Financial Frictions in Macroeconomic Fluctuations

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T he financial crisis that developed starting in the summer of 2007 has made it clear that macroeconomic models need to allocate a more prominent role to the financial sector for understanding the dynamics of the business cycle. Contrary to what has been often reported in popular press, there is a long and well-established tradition in macroeconomics of adding financial market frictions in standard macroeconomic models and showing the importance of the financial sector for business cycle fluctuations. Bernanke and Gertler (1989) is one of the earliest studies. Kiyotaki and Moore (1997) provide another possible approach to incorporating financial frictions in a general equilibrium model. These two contributions are now the classic references for most of the work done in this area during the last 25 years.

Although these studies had an impact in the academic field, formal macroeconomic models used in policy circles have mostly developed while ignoring this branch of economic research. Until recently, the dominant structural model used for analyzing monetary policy was based on the New Keynesian paradigm. There are many versions of this model that incorporate several frictions such as sticky prices, sticky wages, adjustment costs in investment, capital utilization, and various types of shocks. However, the majority of these models are based on the assumption that markets are complete and, therefore, there are no financial market frictions. After the financial crisis hit, it became apparent that these models were missing something crucial about the behavior of the macroeconomy. Since then there have been many attempts

to incorporate financial market frictions in otherwise standard macroeconomic models. What I would like to stress here is that the recent approaches are not new in macroeconomics. They are based on ideas already formalized in the macroeconomic field during the last two and a half decades, starting with the work of Bernanke and Gertler (1989). In this article I provide a systematic description of these ideas.
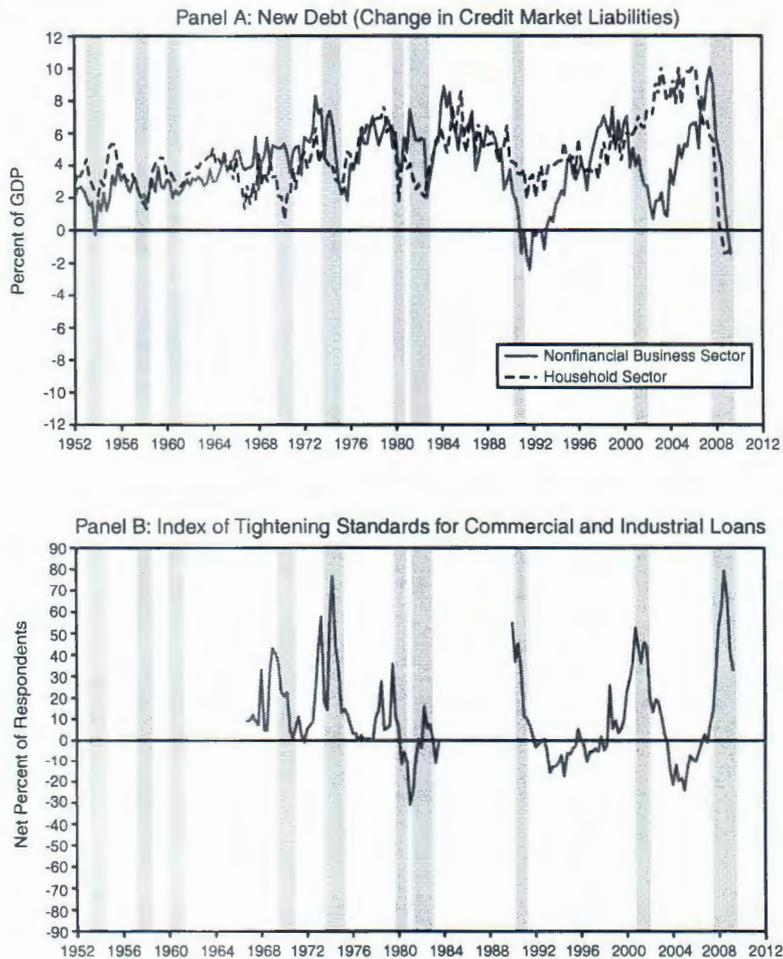
1.   WHY MODELING FRICTIONS IN FINANCIAL MARKETS?

