Appetizer to Math 149S

Warm-up problems and information about the class

Peng Shi

Department of Mathematics
Duke University

August 27, 2009

Warm-up through Game Playing

Game 1

Two players iteratively pick numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. (Player 1 goes first, then player 2, then player 1 again, ...) A number can be picked at most once by any player. A player wins if out of all the numbers he/she has picked, 3 distinct numbers add up to 15. How would you play this game to maximize your wins?

Warm-up through Game Playing

Game 1

Two players iteratively pick numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. (Player 1 goes first, then player 2, then player 1 again, ...) A number can be picked at most once by any player. A player wins if out of all the numbers he/she has picked, 3 distinct numbers add up to 15. How would you play this game to maximize your wins?

Solution:

6	1	8
7	5	3
2	9	4

Warm-up through Game Playing

Game 1

Two players iteratively pick numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. (Player 1 goes first, then player 2, then player 1 again, ...) A number can be picked at most once by any player. A player wins if out of all the numbers he/she has picked, 3 distinct numbers add up to 15. How would you play this game to maximize your wins?

Solution:

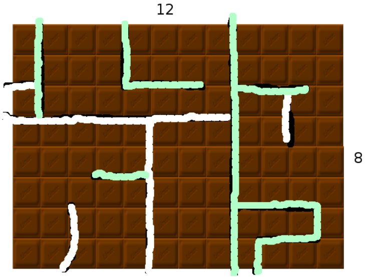
6	1	8
7	5	3
2	9	4

Tic-tac-toe! $\square$

Warm-up through Game Playing

Game 2

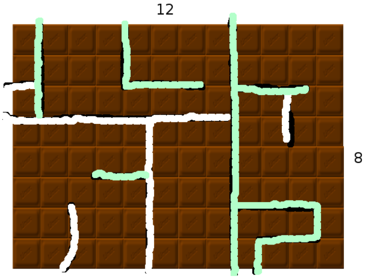
There is a chocolate bar composed of 12×8 squares of chocolate. Two players take turns picking up a pieces of chocolate and cutting it along the grid lines. (A player may choose to cut through and creating a new pieces, or not cut through. The cuts can bend, but they must follow grid lines and must stop at a vertex of some square. One has to cut at least one link each turn.) A player wins by making the last cut to break the chocolate into 96 squares. Do you want to be player 1 or 2? How would you play this game?



Warm-up through Game Playing

Game 2

There is a chocolate bar composed of 12×8 squares of chocolate. Two players take turns picking up a pieces of chocolate and cutting it along the grid lines. (A player may choose to cut through and creating a new pieces, or not cut through. The cuts can bend, but they must follow grid lines and must stop at a vertex of some square. One has to cut at least one link each turn.) A player wins by making the last cut to break the chocolate into 96 squares. Do you want to be player 1 or 2? How would you play this game?



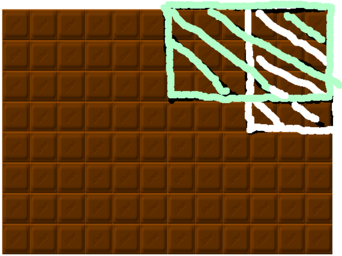
Solution:

Player 1 can always win by first making a cut in the middle to create two 6×8 pieces, hence creating two symmetric "sides". For the following turns, whatever player 2 does to one side (breaking the symmetry), player 1 mirrors on the other side. Because the final configuration is symmetric, player 2 cannot win. $\square$

Warm-up through Game Playing

Game 3 (Exercise)

There is a chocolate bar composed of $m \times n$ squares of chocolate. ($mn > 1$). Two players iteratively choose some rectangle starting from the top right corner, and remove all squares of chocolate in that rectangle. (The rectangle must be of non-zero size and must follow the grids.) The player that takes the last pieces loses. Who can always guarantee a win? Why?



What is Common?

- Mathematical puzzles
- Involves creativity
- Perhaps require a clever idea
- Develops mathematical problem solving/proof making skills.
- Fun to solve

What is Common?

- Mathematical puzzles
- Involves creativity
- Perhaps require a clever idea
- Develops mathematical problem solving/proof making skills.
- Fun to solve

This is what Math 149S is about!

(Although problems in Math 149S may be harder and require more mathematical theory.) We have a range of difficulty so you can learn something regardless of your current skill level (See **Problem set 0**.)

What is Common?

