Math 149S Problem Set 8:

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Problems may not be sorted in level of difficulty. You are NOT expected to solve every problem. Each problem is worth 10 points (if the proof is completely correct and well-written). Points maybe subtracted from a correct solution if the proof is badly written. Depending on the amount of progress, partial points maybe awarded. To get an A you need at least 30 points.

Problem 1. *Solve the following recurrence relations.*

1. $f(n) = 5f(n-1) - 6f(n-2)$, $f(0) = 1$, $f(1) = 5$.
2. $f(n) = 6f(n-1) - 9f(n-2)$, $f(0) = 1$,$f(1) = 2$.
3. $f(n) = 6f(n-1) - 9f(n-2) + 2n$, $f(0) = 1$, $f(1) = 0$.

Problem 2. *Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $\forall x \in \mathbb{R}$,*

$$x^2 f(x) + f(1-x) = 2x - x^4$$

Problem 3. *Find all real valued functions on the reals satisfying (1) $f(0) = 1/2$ (2) for some real a we have $\forall x, y \in \mathbb{R}$,*

$$f(x+y) = f(x)f(a-y) + f(y)f(a-x)$$

Problem 4. *Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $\forall x, y, z, t \in \mathbb{R}$,*

$$[f(x) + f(z)][f(y) + f(t)] = f(xy - zt) + f(xt + yz)$$

Problem 5. *Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $\forall x, y \in \mathbb{R}$,*

$$f(x^2 + f(y)) = y + f(x)^2$$