Before adding complexity to the model, we would like to understand why it is desirable to have meaningful financial markets in macroeconomic models, besides the obvious observation that they seem to have played an important role in the recent crisis. One motivating observation is that the flows of credit are highly pro-cyclical. As shown in the top panel of Figure 1, the change in credit market liabilities moves closely with the cycle. In particular, debt growth drops significantly during recessions. The only exception is perhaps for the household sector in the 2001 recession. However, the growth in debt for the business sector also declined in 2001. Especially sizable is the drop in the most recent recession. The pro-cyclicality of corporate debt is also shown in Covas and Den Haan (2011) using Compustat data.

The cyclical properties of financial markets can be seen not only by the aggregate dynamics of credit flows (as shown in the top panel of Figure 1), but also by indicators of tightening credit standards. The bottom panel of Figure 1 plots the net fraction of senior bank managers reporting tightening credit standards for commercial and industrial loans in a survey conducted by the Federal Reserve Board. Clearly, more and more banks tighten their credit standard during recessions. Other indicators of credit tightening such as credit spreads, that is, interest rate differentials between bonds with differing ratings, convey a similar message as shown in Gilchrist, Yankov, and Zakrajsek (2009).

If markets were complete, the financial structure of individual agents, being households, firms, or financial intermediaries, would be indeterminate. We would then be in a Modigliani and Miller (1958) world and there would not be reasons for the financial flows to follow a cyclical pattern. However, the fact that credit flows are highly pro-cyclical and the index of tightening standards is countercyclical suggests that the complete-market paradigm has some limitations. This is especially true for the index of credit tightening.[1]

Of course, Figure 1 does not tell us whether it is the macroeconomic recession that causes the contraction in credit growth or the credit contraction

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[1] Although the pro-cyclicality of financial flows does not contradict Modigliani and Miller since the financial structure is indeterminate, when markets are complete there is no reason for lenders to change their "credit standards" over the business cycle. Here I interpret the index of credit standards as reflecting the characteristics of an individual borrower that are required to receive a loan. So it is something additional to the market-clearing risk-free interest rate.

Figure 1 Debt and Credit Market Conditions

Panel A: New Debt (Change in Credit Market Liabilities)

Panel B: Index of Tightening Standards for Commercial and Industrial Loans

Notes: Panel A shows change in the volume of credit market instruments in the households and business sector divided by gross domestic product. The data are from the Flow of Funds of the Federal Reserve Board. Panel B shows the index of credit tightening in commercial and industrial loans. The data are from the Senior Loan Officer Opinion Survey on Bank Lending Practices from the Federal Reserve Board. The survey was not conducted from 1984–1990. I thank Egon Zakrajsek for making available the historical data from 1967–1983.

that causes or amplifies the macroeconomic recession. It would then be convenient to distinguish three possible channels linking financial flows to real economic activity.

1. *Real activity causes movements in financial flows.* One hypothesis is that investment and employment respond to changes in real factors such as movements in productivity. In this case, borrowers cut their debt simply because they need less funds to conduct economic transactions. If this was the only linkage between real and financial flows, the explicit modeling of the financial sector would be of limited relevance for understanding movements in real economic activities.

2. *Amplification.* The second hypothesis is that the initial driving force of movements in economic activities are nonfinancial factors such as drops in productivity or monetary policy shocks. However, as investment and employment fall, the credit ability of borrowers deteriorates more than the financing need after the drop in economic activity. This could happen, for instance, if the fall in investment generates a fall in the market value of assets used as collateral. The presence of financial frictions will then generate a larger decline in investment and employment compared to the decline we would observe in absence of financial frictions. Therefore, financial frictions *amplify* the macroeconomic impact of the exogenous changes.

3. *Financial shocks.* A third hypothesis is that the initial disruption arises in the financial sector of the economy. There are no initial changes in the nonfinancial sector. Because of the disruption in financial markets, fewer funds can be channeled from lenders to borrowers. As a result of the credit tightening, borrowers cut on spending and hiring, and this generates a recession. I will refer to these types of exogenous changes as "credit" or "financial" shocks.

