations are negative, and when stock market leverage betas are negative.

5. The consumption/volatility correlation

Our model implies that the correlation between shocks to expected consumption growth and consumption volatility also varies with consumption shock persistence. This is a critical implication, because this correlation links CSP to the equity risk premia. Because shocks to expected consumption growth and consumption volatility are difficult to measure, we instead exam-
ine the relationship in levels. This is justified by the results in Panel (c) of Figure 2, which showed that our model implies a higher correlation between \( x_t \) and \( \sigma_t \) when \( \rho_t \) is low. Equivalently, the correlation between \( x_t \) and \( \sigma_t \) will be higher when the SB correlation or the stock market leverage beta are high.

Because \( \Delta c_{t+1} \) is equal to \( x_t \) in expectation, we test this hypothesis using full and restricted versions of the predictive regression

\[
\Delta c_{t+1} = \beta_0 + \beta_1 \text{UN}\text{C}_t + \beta_2 R_t \times \text{UN}\text{C}_t + \beta_3 R_t + \beta_4 \Delta c_t + \epsilon_{t+1},
\]

where \( R_t \) is either the SB correlation or the stock market leverage beta and \( \text{UN}\text{C}_t \) is one of four measures of uncertainty. Three of these measures are the macro uncertainty measures used in the previous section. The final one is an estimate of stock market volatility from the two-component model of Brandt and Jones (2006). If \( R_t \) is closely related to consumption leverage, then we should obtain a positive estimate for the \( \beta_2 \) parameter.

Table V summarizes the results of these regressions. In each of the four panels, we use a different proxy for \( R_t \). We include regressions with and without the controls \( R_t \) and \( \Delta c_t \). Overall, the table provides reasonably strong support for our hypothesis. A positive \( \beta_2 \) is found in each regression, and it is statistically significant (at the 10 percent level) in most cases. Results are strongest in Panel A, which uses the daily SB correlation to proxy for CSP. Other panels use CSP proxies that are either based on lower frequency data (Panels B and C) or are available only over a shorter sample (Panel D).

6. Stock return predictability

One final implication of the model is the time-varying negative relationship between bond yields and future wealth portfolio returns, as proxied by the stock index. The relationship between bond yields and stock market returns has been studied in a number of papers. Fama and Schwert (1977) estimate
a simple predictive regression of future stock returns on lagged bond yields and find a negative slope, which they interpret as the result of stocks being inflation hedges. Breen, Glosten, and Jagannathan (1989) further confirm the economic significance of this predictability. More recently, Ang and Bekaert (2007) find that short-term Treasury yields, along with dividend yields, jointly predict stock returns in many international markets. They argue that the yields represent a component of the discount rate used by investors to value equities. Campbell and Thompson (2008) also document statistically significant in-sample predictability and positive out-of-sample $R^2$s using T-Bill rates.

Our model suggests that the extent to which bond yield predict stock returns depends on the relationship between current and expected consumption growth. A higher CSP associated with a stronger, more negative predictive slope between bond yields and future returns. We test this hypothesis in the monthly regression

$$R_{e,S,t,t+\tau}^e = \beta_0 + \beta_1 y_t + \beta_2 y_t \times R_t + \epsilon_{t+1},$$

(11)

where $R_{e,S,t,t+\tau}^e$ is the $\tau$-month excess market return, $y$ is the one-year constant maturity Treasury yield, and $R_t$ is the estimated SB correlation or the stock market leverage beta. We show the result for one, three, six, and 12 month forecast horizons ($\tau$) and across our four proxies of CSP.

Table VI summarizes the results of these regressions. Panel A shows the results of simple predictive regressions, in which only the lagged bond yield is used to predict excess stock returns. Although the regression coefficients are all negative, they are only marginally statistically significant for the one-month horizon. This is qualitatively consistent with but notably weaker than the results of early studies by Fama and Schwert (1977) and Breen, Glosten, and Jagannathan (1989).
The novel implication of our model is that the slope should be more negative when the SB correlation or the stock market leverage is lower, implying $\beta_2 > 0$. In Panel B, we test for this effect using the SB correlations as the proxy for CSP, while Panel C uses the stock market leverage beta in place of $R_i$. We find evidence of this hypothesis in both panels of the table, as evidenced by the consistently positive coefficients on the $y_{j,t} \times R_t$ terms. The evidence is more substantial for SB correlations and daily stock market leverage betas. The relatively weaker result for monthly SB correlations and daily leverage betas is partly expected from previous tables, as they tend to be more noisy measures of the current/expected consumption growth correlations.

To understand the degree with which return predictability varies, consider forecasts based on one-year Treasury yields. If the SB correlation were 0.4, the conditional slope of one-month market excess returns on yields would be a paltry $-0.037 (-0.271 + 0.585 \times 0.4)$, implying that yields have essentially no predictive power for future returns. Similar conclusions hold for longer investment horizons as well. However, were the return correlation instead $-0.5$, a 1% increase in the one-year Treasury yield would be associated with a 0.6% decline in monthly stock returns, a 1.7% decline in three-month returns, 2.0% decline in six-month returns, and 4.7% decline in 12-month returns. Economic magnitudes are similar when based on the stock market leverage betas.

In Panel D and E, we repeat the exercise with our estimated real yields rather than using nominal yields. Overall, we see similar results, albeit slightly weaker results for short-term predictability. The six-month and 12-month interactive coefficients are all highly statistically significant. In terms of economic magnitude, the results are similar to those using nominal yields, but results using real yields are statistically weaker.

Many asset pricing models imply a negative relationship between bond
yields and stock risk premium, as high uncertainty both means lower bond yields and higher risk premium. It is therefore puzzling why the empirical relationship is so weak. Our results show that the predictive relationship is stronger than it appears, but only during periods when proxies indicate that CSP is high.

IV. Conclusion

While the exogenous consumption process examined by Bansal and Yaron (2004) is highly successful in replicating key moments of asset returns, its assumption of independent shocks is inconsistent both with macroeconomic theory and with consumption data. In particular, the model does not account for the relationship between shocks to current consumption growth and expected future consumption growth, which we term consumption shock persistence (CSP). In theory, this relationship may be positive or negative, depending on whether permanent or transient shocks to income or productivity are more prevalent. The model also does not account for the negative correlation between shocks to consumption growth and consumption volatility, which likely arises due to a precautionary savings motive.

