Computing Integrals

$2 + 2 = 4$ *on crack*

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Introduction

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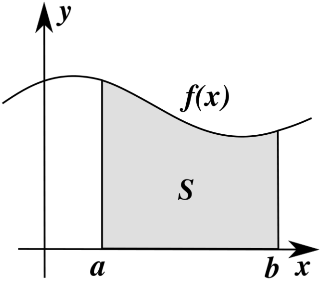

(For definition, see Math 139, 203, 241.)

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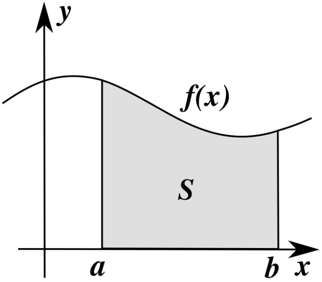

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We will review standard techniques and learn to integrate cleverly through a few examples.

Basic Integration Techniques

- Integration by Parts:

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Ex: $\int_0^1 \frac{1}{1-x^2} dx = \int_0^{\pi/2} \frac{\cos\theta}{\cos\theta} d\theta = \frac{\pi}{2}$.

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- Partial fractions. Ex:

$$\int \frac{1}{x^2-1} dx = \frac{1}{2}\left(\int \frac{1}{x-1} dx - \int \frac{1}{x+1} dx\right) = \frac{1}{2} \ln\left(\frac{x-1}{x+1}\right)$$

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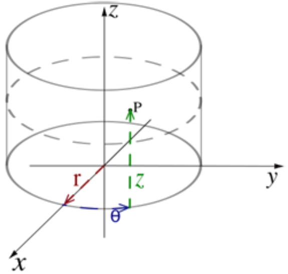
See Math 41.

Change of Coordinates

Cylindrical coordinates:

$$\begin{array}{rcl} x & \mapsto & r\cos\theta \\ y & \mapsto & r\sin\theta \\ z & \mapsto & z \\ dxdydz & \mapsto & rdrd\theta dz \end{array}$$



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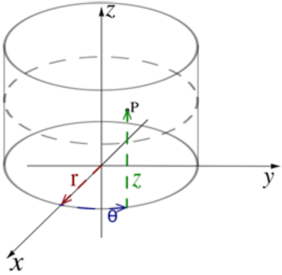
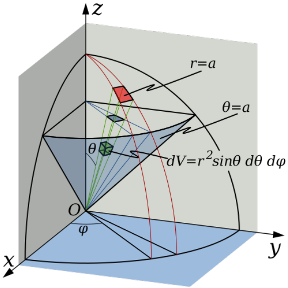


Spherical coordinates:

$$\begin{aligned} x &\mapsto \rho\sin\phi\cos\theta \\ y &\mapsto \rho\sin\phi\sin\theta \\ z &\mapsto \rho\cos\theta \\ dxdydz &\mapsto \rho^2\sin\phi d\rho d\theta d\phi \end{aligned}$$



Change of Coordinates

In general, define the Jacobian

$$\frac{\partial(y_1, \cdots, y_n)}{\partial(x_1, \cdots, x_n)} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_n}{\partial x_1} & \cdots & \frac{\partial y_n}{\partial x_n} \end{pmatrix}$$

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See Math 105, Math 204.

Advanced Techniques

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While standard theories are powerful, sometimes we can make our lives a lot easier by little bit of cleverness. Let us look at some examples!

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Evaluate

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$$\implies I = \sqrt{\pi}$$

$\square$

Exploit Symmetry

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(Putnam '87 B-1) Evaluate

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$$\Longrightarrow 2I = \int_2^4 1 dx = 2$$

$$\Longrightarrow I = 1$$

□

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Let $m = 3\tan\frac{\theta}{2}$, $dm = \frac{3}{2}\sec^2\frac{\theta}{2} d\theta$.

$$\begin{aligned} I &= \tfrac{2}{3}\int_0^{\sqrt{3}} \tfrac{dm}{1+m^2} \\ &= \tfrac{2}{3}(\tan^{-1} x \mid_0^{\sqrt{3}}) \\ &= \tfrac{2\pi}{9} \end{aligned}$$

Understand Underlying Geometry

Example

(Putnam '06 A1) Find the volume of the region of points (x, y, z) such that

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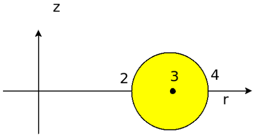


By Pappus' theorem, the volume is

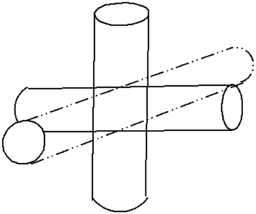
$$(3 \cdot 2\pi)(\pi \cdot 1^2) = 6\pi^2$$

□

Divide Regions Cleverly

Problem

Three infinitely long circular cylinders each with unit radius have their aes along the x, y, and z-axes. Determine the volume of the region common to all three cylinders. (Thus one needs the volume common to $\{y^2 + z^2 \leq 1\}$, $\{z^2 + x^2 \leq 1\}$, $\{x^2 + y^2 \leq 1\}$.)



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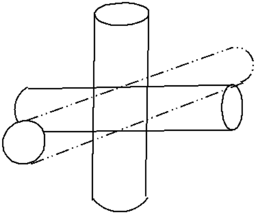

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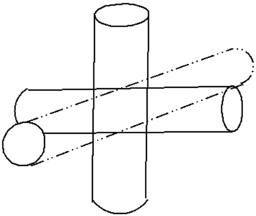

Hard to express condition Insight:

compute for $|y| \geq |x| \geq |z|$; final volume will be 3! times as large.

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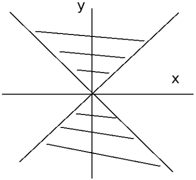

Suffices to compute for $|y| \geq |x| \geq |z|$. Use cylindrical coordinates, then the conditions become $r^2 \leq 1$, and $|\sin\theta| \geq |\cos\theta|$

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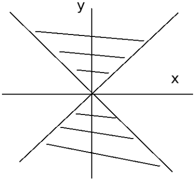
Solution.



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\begin{aligned}
\frac{I}{6} &= 2\int_{\pi/4}^{3\pi/4}\int_0^1\int_{-r|\cos\theta|}^{r|\cos\theta|} r\,dz\,dr\,d\theta \\
&= 8\left(\int_{\pi/4}^{\pi/2}\cos\theta\, d\theta\right)\left(\int_0^1 r^2 dr\right) \\
&= 8\left(1-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{3}\right)
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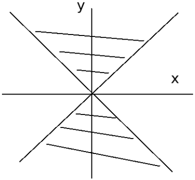

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$$\implies I = 8(2-\sqrt{2})$$

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- A lot of techniques available
- Don't just brute force: a little bit of cleverness makes life a lot easier.
- If possible, draw diagrams and try to visualize what's going on.

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- Be as creative as possible! Have fun!