Most of the literature in dynamic macrofinance has focused on the second channel, that is, on the "amplification" mechanism generated by financial market frictions. More specifically, the central hypothesis is that financial frictions "exacerbate" a recession but are not the "cause" of the recession. Something wrong (a negative shock) first happens in the nonfinancial sector. This could be caused by "exogenous" changes in productivity, monetary aggregates, interest rates, preferences, etc. These shocks would generate a macroeconomic recession even in absence of financial market frictions. With financial frictions, however, the magnitude of the recession becomes much bigger.

The third channel, that is, the analysis of financial shocks as a "source" of business cycle fluctuations, has received less attention in the literature. More recently, however, a few studies have explored this possibility. In this article I will present the main theoretical ideas about the second and third channels, that is, "amplification" and "financial shocks." I will not focus on the first hypothesis only because, as observed above, if this was the most relevant channel of linkage between real and financial flows, the explicit modeling of the

financial sector would be of limited relevance for understanding movements in real macroeconomic activities.

2.   MODELING FINANCIAL FRICTIONS

Technically, financial frictions emerge when trade in certain assets cannot take place. In an Arrow-Debreu world with state-contingent trades, markets for some contingencies are missing and, therefore, there is a limit to the feasible range of intertemporal and intratemporal trades. In practical terms this implies that agents are unable to anticipate or postpone spending (for consumption or investment) or insure against uncertain events (to smooth consumption or investment). Of course, this becomes relevant only if agents are heterogeneous. Therefore, any models with financial frictions share the following features:

1. *Missing markets:* Some asset trades are not available or feasible.

2. *Heterogeneity:*   Agents   are   heterogeneous   in   some   important dimension.

I should clarify that these two features are necessary but not sufficient for incomplete markets to play an important role. That we need heterogeneity is obvious. If all agents are homogeneous, there is no reason to trade claims intertemporally or intratemporally. So the fact that some markets are missing becomes irrelevant. Also, if agents could trade any type of contingency, we would have an economy with complete markets. On the other hand, the fact that some markets are missing may be irrelevant if in equilibrium agents choose voluntarily not to trade in these markets. Therefore, market incompleteness and heterogeneity must take specific configurations. In the next two subsections, I will describe first the most common approaches to modeling missing markets and then I will discuss the most common approaches used to generate heterogeneity.

Missing Markets

The approaches used to model missing markets can be divided into two categories: "exogenous" market incompleteness and "endogenous" market incompleteness.

1. *Exogenous market incompleteness.* The first category includes models that impose exogenously that certain assets cannot be traded. For example, it is common to assume that agents can hold bonds (issue debt if negative) but they cannot hold assets with payoffs contingent on information that becomes available in the future. This approach does not attempt to explain why certain assets cannot be traded but it takes a

more pragmatic approach. Since a large volume of financing observed in the real economy is in the form of standard debt contracts, while the volume of state contingent contracts is limited, it makes sense to assume that debt contracts are the only financial instruments that are available. A further restriction, which is also exogenously imposed, is that the total amount of debt cannot exceed a certain limit (exogenous borrowing constraint). Of course, the goal of this literature is not to explain why markets are incomplete but to understand the consequences of market incompleteness.

2. *Endogenous market incompleteness.* The second category includes models in which the set of feasible contracts are derived from agency problems. The idea is that markets are missing because parties are not willing to engage in certain trades because they are not enforceable or incentive-compatible. What this means is that the borrower is unable to borrow or insure against the risk because, with high liabilities and full insurance, he or she would act against the interests of the lender. Typically, endogenous market incompleteness is derived from two agency problems:

  (a) *Limited enforcement.* The idea of limited enforcement is that the lender is fully capable of observing whether or not the borrower is fulfilling his or her contractual obligations. However, there are no tools the lender can use to enforce the contractual obligations. For example, even if the lender knows that the borrower is not exerting effort or is diverting funds, it may be difficult to prove it in court. There could also be legal limits to what the lender can enforce. For example the law does not allow the lender to force the borrower to work in order to repay the debt (no slavery).

  (b) *Information asymmetry.* Information asymmetries also limit the ability of lenders to force the borrowers to fulfil their obligations. In this case, the limit derives from the inability to observe the borrower's action. For example, if the repayment depends on the performance of the business and the performance depends on unobservable effort, the borrower may have an incentive to choose low effort.