Because of these assumptions, the model is unable to match a number of well documented features of financial markets. The correlation between stocks and bonds is highly time-varying in the data, and in addition appears to vary with the level of stock market volatility. These effects are absent in the model of Bansal and Yaron, which features a constant stock/bond correlation. The model also implies a constant stock market leverage effect, which is inconsistent with evidence shown by Pyun (2019).

We propose a model that allows for a significantly more realistic depen-
dence structure. Shocks to current and expected future consumption growth are stochastically correlated, which we view as a reduced form approach to modeling the relative importance of transitory and permanent shocks. Shocks to current consumption and consumption growth are negatively correlated at a fixed value, which maintains parsimony and reflects the likely importance of the precautionary savings motive.

The model implies that the correlation between stock and bond returns is decreasing in CSP. So is the stock market leverage beta. Empirically, we see that during periods of more negative stock/bond correlations or leverage betas, consumption growth tends to become more serially correlated. This result provides evidence of time variation in CSP, and it also links it to correlations that are readily estimable from high-frequency asset price data. We also see strong evidence that the SB correlation and the market leverage beta are positively related, which is implied by our model and new to this paper.

Our model also predicts the negative relation between stock market volatility and the SB correlation that has been observed in prior studies, such as Connolly, Stivers, and Sun (2005) or Baele, Bekaert, and Inghelbrecht (2010). This is because high consumption persistence makes cash flows and discount rates negatively correlated, which amplifies the effects of these shocks. Empirically, we find strong evidence for this relation.

We also find evidence of a time-varying relation between current uncertainty and future consumption growth. Nakamura, Sergeyev, and Steinsson (2017) show that this relation is generally negative, particularly during economic contractions. Our model implies that the correlation should be more negative when CSP is high, or equivalently when the SB correlation or the stock market leverage beta are negative, which we confirm in the data.
Finally, the model implies that the slope coefficient of the predictive relationship between current bond yields and future stock returns also varies as a function of CSP. Using our CSP proxies, we confirm this prediction in the data. Stock returns are strongly related to lagged bond yields, but only in environments where the SB correlation or leverage beta is negative. We also show that the source of this predictability is the real yield rather than the inflation component.

Thus, consumption shock persistence appears to account for a variety of stylized facts that are typically not linked together and whose explanations are still not fully understood. Furthermore, it does so using an intuitive and relatively modest generalization of the standard long risk framework. As researchers examine the conditional implications of long-run risk more closely, it seems natural that time-varying correlations should play an important role.
References


Ang, Andrew, Geert Bekaert, and Min Wei, 2007, Do macro variables, asset markets, or surveys forecast inflation better?, *Journal of Monetary Economics* 54, 1163–1212.


Figure 1. Consumption Persistence and Model-based Correlations

This figure shows the relationships between CSP and either the stock/bond return correlations or the stock market leverage betas under the consumption and dividend dynamics provided in the main text.
Regression of $\Delta c_t$ on $x_t$ shocks

Regression of $\Delta c_{t+1}$ on $\Delta c_t$

Regression of $x_t$ on $\sigma_i^2$

Regression of $MRP_t$ on $y_t$

**Figure 2.** Simulation-based Regression Betas Conditional on Consumption Growth Leverage

This figure describes the relationship between the slope coefficients of various simple linear regressions and CSP. $MRP_t$ denotes the market risk premium at time $t$.

Stock/bond correlations (daily)

Stock/bond correlations (monthly)

Stock market leverage (monthly)

Stock market leverage (daily)

**Figure 3.** Interactive Beta of Consumption Growth Regressions For Multiple Lags

This figure plots the slope estimates ($\hat{\alpha}_{3,k}$) of the interactive regressions

$$\Delta c_{t+k} = \alpha_{0,k} + \alpha_{1,k} \Delta c_t + \alpha_{2,k} R_t \times \Delta c_t + \alpha_{3,k} R_t + \epsilon_{t+k},$$

for different values of the interval $(k)$, where $R_t$ is the stock/bond return correlation or the stock market leverage beta estimated using daily or monthly observations. The dotted lines show the 90% confidence interval.
Table I  
Model Parameters and Simulations

This table summarizes the value of the model parameters, asset pricing moments implied by these parameters, as well as the relationship between macro and financial variables. Panel A shows the values of the parameters, and Panel B shows the moments obtained via simulating the dynamics. \( y \) denotes bond yields, \( R_{TW/m} \) is the return of the wealth (consumption only) or the market portfolio (full model), \( \sigma \) is the volatility of the wealth/market portfolio, SB Corr denotes the stock/bond return correlation, and Lev is the beta of market/wealth portfolio returns regressed on the first difference of market volatility. Values in Panel B are scaled to the annual level. Panel C summarizes the correlations between macroeconomic (\( \Delta c_{t+1}, x_{t+1}, \) and \( \sigma_{t+1} \)) and asset pricing variables (the first-order differences in bond yields (\( y_{t+1} \)) and the variance of the wealth/market portfolio (\( \sigma_{TW/m,t+1} \)) as well as the returns of the total wealth/market portfolio (\( R_{TW/m} \)). Panel D shows the relationship between CSP (\( \rho_t \)) and the model-based stock/bond correlations (SB Corr) or the stock market leverage (Lev).