- Mathematical puzzles
- Involves creativity
- Perhaps require a clever idea
- Develops mathematical problem solving/proof making skills.
- Fun to solve

This is what Math 149S is about!

(Although problems in Math 149S may be harder and require more mathematical theory.) We have a range of difficulty so you can learn something regardless of your current skill level (See **Problem set 0**.)

Goals of the course:

1. To improve your problem solving skills
2. To sharpen your proof-writing skills

What is Common?

- Mathematical puzzles
- Involves creativity
- Perhaps require a clever idea
- Develops mathematical problem solving/proof making skills.
- Fun to solve

This is what Math 149S is about!

(Although problems in Math 149S may be harder and require more mathematical theory.) We have a range of difficulty so you can learn something regardless of your current skill level (See **Problem set 0**.)

Goals of the course:

1. To improve your problem solving skills
2. To sharpen your proof-writing skills
3. To prepare you for the Putnam competition

The Putnam Mathematics Competition

The Putnam Mathematics Competition is

- the premier math contest for college students.
- 6 hours 12 problems
- significant monetary prizes
- highly regarded by graduate schools in math, sciences and engineering
- highly regarded by hedge funds, and other math-literate employers

The Putnam Mathematics Competition

The Putnam Mathematics Competition is

- the premier math contest for college students.
- 6 hours 12 problems
- significant monetary prizes
- highly regarded by graduate schools in math, sciences and engineering
- highly regarded by hedge funds, and other math-literate employers

Year	Duke's national rank
1999	3
2000	1
2001-2005	3
2006	8
2007	5
2008	6

The Putnam Mathematics Competition

The Putnam Mathematics Competition is

- the premier math contest for college students.
- 6 hours 12 problems
- significant monetary prizes
- highly regarded by graduate schools in math, sciences and engineering
- highly regarded by hedge funds, and other math-literate employers

Year	Duke's national rank
1999	3
2000	1
2001-2005	3
2006	8
2007	5
2008	6

We need your help! (Traditionally Putnam winnings fund our free pizza.)

About Math 149S

- Meets once a week for 75 min (some weekday evening)
- Each class: 30 min powerpoint lecture, 30 min problem solving, 15 min student presentations, FREE PIZZA!
- Grade based on solving weekly problem sets: typically 3 well-written solutions a week = A
- Half-credit. Satisfies QS, S.

About Math 149S

- Meets once a week for 75 min (some weekday evening)
- Each class: 30 min powerpoint lecture, 30 min problem solving, 15 min student presentations, FREE PIZZA!
- Grade based on solving weekly problem sets: typically 3 well-written solutions a week = A
- Half-credit. Satisfies QS, S.

Topics:

- pigeonhole principle
- invariance/extremal arguments
- induction
- recurrences
- counting
- probability
- polynomials
- number theory
- calculus
- linear algebra
- other tricks

About Math 149S

Student Instructors:

Peng Shi (Senior Math and Compsci major)

Math Rognlie (Senior Math and Econ major)

Recommended book: *Problem-Solving Strategies* by Arthur Engel.

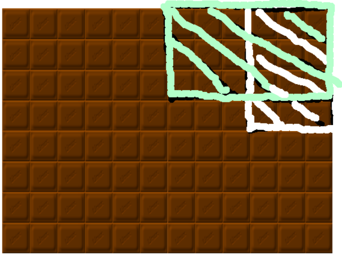
Any questions?

If you are interested in taking the class, speak to us after and we will give you permission numbers. If you can fully solve 3-4 problems on Problem Set 0, you are in a position to, with some work, get A in our class.

Solution to Game 3

Game 3 (Exercise)

There is a chocolate bar composed of $m \times n$ squares of chocolate. ($mn > 1$). Two players iteratively choose some rectangle starting from the top right corner, and remove all squares of chocolate in that rectangle. (The rectangle must be of non-zero size and must follow the grids.) The player that takes the last pieces loses. Who can always guarantee a win? Why?



Solution:

Player 1 can always win by the following argument: Because this is a finite game with no ties, exactly one of the player must have a winning strategy. Suppose player 1 removes the top-right square, and player 2 force a win by removing squares in some rectangle A. Then player 1 can, instead of removing the top-right square, removed A in his/her first turn, thus stealing the opponent's winning strategy. Hence, player 2 cannot have a winning strategy. Therefore player 1 can always win! (although we are not sure how without more extensive calculations.) □