From a technical point of view, models with limited enforcement are typically easier to analyze than models with information asymmetries. Both models, however, share a common property: higher is the net worth of borrowers and higher is the (incentive-compatible) financing that can be raised externally—a recurrent factor in the theoretical analysis that will be conducted in the remaining sections of this article.

Heterogeneity

There are many approaches used in the literature to generate heterogeneity. One popular approach is that agents are ex-ante identical but they are subject to idiosyncratic shocks. Therefore, the heterogeneity derives from the assumption that at any point in time each agent receives a different shock. For example, in the Bewley (1986) economy agents receive stochastic endowments. Because at any point in time there are agents with low endowments while others have high endowments, it will be optimal to sign state-contingent contracts that insure the endowment risks and allow for consumption smoothing. With these contracts, agents receive payments when their endowments are low and make payments when their endowments are high.

If markets were complete, the analysis of this model would be simple. With incomplete markets, however, the model generates high dimensional heterogeneity. Even if agents are initially or ex-ante homogeneous, in the long run there will be a continuum of asset holdings. Because the state dimensionality makes the characterization of the equilibrium challenging, the majority of applications of Bewley-type economies have abstracted from aggregate uncertainty and business cycle fluctuations. An exception is Krusell and Smith (1998). Other exceptions include Cooley, Marimon, and Quadrini (2004), where the heterogeneity is on the production side and, more recently, Guerrieri and Lorenzoni (2010) and Khan and Thomas (2011). In general, however, the majority of studies investigating the importance of financial frictions for macroeconomic fluctuations have tried alternative approaches that keep the degree of heterogeneity small.

A common approach is to assume that there are only two types of agents with permanent differences in preferences and/or technology. In equilibrium one agent ends up being the *borrower* and the other the *lender*. Alternatively, there could be a continuum of heterogeneous agents but their aggregate behavior can be characterized by a single representative agent thanks to linear aggregation. This is the case, for example, in Carlstrom and Fuerst (1997); Bernanke, Gertler, and Gilchrist (1999); and Miao and Wang (2010). Although entrepreneurs face uninsurable idiosyncratic risks and there is a distribution of entrepreneurs over net worth, the linearity of technology and preferences allows for the derivation of aggregate policies that are independent of the distribution. So, effectively, the reduced form in these models is also characterized by only two representative agents: households/workers and entrepreneurs.

Still, the fact that firms are owned by entrepreneurs and external financing is limited is not enough for financial frictions to play an important role. Even if entrepreneurs (firms) are temporarily financially constrained, that is, they would like to borrow more than they are allowed, over time they could save enough resources to make the financial constraints nonbinding. Therefore, further assumptions need to be made in order for the borrowing constraints to also be relevant in the long run. This is achieved in different ways.

1. *Finite life span.* A common modeling approach is based on the assumption that borrowers have a finite life span. For example, in overlapping generations models, it is commonly assumed that newborn agents have no initial assets and, therefore, they are financially constrained in the first stage of their lives. Over time agents accumulate assets and become unconstrained. However, since there is a continuous entrance of newborns, at any point in time there are always some agents who face binding financial constraints. A similar idea is applied in industry dynamics models where exiting firms are replaced by new entrant firms.

2. *Different discounting.* Another common approach is to assume that borrowers are infinitely lived but they discount the future more heavily than lenders. What this implies is that the cost of external financing is lower than the cost of internal funds. As a result, debt is preferred to internal funds. This insures that borrowers do not save enough to make the borrowing constraint irrelevant. Then, unanticipated shocks could lead to a larger spending response of borrowers because of the binding constraint.

3. *Tax benefits.* A similar approach to the differential discounting is the assumption that there are tax benefits of debt. For example, the tax deductibility of interest payments from corporate earnings generates a preference for debt over equity, and corporations tend to leverage up. However, if the firm is unexpectedly required to de-leverage and it is difficult to replace debt with equity in the short term, the result could be large drops in investment and employment.

4. *Bargaining position.* A further assumption proposed in the literature is that external financing (debt/outside equity) is preferred to inside financing (entrepreneurial equity), not because of differential discounting or tax benefits, but because it affects the bargaining position of firms in the negotiation of wages and/or executive compensation. The idea is that, if the compensation of workers and managers is determined through bargaining (in the case of workers the bargaining could be with unions), high-leveraged firms would be able to bargain lower compensations simply because the bargaining surplus is reduced by the debt.