### Panel A. Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Preference Parameters</th>
<th>Consumption Parameters</th>
<th>Correlation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>10</td>
<td>( \mu )</td>
<td>0.0015</td>
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<tr>
<td>( \psi )</td>
<td>1.5</td>
<td>( p_x )</td>
<td>0.979</td>
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<tr>
<td>( \beta )</td>
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<td>( \omega_c )</td>
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<tr>
<td>( \sigma )</td>
<td></td>
<td>( \sigma_p )</td>
<td>4.6 \times 10^{-4}</td>
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### Panel B. Simulated Moments

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<tr>
<th>Variable</th>
<th>Baseline Model</th>
<th>Consumption-Only</th>
<th>Full Model</th>
<th>Data</th>
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</thead>
<tbody>
<tr>
<td>( E[R_{TW/m}] )</td>
<td>4.09%</td>
<td>4.11%</td>
<td>8.79%</td>
<td>10.63%</td>
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<tr>
<td>( y )</td>
<td>2.25%</td>
<td>2.55%</td>
<td>2.54%</td>
<td>1.03%</td>
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<tr>
<td>( \sigma_{TW/m} )</td>
<td>3.19%</td>
<td>3.28%</td>
<td>16.45%</td>
<td>14.13%</td>
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<tr>
<td>( SD(y) )</td>
<td>0.41%</td>
<td>0.40%</td>
<td>0.40%</td>
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</tr>
<tr>
<td>SB Corr</td>
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<tr>
<td>Lev</td>
<td>-296.22</td>
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<td>-129.42</td>
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### Panel C. Relationships between simulated variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>( \Delta y_{t+1} )</th>
<th>( R_{TW/m,t+1} )</th>
<th>( \Delta \sigma_{TW/m,t+1}^2 )</th>
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<tbody>
<tr>
<td>( \Delta c_{t+1} )</td>
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<td></td>
<td>Consumption-Only</td>
<td>0.019</td>
<td>0.859</td>
<td>-0.173</td>
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<tr>
<td></td>
<td>Full Model</td>
<td>0.019</td>
<td>0.628</td>
<td>-0.197</td>
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<tr>
<td>( x_{t+1} - E_t[x_{t+1}] )</td>
<td>Baseline</td>
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<td>0.000</td>
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<tr>
<td></td>
<td>Consumption-Only</td>
<td>0.949</td>
<td>0.495</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Full Model</td>
<td>0.949</td>
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<td>0.000</td>
</tr>
<tr>
<td>( \sigma_{t+1}^2 - E_t[\sigma_{t+1}^2] )</td>
<td>Baseline</td>
<td>-0.183</td>
<td>-0.104</td>
<td>0.996</td>
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<td></td>
<td>Consumption-Only</td>
<td>-0.138</td>
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<td></td>
<td>Full Model</td>
<td>-0.138</td>
<td>-0.383</td>
<td>0.695</td>
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</table>

### Panel D. Correlation between \( \rho_t \) and proxies

<table>
<thead>
<tr>
<th>Model</th>
<th>SB Corr</th>
<th>Lev</th>
</tr>
</thead>
<tbody>
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<td>Consumption-Only</td>
<td>-0.989</td>
<td>-1.000</td>
</tr>
<tr>
<td>Full Model</td>
<td>-0.997</td>
<td>-1.000</td>
</tr>
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</table>
Table II
Predictability of Consumption Growth (I)

This table summarizes the slopes and the Newey-West adjusted t-statistics of quarterly regressions that examine the relationship between CSP and the asset correlations. Panel A summarizes the results of

$$\Delta c_{t+1} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 R_t \times \Delta c_t + \alpha_3 R_t + \epsilon_{1,t+1},$$

where $R$ is either the stock/bond return correlation (SB Cor) or the stock market leverage beta divided by 100 (Lev). Panel B shows the results of the contemporaneous regression

$$\Delta c_t = \alpha'_0 + \alpha'_1 \Delta y_t + \alpha'_2 R_t \times \Delta y_t + \alpha'_3 R_t + \epsilon_{2,t},$$

where $y_t$ is the nominal 10-year bond yield. In Panel C, we replace the nominal yield with the real yield ($r_t$).

<table>
<thead>
<tr>
<th>Panel A. Serial Correlation of Consumption Growth</th>
<th>Panel B. Consumption Growth and Bond Yield Innovations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable: $\Delta c_{t+1}$</td>
<td>Dependent Variable: $\Delta c_t$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta c_t$</td>
<td>$\Delta y_t$</td>
</tr>
<tr>
<td>0.486</td>
<td>0.096</td>
</tr>
<tr>
<td>(17.1)</td>
<td>(3.83)</td>
</tr>
<tr>
<td>$\Delta c_t \times R_t$</td>
<td>$\Delta y_t \times R_t$</td>
</tr>
<tr>
<td>$-0.446$</td>
<td>$-0.401$</td>
</tr>
<tr>
<td>(-11.16)</td>
<td>(-3.76)</td>
</tr>
<tr>
<td>$R_t$</td>
<td>$R_t$</td>
</tr>
<tr>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>(2.97)</td>
<td>(1.41)</td>
</tr>
<tr>
<td>$\text{Adj-R}^2$ 0.233</td>
<td>$\text{Adj-R}^2$ 0.043</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C. Consumption Growth and Real Bond Yield Innovations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable: $\Delta c_t$</td>
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<tr>
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</tr>
<tr>
<td>0.012</td>
</tr>
<tr>
<td>(4.50)</td>
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<tr>
<td>$\Delta c_t \times R_t$</td>
</tr>
<tr>
<td>$-0.288$</td>
</tr>
<tr>
<td>(-2.29)</td>
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<tr>
<td>$R_t$</td>
</tr>
<tr>
<td>0.003</td>
</tr>
<tr>
<td>(1.44)</td>
</tr>
<tr>
<td>$\text{Adj-R}^2$ 0.073</td>
</tr>
</tbody>
</table>

| Table III
Stock/Bond Return Correlations and the Stock Market Leverage Effect

This table summarizes the slopes and Newey-West adjusted standard errors of contemporaneous regressions of realized stock/bond correlations on stock market leverage betas (Lev), with or without a control for market volatility ($\hat{\sigma}_t$). Daily measures use daily data on stock and bond returns to compute the SB correlation and daily data on stock returns and VXO changes to compute the leverage beta and are estimated using 1-month and 12-month non-overlapping windows. Monthly measures use monthly data for returns and compute volatility changes using the long-run volatility estimate ($\hat{\sigma}_t$) from the Brandt and Jones (2006) two-factor EGARCH model and are estimated using non-overlapping 12-month and 60-month windows.