But independent of the particular modeling approach, all models with financial market frictions are characterized by the presence of at least two groups of agents—one group that would like to raise external funds and one group that provides at least some of the funds.

3. A SIMPLE THEORETICAL FRAMEWORK

The discussion conducted so far has provided an informal description of the basic features of models used to study the importance of financial market frictions for the business cycle. Now I provide a more analytical description using a formal model that is rich enough to capture the various ideas proposed in the literature but remains analytically tractable.

To achieve this goal, I assume that there are only two periods—period 1 and period 2—and two types of agents—a unit mass of workers and a unit mass of entrepreneurs. Variables that refer to period 2 will be indicated with a prime superscript.

The lifetime utility of workers is

$$E\left\{ c - \frac{h^2}{2} + \delta c' \right\},$$

where c and h are consumption and labor in period 1 and c' is consumption in period 2. The lifetime utility of entrepreneurs is

$$E\left\{ c + \beta c' \right\}.$$

Thus, entrepreneurs' utility is also linear in consumption but there is no disutility from working. The assumption of risk neutrality is not essential but it simplifies the analysis. When relevant, I will comment on the importance of risk neutrality.

I now describe what happens in each of the two periods.

- *Period 1.* Entrepreneurs enter period 1 with capital K and debt B owed to workers. In principle, B could be negative. However, as we will see, this case is not of theoretical interest.

  There are two production stages during the first period. In the first stage, intermediate goods are produced with capital and labor. In the second stage, the intermediate goods are used as inputs in the production of consumption and new capital goods.

  - *Stage 1: Production of intermediate goods.* Intermediate goods are produced by entrepreneurs with the production function

    $$y = AK^\theta h^{1-\theta},$$

    where A is the aggregate level of productivity, K is the input of capital, and h is the input of labor supplied by workers.

  - *Stage 2: Production of final goods.* In this stage, intermediate goods are used as inputs in the production of consumption and new capital goods. The transformation in consumption goods is simple: One unit of intermediate goods is transformed into one

unit of consumption goods. New capital goods are produced by individual entrepreneurs using the technology

$$k^n = \omega i,$$

where i is the quantity of intermediate goods used in the production of new capital goods and ω is the idiosyncratic productivity realized after the choice of i. The cumulative density function is denoted by $\Phi(\omega)$. Later we will consider two cases: $E\omega = 1$ and $E\omega = 0$. In the second case there is no production of investment goods and, therefore, the aggregate stock of capital in period 2 is the same as in period 1.

- *Period 2.* Second period production takes place only with the input of capital. Since this is the terminal period, only consumption goods are produced. There are two sectors of production.

  - *Sector 1: Entrepreneurial sector.* This is composed of firms owned by individual entrepreneurs with technology $y' = A'k'$, where k' is the input of capital acquired by the entrepreneur in period 1.

  - *Sector 2: Residual sector.* The second sector is formed by frictionless firms directly owned by workers with technology $y' = A'G(k')$. The function $G(.)$ is strictly increasing and concave and satisfies $G'(0) = 1$.

  The key difference between the entrepreneurial sector and the residual sector is that the former is more productive than the latter, that is, $G'(k') < 1$ for $k' > 0$. As we will see, in absence of financial frictions, production will take place only in the entrepreneurial sector. With frictions, part of the production could also take place in the less productive but frictionless residual sector. For simplicity I assume that A' is known in period 1 and, therefore, there is no aggregate uncertainty.

Before proceeding I impose the following conditions:

Assumption 1 *Entrepreneurs and workers have the same discounting, $\delta = \beta$. Furthermore, $\beta A' > 1$.*

It is often assumed in the literature that $\delta > \beta$, that is, entrepreneurs (borrowers) are more impatient than workers (lenders). This is an important assumption in an infinite horizon model. With only two periods, however, the discount differential does not play an important role, which motivates the assumption $\delta = \beta$. The condition $\beta A' > 1$, instead, guarantees that postponing consumption through investment is efficient since the discounted value of the productivity of capital in period 2 is greater than 1.

Timing Summary

The structure of the model described so far, although stylized, is fairly complex. There are important timing assumptions that are made to keep the model analytically tractable. To make sure that these assumptions are clear, it would be helpful to summarize the timing sequence.

1. Entrepreneurs start period 1 with capital K and debt B. Workers start with wealth B.

2. Entrepreneurs hire workers to produce intermediate goods with the technology $y = AK^\theta h^{1-\theta}$. The labor market is competitive and clears at the wage rate w.

3. Entrepreneurs purchase intermediate goods i to produce new capital goods using the technology $k^n = \omega i$. The choice of i is made before observing the idiosyncratic productivity ω.

4. At this point we are at the end of period 1. The idiosyncratic productivities are observed and all incomes are realized. Entrepreneurs and workers allocate their end-of-period wealth between current consumption and savings in the form of capital goods and/or financial instruments (bonds).

5. We are now in period 2. Production takes place with the capital inputs accumulated in the previous period.

6. Entrepreneurs repay the debt to workers and both agents consume their residual wealth.

Plan for the Theoretical Analysis

I have now completed the description of preferences, technology, and timing. What is left to describe are the financial frictions that impose additional constraints on the choices of debt. These will be specified in the analysis of the various cases reviewed in this article. The presentation will be organized in four main sections:

- Section 4 characterizes the equilibrium in the frictionless model. This provides the baseline framework to which I compare the various versions of the model with financial frictions.

- Section 5 presents the costly state verification model based on information asymmetries where the financial frictions have a direct impact on investment.

- Section 6 presents the collateral/limited enforcement model. I first show the properties of this model when the frictions have a direct impact

only on investment. I then extend the analysis to the case in which the frictions also have a direct impact on the demand of labor.

- Section 7 analyzes the impact of credit shocks. I first present the model with exogenous credit shocks and then I propose one possible approach to make these shocks endogenous through a liquidity channel. In this section I also show the importance of credit shocks in an open economy framework.

4.   BASELINE MODEL WITHOUT FINANCIAL FRICTIONS

I start with the characterization of the problem solved by workers

$$\max_{c,c',k',b'} \quad \left\{ c - \frac{h^2}{2} + \delta c' \right\}$$

subject to:

$$B + wh = c + \frac{b'}{R} + qk'$$

$$A'G(k') + b' = c'$$

$$c \geq 0, \quad c' \geq 0, \tag{1}$$

where B is the initial ownership of bonds, R is the gross interest rate, w is the wage rate, and q is the price of capital. Since A' is known in period 1, workers do not face any uncertainty.

The first two constraints are the budget constraints in period 1 and 2, respectively. They equalize the available resources (left-hand side) to the expenditures (right-hand side). The problem is also subject to the non-negativity of consumption in both periods. However, thanks to Assumption 1, c' will always be positive and we have to worry only about the non-negativity of consumption in period 1. Intuitively, since capital is very productive in period 2 and preferences are linear, agents may choose to maximize their savings in period 1.

The first-order conditions are

$$h = w(1 + \lambda) \tag{2}$$

$$(1 + \lambda)q = \delta A'G'(k') \tag{3}$$

$$1 + \lambda = \delta R, \tag{4}$$

where λ is the Lagrange multiplier associated with the non-negativity constraint on consumption in period 1.

The problem solved by entrepreneurs can be written as

$$\max_{h,i,c,k',b',c'} \quad E\left\{c + \beta c'\right\}$$

subject to:

$$AK^\theta h^{1-\theta} - wh + qK + (qE\omega - 1)i + \frac{b'}{R} = B + c + qk'$$

$$A'k' = b' + c'$$

$$c \geq 0, \quad c' \geq 0, \quad i \geq 0.$$

The first two constraints are the budget constraints in period 1 and 2, respectively. They equalize the available resources (left-hand side) to the expenditures (right-hand side). The terms $AK^\theta h^{1-\theta} - wh$ and $(qE\omega - 1)i$ are, respectively, the profit earned by the entrepreneur in the production of intermediate goods and the (expected) profit earned in the production of new capital goods.