<table>
<thead>
<tr>
<th>Stock/Bond Correlation</th>
<th>Daily Measures</th>
<th>Monthly Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1M Estimation</td>
<td>12M Estimation</td>
</tr>
<tr>
<td>$\hat{\sigma}_t$</td>
<td>-7.850</td>
<td>-8.047</td>
</tr>
<tr>
<td>(4.50)</td>
<td>(-3.82)</td>
<td>(-3.50)</td>
</tr>
<tr>
<td>$\text{Lev}_t$</td>
<td>0.026</td>
<td>0.017</td>
</tr>
<tr>
<td>(4.90)</td>
<td>(3.14)</td>
<td>(2.80)</td>
</tr>
<tr>
<td>$\text{Adj-R}^2$</td>
<td>0.058</td>
<td>0.147</td>
</tr>
</tbody>
</table>
Table IV: Market Variance Predictability

This table summarizes the relationship between stock/bond correlations, the stock market leverage effect, and the market variance. In all regressions, the dependent variable is the realized variance of stock returns ($RV_{t+1}$) estimated using the sum of daily squared returns in the following month. Independent variables include the macro uncertainty measure of Jurado, Ludvigson, and Ng (2015), the monetary policy uncertainty measure of Baker, Bloom, and Davis (2016), and the consumption growth volatility estimate of Schorfheide, Song, and Yaron (2018). Uncertainty measures are interacted with either the monthly or daily measure of stock/bond return correlation or the stock market leverage beta.

<table>
<thead>
<tr>
<th>Panel A. Uncertainty is Macro Uncertainty</th>
<th>Dependent Variable: $RV_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.439</td>
</tr>
<tr>
<td></td>
<td>(3.11)</td>
</tr>
<tr>
<td>Uncertainty,</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(2.62)</td>
</tr>
<tr>
<td>Uncertainty $\times$ SB Cor, (D)</td>
<td>−0.005</td>
</tr>
<tr>
<td></td>
<td>(−2.59)</td>
</tr>
<tr>
<td>SB Cor, (M)</td>
<td>−0.004</td>
</tr>
<tr>
<td></td>
<td>(−1.85)</td>
</tr>
<tr>
<td>Lev, (M)</td>
<td>−0.034</td>
</tr>
<tr>
<td></td>
<td>(−2.43)</td>
</tr>
<tr>
<td>Lev, (D)</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(−0.28)</td>
</tr>
<tr>
<td>Adj-$R^2$</td>
<td>0.158</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Uncertainty is Monetary Policy Uncertainty</th>
<th>Dependent Variable: $RV_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.443</td>
</tr>
<tr>
<td></td>
<td>(3.21)</td>
</tr>
<tr>
<td>Uncertainty,</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>(1.17)</td>
</tr>
<tr>
<td>Uncertainty $\times$ SB Cor, (D)</td>
<td>−0.584</td>
</tr>
<tr>
<td></td>
<td>(−2.11)</td>
</tr>
<tr>
<td>SB Cor, (M)</td>
<td>−0.506</td>
</tr>
<tr>
<td></td>
<td>(−1.49)</td>
</tr>
<tr>
<td>Lev, (M)</td>
<td>−3.697</td>
</tr>
<tr>
<td></td>
<td>(−1.98)</td>
</tr>
<tr>
<td>Lev, (D)</td>
<td>−0.123</td>
</tr>
<tr>
<td></td>
<td>(−1.61)</td>
</tr>
<tr>
<td>Adj-$R^2$</td>
<td>0.143</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C. Uncertainty is Consumption Growth Volatility</th>
<th>Dependent Variable: $RV_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.441</td>
</tr>
<tr>
<td></td>
<td>(3.05)</td>
</tr>
<tr>
<td>Uncertainty,</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(2.79)</td>
</tr>
<tr>
<td>Uncertainty $\times$ SB Cor, (D)</td>
<td>−0.006</td>
</tr>
<tr>
<td></td>
<td>(−2.40)</td>
</tr>
<tr>
<td>SB Cor, (M)</td>
<td>−0.004</td>
</tr>
<tr>
<td></td>
<td>(−1.65)</td>
</tr>
<tr>
<td>Lev, (M)</td>
<td>−0.026</td>
</tr>
<tr>
<td></td>
<td>(−2.19)</td>
</tr>
<tr>
<td>Lev, (D)</td>
<td>−0.002</td>
</tr>
<tr>
<td></td>
<td>(−1.83)</td>
</tr>
<tr>
<td>Adj-$R^2$</td>
<td>0.157</td>
</tr>
</tbody>
</table>
Table V
Predictability of Consumption Growth (II)

This table summarizes the results of the predictive regression
\[ \Delta c_{t+1} = \beta_0 + \beta_1 \text{UNC}_t + \beta_2 R_t \times \text{UNC}_t + \beta_3 \text{R}_t + \beta_4 \text{c}_t + \epsilon_{t+1}, \]
where \( R \) is either the stock/bond correlation or the stock market leverage beta, and the proxies of uncertainty (UNC) are defined as in previous tables.

Panel A. Stock/Bond Return Correlation (Daily)

<table>
<thead>
<tr>
<th>Dependent Variable: ( \Delta c_{t+1} )</th>
<th>UNC = MU</th>
<th>UNC = MPU</th>
<th>UNC = HX</th>
<th>UNC = ( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNC, ( t )</td>
<td>-0.025</td>
<td>-0.016</td>
<td>-0.028</td>
<td>-0.004</td>
</tr>
<tr>
<td>UNC, ( R_t ) \times \text{unc}</td>
<td>0.005</td>
<td>0.003</td>
<td>0.030</td>
<td>0.005</td>
</tr>
<tr>
<td>( t )</td>
<td>(3.05)</td>
<td>(2.65)</td>
<td>(1.90)</td>
<td>(2.26)</td>
</tr>
<tr>
<td>( t )</td>
<td>(1.76)</td>
<td>(2.26)</td>
<td>(1.87)</td>
<td>(2.27)</td>
</tr>
<tr>
<td>( t )</td>
<td>(2.27)</td>
<td>(2.11)</td>
<td>(1.65)</td>
<td>(1.18)</td>
</tr>
<tr>
<td>Adj-R(^2)</td>
<td>0.216</td>
<td>0.305</td>
<td>0.224</td>
<td>0.079</td>
</tr>
</tbody>
</table>

Panel B. Stock/Bond Return Correlation (Monthly)

<table>
<thead>
<tr>
<th>Dependent Variable: ( \Delta c_{t+1} )</th>
<th>UNC = MU</th>
<th>UNC = MPU</th>
<th>UNC = HX</th>
<th>UNC = ( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNC, ( t )</td>
<td>-0.013</td>
<td>-0.013</td>
<td>-0.022</td>
<td>-0.003</td>
</tr>
<tr>
<td>UNC, ( R_t ) \times \text{unc}</td>
<td>0.050</td>
<td>0.004</td>
<td>0.031</td>
<td>0.006</td>
</tr>
<tr>
<td>( t )</td>
<td>(2.62)</td>
<td>(3.06)</td>
<td>(1.85)</td>
<td>(3.11)</td>
</tr>
<tr>
<td>( t )</td>
<td>(2.90)</td>
<td>(1.61)</td>
<td>(2.35)</td>
<td>(2.47)</td>
</tr>
<tr>
<td>( t )</td>
<td>(2.1)</td>
<td>(1.24)</td>
<td>(2.42)</td>
<td>(2.58)</td>
</tr>
<tr>
<td>Adj-R(^2)</td>
<td>0.113</td>
<td>0.251</td>
<td>0.215</td>
<td>0.154</td>
</tr>
</tbody>
</table>

Panel C. Stock Market Leverage (Daily)

<table>
<thead>
<tr>
<th>Dependent Variable: ( \Delta c_{t+1} )</th>
<th>UNC = MU</th>
<th>UNC = MPU</th>
<th>UNC = HX</th>
<th>UNC = ( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNC, ( t )</td>
<td>-0.030</td>
<td>-0.025</td>
<td>0.005</td>
<td>-0.002</td>
</tr>
<tr>
<td>UNC, ( R_t ) \times \text{unc}</td>
<td>0.020</td>
<td>0.013</td>
<td>0.447</td>
<td>0.049</td>
</tr>
<tr>
<td>( t )</td>
<td>(0.95)</td>
<td>(0.91)</td>
<td>(1.20)</td>
<td>(2.09)</td>
</tr>
<tr>
<td>( t )</td>
<td>(1.69)</td>
<td>(1.25)</td>
<td>(2.01)</td>
<td>(1.84)</td>
</tr>
<tr>
<td>( t )</td>
<td>(1.50)</td>
<td>(1.50)</td>
<td>(1.47)</td>
<td>(1.64)</td>
</tr>
<tr>
<td>Adj-R(^2)</td>
<td>0.178</td>
<td>0.283</td>
<td>0.179</td>
<td>0.065</td>
</tr>
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</table>

Panel D. Stock Market Leverage (Monthly)

<table>
<thead>
<tr>
<th>Dependent Variable: ( \Delta c_{t+1} )</th>
<th>UNC = MU</th>
<th>UNC = MPU</th>
<th>UNC = HX</th>
<th>UNC = ( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNC, ( t )</td>
<td>-0.024</td>
<td>-0.015</td>
<td>-0.025</td>
<td>-0.003</td>
</tr>
<tr>
<td>UNC, ( R_t ) \times \text{unc}</td>
<td>0.054</td>
<td>0.039</td>
<td>0.270</td>
<td>0.042</td>
</tr>
<tr>
<td>( t )</td>
<td>(4.10)</td>
<td>(3.74)</td>
<td>(2.03)</td>
<td>(3.09)</td>
</tr>
<tr>
<td>( t )</td>
<td>(3.74)</td>
<td>(2.03)</td>
<td>(1.72)</td>
<td>(2.84)</td>
</tr>
<tr>
<td>( t )</td>
<td>(2.47)</td>
<td>(1.37)</td>
<td>(2.27)</td>
<td>(2.19)</td>
</tr>
<tr>
<td>Adj-R(^2)</td>
<td>0.242</td>
<td>0.292</td>
<td>0.251</td>
<td>0.110</td>
</tr>
</tbody>
</table>
This table summarizes the results of the regression

\[ R_{t+1}^e = \beta_0 + \beta_1 \text{Yield}_t + \beta_2 \text{Yield}_t \times R_t + \epsilon_{t+1}, \]

where \( R_e^e \) and \( \text{Yield} \) are the value-weighted market excess return and the nominal \((y_t)\) or estimated real yields \((r_t)\) of the one-year constant maturity Treasury, respectively. \( R_t \) is either the estimated correlation between stock and bond returns (SB Corr) or the beta of the market returns regressed on first order difference in market volatility divided by 100 (Lev). The correlations and the leverage betas are estimated using daily (columns “Daily”) or monthly (columns “Monthly”) observations. The t-statistics are adjusted for heteroscedasticity and autocorrelation using Newey-West standard errors.

### Panel A. Simple predictive regressions

<table>
<thead>
<tr>
<th></th>
<th>One-month</th>
<th>Three-month</th>
<th>Six-month</th>
<th>Twelve-month</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_t )</td>
<td>-0.088</td>
<td>-0.201</td>
<td>-0.339</td>
<td>-0.621</td>
</tr>
<tr>
<td></td>
<td>(-1.64)</td>
<td>(-1.56)</td>
<td>(-1.28)</td>
<td>(-1.29)</td>
</tr>
<tr>
<td>Adj-( R^2 )</td>
<td>0.003</td>
<td>0.006</td>
<td>0.009</td>
<td>0.008</td>
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</table>

### Panel B. Interactive predictive regressions using stock/bond return correlations

<table>
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<th>Three-month</th>
<th>Six-month</th>
<th>Twelve-month</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_t )</td>
<td>-0.271</td>
<td>-0.170</td>
<td>-0.781</td>
<td>-1.410</td>
</tr>
<tr>
<td></td>
<td>(-3.21)</td>
<td>(-2.69)</td>
<td>(-3.61)</td>
<td>(-3.60)</td>
</tr>
<tr>
<td>( y_t \times \text{SB Corr} )</td>
<td>0.585</td>
<td>0.291</td>
<td>1.757</td>
<td>3.420</td>
</tr>
<tr>
<td></td>
<td>(2.47)</td>
<td>(1.87)</td>
<td>(1.90)</td>
<td>(2.07)</td>
</tr>
<tr>
<td>Adj-( R^2 )</td>
<td>0.014</td>
<td>0.005</td>
<td>0.040</td>
<td>0.014</td>
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</table>

### Panel C. Interactive predictive regressions using stock market leverage betas

<table>
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<th>Three-month</th>
<th>Six-month</th>
<th>Twelve-month</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_t )</td>
<td>0.123</td>
<td>-0.150</td>
<td>0.443</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td>(0.73)</td>
<td>(-2.43)</td>
<td>(1.32)</td>
<td>(1.63)</td>
</tr>
<tr>
<td>( y_t \times \text{Lev} )</td>
<td>0.287</td>
<td>0.024</td>
<td>1.022</td>
<td>2.281</td>
</tr>
<tr>
<td></td>
<td>(1.25)</td>
<td>(2.07)</td>
<td>(2.15)</td>
<td>(2.55)</td>
</tr>
<tr>
<td>Adj-( R^2 )</td>
<td>0.001</td>
<td>0.009</td>
<td>0.015</td>
<td>0.021</td>
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</table>

### Panel D. Interactive predictive regressions using stock/bond return correlations

<table>
<thead>
<tr>
<th></th>
<th>One-month</th>
<th>Three-month</th>
<th>Six-month</th>
<th>Twelve-month</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_t )</td>
<td>-0.190</td>
<td>-0.218</td>
<td>-0.514</td>
<td>-1.035</td>
</tr>
<tr>
<td></td>
<td>(-2.25)</td>
<td>(-2.79)</td>
<td>(-2.47)</td>
<td>(-2.84)</td>
</tr>
<tr>
<td>( r_t \times \text{SB Corr} )</td>
<td>0.101</td>
<td>0.372</td>
<td>1.765</td>
<td>3.306</td>
</tr>
<tr>
<td></td>
<td>(0.28)</td>
<td>(1.18)</td>
<td>(1.08)</td>
<td>(2.32)</td>
</tr>
<tr>
<td>Adj-( R^2 )</td>
<td>0.005</td>
<td>0.005</td>
<td>0.015</td>
<td>0.020</td>
</tr>
</tbody>
</table>

### Panel E. Interactive predictive regressions using stock market leverage betas

<table>
<thead>
<tr>
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<th>One-month</th>
<th>Three-month</th>
<th>Six-month</th>
<th>Twelve-month</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_t )</td>
<td>0.241</td>
<td>-0.171</td>
<td>0.791</td>
<td>2.307</td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
<td>(-2.21)</td>
<td>(1.13)</td>
<td>(1.77)</td>
</tr>
<tr>
<td>( r_t \times \text{Lev} )</td>
<td>0.451</td>
<td>0.025</td>
<td>1.452</td>
<td>3.871</td>
</tr>
<tr>
<td></td>
<td>(1.01)</td>
<td>(1.10)</td>
<td>(1.65)</td>
<td>(2.24)</td>
</tr>
<tr>
<td>Adj-( R^2 )</td>
<td>0.000</td>
<td>0.000</td>
<td>0.010</td>
<td>0.010</td>
</tr>
</tbody>
</table>
A. Technical appendix

1. The wealth-consumption ratio

Following the Campbell-Shiller decomposition, the returns to total wealth portfolio can be represented by

\[ R_{TW,t+1} = \kappa_0 + \Delta c_{t+1} + A_0(\kappa_1 - 1) + A_1(\kappa_1 x_{t+1} - x_t) + A_2(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_3(\kappa_1 p_{t+1} - p_t). \]

The intertemporal marginal rate of substitution (IMRS) is

\[ m_{t+1} = \theta \log \beta - \gamma \Delta c_{t+1} + (\theta - 1)[\kappa_0 + A_0(\kappa_1 - 1) + A_1(\kappa_1 x_{t+1} - x_t) + A_2(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_3(\kappa_1 p_{t+1} - p_t)]. \]

The unexpected component of the IMRS is represented by

\[ m_{t+1} - E_t[m_{t+1}] = \lambda_c \sigma_t \epsilon_{c,t+1} + \lambda_x \sigma_t \epsilon_{x,t+1} + \lambda_\epsilon \sigma_t \epsilon_{x,t+1} + \lambda_\delta \sigma_t \epsilon_{p,t+1}, \]

where \( \lambda_c = -\gamma \), \( \lambda_x = (\theta - 1)\kappa_1 A_1 \varphi_x \), \( \lambda_\epsilon = (\theta - 1)\kappa_1 A_2 \sigma_v \), and \( \lambda_\delta = (\theta - 1)\kappa_1 A_3 \sigma_p \).

We solve for \( A_0 \), \( A_1 \), \( A_2 \), and \( A_3 \) using equation using the Euler equation \( E_t[m_{t+1} + R_{TW,t+1}] + \text{Var}_t[m_{t+1} + R_{TW,t+1}] = 0 \). For \( A_1 \), we collect all terms associated with \( x_t \):

\[ A_1 = \frac{1 - \psi}{1 - \kappa_1 p_t}. \]

Collecting the terms from the Euler equation that are functions of \( \sigma_t^2 \) and \( p_t \), it can be seen that \( A_2 \) and \( A_4 \) must jointly satisfy the conditions

\[ 2A_2(\kappa_1 s_1 - 1) + \theta((A_1 \kappa_1 \varphi_x)^2 + (A_2 \kappa_1 \sigma_v)^2 + (A_3 \kappa_1 \sigma_p)^2 + (1 - \frac{1}{\psi})^2) + 2(1 - \gamma) \kappa_1 A_2 \sigma_v \varphi_p = 0 \]

\[ A_3 = A_{30} + A_{32} A_2, \]

where \( A_{30} = \frac{(1-\gamma)\kappa_1 A_1 \varphi_x}{1-\kappa_1 \omega_1} < 0 \) and \( A_{32} = \frac{\theta \sigma_v \kappa_1^2 A_1 \varphi_x \sigma_v}{1-\kappa_1 \omega_1} > 0 \).

\( A_2 \) can be obtained by solving a quadratic equation after plugging the second equation into the first. It can also be shown that \( A_2 < 0 \) when \( \gamma > 1 \) and \( \psi > 1 \) by evaluating the characteristics of the quadratic equation. We obtain two values for \( A_2 \). We choose the value that is closer to the baseline model. The second value generates unrealistic moments of asset returns. The negative sign of \( A_2 \) also implies \( A_3 < 0 \).
Finally, \( A_0 \) satisfies

\[
A_0 = \frac{1}{1 - \kappa_1} \left[ \log \beta + \kappa_0 + (1 - \frac{1}{\psi})\mu + k_1(A_2 s_0 + A_3 \omega_0) \right].
\]

2. The price-dividend ratio

Similar to the wealth-consumption ratio, market returns can be expressed as

\[
R_{m,t+1} = \kappa_0 + \Delta d_{t+1} + A_{m,0}(\kappa_1 - 1) + A_{m,1}(\kappa_1 x_{t+1} - x_t) + A_{m,2}(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_{m,3}(\kappa_1 p_{t+1} - p_t).
\]

We again solve for the coefficients using the Euler equation \( E_t[m_{t+1} + R_{m,t+1}] + 0.5 \text{Var}_t[m_{t+1} + R_{m,t+1}] = 0 \). Collecting the terms associated with \( x_t, \sigma_t^2 \), and \( p_t \), we can solve for \( A_{m,0}, A_{m,1}, A_{m,2}, \) and \( A_{m,3} \). First, we have

\[
A_{m,1} = \frac{\phi_d - \frac{1}{\psi}}{1 - \kappa_1 p_t}.
\]

As in the wealth-consumption ratio, \( A_{m,2} \), and \( A_{m,3} \) must jointly satisfy the conditions

\[2 A_{m,2}(\kappa_1 s_1 - 1) + 2(\theta - 1)(\kappa_1 s_1 - 1)A_2 + 2(\varphi_{cd} + \lambda_s)(\kappa_1 A_{m,2} \sigma_s + \lambda_s) \rho_{ps} + (\kappa_1 A_{m,1} \sigma_x + \lambda_x) \right] + 2(\varphi_{cd} + \lambda_c) \right)^2 + (\psi_2)^2 = 0.
\]

\[
A_{m,3} = A_{m,30} + A_{m,32} A_{m,2},
\]

where \( A_{m,30} = \frac{1}{1 - \kappa_1 \omega_1}((\varphi_{cd} + \lambda_s)(\kappa_1 A_{m,1} \varphi_x + \lambda_x) + (\theta - 1)(\kappa_1 \omega_1 - 1)A_3 + \lambda_v(\kappa_1 A_{m,1} \varphi_x + \lambda_x) \rho_{ps}) \) and \( A_{m,32} = \frac{1}{1 - \kappa_1 \omega_1} \kappa_1 \sigma_x(\kappa_1 A_{m,1} \varphi_x + \lambda_x) \rho_{ps} \). Evaluating the characteristics of the quadratic function, similar to the earlier case, \( A_{m,2} < 0 \) when \( \gamma > \varphi_{cd} > 1 \), which is consistent with a general long-run risk specification. Also, one can show that \( A_{m,30} < 0 \) and \( A_{m,32} > 0 \), under the condition of \( \gamma > \phi_d \) and \( \varphi_{cd} > 1 \), which implies \( A_{m,3} < 0 \).

Finally, \( A_{m,0} \) satisfies

\[
A_{m,0} = \frac{1}{1 - \kappa_1} \left( \theta \log \beta + \theta \kappa_0 + (1 - \gamma) \mu \right.
\]

\[
+ \kappa_1 s_0(A_2(\theta - 1) + A_{m,2}) + \kappa_1 \omega_0(A_3(\theta - 1) + A_{m,3}) + (\theta - 1)(\kappa - 1) A_0 \right).
\]

3. The stock/bond correlation

The interest rate on a riskless bond is derived by solving

\[
E_t[m_{t+1}] + 0.5 \text{Var}_t[m_{t+1}] = 0.
\]

It can be shown that the yield of the bond is represented by

\[
y_t = Y_0 + Y_1 x_t + Y_2 \sigma_t^2 + Y_3 p_t,
\]

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where
\[
Y_0 = -\theta \log \beta + \gamma \mu - (\theta - 1) (\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1 s_A A_2 + \kappa_1 \omega_0 A_3)
\]
\[
Y_1 = \frac{1}{\psi}
\]
\[
Y_2 = - (\theta - 1)(\kappa_1 s_A - 1)A_2 - \frac{1}{2} (\lambda^2 + \lambda^2 + \lambda^2) - \lambda c \lambda x \theta ps
\]
\[
Y_3 = - (\theta - 1)(\kappa_1 \omega_1 - 1)A_3 - \lambda x \lambda x \theta ps - \lambda c \lambda x.
\]

The unexpected return of the total wealth portfolio and the market return are derived using the Campbell-Shiller decomposition:
\[
R_{TW,t+1} - E_t[R_{TW,t+1}] = \kappa_1 \phi_x A_1 \sigma_t \epsilon_{x,t+1} + \kappa_1 \sigma_x A_2 \sigma_t \epsilon_{c,t+1} + \kappa_1 \sigma_p A_3 \epsilon_{p,t+1} + \sigma_t \epsilon_{c,t+1}
\]
\[
R_{m,t+1} - E_t[R_{m,t+1}] = \kappa_1 \phi_x A_1 \sigma_t \epsilon_{x,t+1} + \kappa_1 \sigma_x A_2 \sigma_t \epsilon_{c,t+1} + \kappa_1 \sigma_p A_3 \epsilon_{p,t+1} + \varphi \sigma_t \epsilon_{c,t+1} + \varphi \sigma_t \epsilon_{d,t+1}
\]

We represent the above relationship by:
\[
S_{j,1} \sigma_t \epsilon_{x,t+1} + S_{j,2} \sigma_t \epsilon_{v,t+1} + S_{j,3} \sigma_t \epsilon_{p,t+1} + S_{j,4} \sigma_t \epsilon_{d,t+1} + S_{j,5} \sigma_t \epsilon_{d,t+1},
\]
where \( j \) is either \( TW \) for the wealth portfolio or \( m \) for the market portfolio. From the above equation, we can derive the stock/bond return correlation by taking the negative of conditional correlation between wealth portfolio/market returns and bond yields.

The conditional covariance can be expressed as
\[
\text{Cov}_t(R_{j,t+1}, y_{t+1}) = (Y_1 S_{j,1} \varphi_x + Y_2 S_{j,2} \sigma_v + Y_3 S_{j,3} \sigma_p + Y_2 S_{j,4} \sigma_v \theta ps) \sigma_t^2
\]
\[
+ (Y_1 \varphi_x S_{j,2} + Y_2 S_{j,1} \sigma_v \theta ps + Y_1 S_{j,3} \varphi_x) p_t.
\]

The conditional variance of the bond yield is
\[
\text{Var}_t(y_{t+1}) = (Y_1 \varphi_x)^2 + (Y_2 \sigma_v)^2 + (Y_2 \sigma_p)^2 + 2Y_1 Y_2 \varphi_x \sigma_v p_t.
\]

Similarly, the conditional variance of the wealth portfolio/market returns is
\[
\sigma_{j,t+1} = (V_{j,2} + V_{j,3} \rho_t) \sigma_t^2
\]
for \( j = \{TW, m\} \), where \( V_{j,2} = \sum_{k=1}^{3} (S_{j,k}^2 + S_{j,2}^2) + 2S_{j,2} S_{j,2} \theta ps \) and \( V_{j,3} = 2S_{j,1} S_{j,2} \theta ps + 2S_{j,2} S_{j,1} \).

4. The stock market leverage effect

The leverage correlation is the conditional covariance between the returns and variance shocks of the wealth portfolio divided by the conditional standard deviations of each. The
covariance can be represented by
\[
\text{Cov}_t(R_{j,t+1}, \sigma^2_{j,t+1}) = [(S_2 + S_{c,ps})V_2\sigma_v + S_3V_3\sigma_p + S_1V_2\sigma_v\rho_t]\sigma_t^2,
\]
for \( j = \{TW, m\} \). Dividing the above by the variance of variance shocks yields the stock market leverage effect. The variance of the market variance shocks is
\[
((V_2\sigma_v)^2 + (V_3\sigma_p)^2)\sigma_t^2.
\]

5. The market risk premium

The risk premium of the wealth/market portfolio can be expressed as
\[
\text{Cov}_t(-m_{t+1}, R_{j,t+1}) = (-\lambda_c(S_{j,c} + S_{j,2}\rho_{ps}) - \lambda_cS_{j,1} - \lambda_vS_{j,2} - \lambda_3S_{j,3} - S_{j,c}\lambda_v\rho_{ps})\sigma_t^2
\]
\[
(-\lambda_cS_{j,2}\rho_{ps} - \lambda_cS_{j,1}\rho_{ps} - \lambda_vS_{j,1} - \lambda_vS_{j,c})\rho_t
\]
for \( j = \{TW, m\} \).

B. Data

Quarterly consumption data is obtained from the national income and product accounts (NIPA) provided by the Bureau of Economic Analysis. We measure consumption at the quarterly frequency as the sum of the real personal consumption expenditure on non-durables and services. We take the quantity index of NIPA Table 2.3.3 and divide it by the total population obtained from NIPA Table 7.1. Consumption growth is defined as the first log difference and is computed from 1962 to 2019.

Bond yields are obtained from the website of the Federal Reserve Bank of St. Louis and are available from 1962 to 2019. We use the 10-year yield, though changing the maturity does not affect our results qualitatively. Real bond yields are calculated by subtracting the expected inflation rate from the nominal yield. Expected inflation is estimated on an out-of-sample basis using a first order AR(1) process applied to quarterly seasonally-adjusted first differences in the Consumer Price Index (CPI), where we use a 10-year rolling window for the estimation. The CPI data is obtained from the Bureau of Labor Statistics. Excess market returns are from Ken French’s data library.

\footnote{Ang, Bekaert, and Wei (2007) show that an ARMA or even an AR model performs relatively well in forecasting future inflation rates.}
We measure macroeconomic uncertainty in three different ways. First, we use the 12-month macro uncertainty measure from Jurado, Ludvigson, and Ng (2015), which is obtained from Sydney Ludvigson’s website and is available from 1961 to 2019. These data are available on a monthly basis, and we convert to quarterly by choosing the last value of each quarter. Second, we use the monetary policy uncertainty from Baker, Bloom, and Davis (2016). This uncertainty index is estimated using textual analysis of newspaper articles and is substantially different from those estimated from macroeconomic aggregates. The data covers the period from 1985 to 2019 and can be downloaded at the authors’ Economic Policy Uncertainty website. Third, we use the volatility estimate of expected consumption growth estimated using the long-run risk model of Schorfheide, Song, and Yaron (2018). That series, which was provided by the authors of the paper, is available from 1962 to 2014.

We also use several different measures of stock market volatility. The first is a monthly measure, the so-called “realized variance” computed as the squared daily excess market returns. The second volatility measure is the VXO index of the Chicago Board Options Exchange (CBOE). VXO is the predecessor of the VIX and measures the implied volatility of options on the S&P 100 Index (as opposed to the VIX, which is the model-free implied volatility of S&P 500 Index options). We choose it because it is available going back to 1986, while the VIX starts in 1990. Finally, we estimate measure of equity market volatility using the two-factor EGARCH model of Brandt and Jones (2006), which is closely related to the model of Engle and Lee (1999). Specifically, we use the long-run factor from the most general specification of Brandt and Jones, which we fit using daily market returns from 1950 to 2019. By focusing on the the long-run factor, we are excluding volatility fluctuations with very low persistence, which we believe are less relevant for explaining macroeconomic dynamics at horizons of one quarter or more.