As for workers, I do not have to worry about the non-negativity constraint on c'. The first-order conditions are

$$w = (1-\theta)AK^\theta h^{-\theta} \tag{5}$$

$$qE\omega = 1 \leq 1, \qquad (= \text{if } i > 0) \tag{6}$$

$$(1+\gamma)q = \beta A' \tag{7}$$

$$1 + \gamma = \beta R, \tag{8}$$

where γ is the Lagrange multiplier on the non-negativity constraint on consumption in period 1. Since $\delta = \beta$ (by Assumption 1), equations (4) and (8) imply $\lambda = \gamma$. What this means is that the non-negativity of consumption in period 1 is either binding for both agents or it is not binding for both of them.

Substituting the labor supply (2) in the demand of labor (5), we get the wage equation

$$w = (1-\theta)^{\frac{1}{1+\theta}} A^{\frac{1}{1+\theta}} K^{\frac{\theta}{1+\theta}} (1+\lambda)^{\frac{-\theta}{1+\theta}}. \tag{9}$$

Substituting back in the supply of labor, working hours can be expressed as

$$h = (1-\theta)^{\frac{1}{1+\theta}} A^{\frac{1}{1+\theta}} K^{\frac{\theta}{1+\theta}} (1+\lambda)^{\frac{1}{1+\theta}}. \tag{10}$$

Entrepreneurs' income in period 1, after the production of intermediate goods is

$$Y^e = AK^\theta h^{1-\theta} - wh, \tag{11}$$

where w and h are determined in (9) and (10). Therefore, the supply of labor and entrepreneurial income depend on the multiplier λ. The value of this variable depends on the assumption about $E\omega$. When I introduce financial frictions I will consider two cases: $E\omega = 1$ and $E\omega = 0$. The first case defines an economy with capital accumulation while the second case defines an economy with fixed capital.

- *Case 1: $E\omega = 1$.* Because $\beta A' > 1$, the intermediate goods produced
  in period 1 are all used in the production of capital goods. Thus, current
  consumption is zero for both entrepreneurs and workers. This implies
  that the multiplier $\lambda = \gamma$ is positive. Since investment i is positive,
  condition (6) is satisfied with equality, and therefore, $q = 1$. Then
  equations (7) and (8) imply that $R = A'$. Agents anticipate that the
  productivity of capital is high next period and it becomes convenient
  to save the whole income to take advantage of the higher return. The
  labor supply is higher than the wage since $\lambda = \gamma > 0$ (see equation
  [2]) and the demand of labor is determined by its marginal product (see
  equation [5]). The whole capital produced in period 1 is accumulated by
  entrepreneurs since the entrepreneurial sector is more productive than
  the residual sector and there are no agency problems in the repayment
  of the intertemporal debt b'.

  It is now easy to see the impact of productivity changes. An increase
  in current productivity A generates an increase in the supply of labor
  and output as we can see from equations (9)–(11), after replacing $1 +
  \gamma = \beta A'$ from equation (7), taking into account that $\lambda = \gamma$. Since
  the increase in income is saved, the productivity boom also generates
  an investment boom. There is no impact in current consumption but
  this is a consequence of assuming risk neutrality. With risk aversion,
  consumption in period 1 is also likely to increase in response to a
  persistent productivity improvement.

  An increase in A' also generates an increase in the current supply of
  labor (see equation [10] after substituting $1 + \gamma = \beta A'$), which in turn
  generates an increase in output and savings. Therefore, the model has
  the typical properties of the neoclassical business cycle model.

- *Case 2: $E\omega = 0$.* Since $E\omega = 0$, we can see from equation (6) that
  $i = 0$, that is, there is no capital accumulation. The whole capital K is
  acquired by entrepreneurs because the entrepreneurial sector is more
  productive than the residual sector and there are no agency problems in
  the repayment of the intertemporal debt b'. Since consumption cannot
  be zero for both workers and entrepreneurs, $\lambda = \gamma = 0$ (in absence
  of investment aggregate consumption in period 1 must be equal to
  aggregate production in period 1). This implies that the price of capital
  is $q = \beta A'$ (see equations [3] and [7]). In this way both agents are
  indifferent between current and future consumption and the new debt
  b' is undetermined.

  An increase in current productivity A generates an increase in the supply
  of labor and output as we can see from (9)–(11) after substituting $\lambda =
  0$. However, the productivity change in period 1 does not affect next
  period production since there is no capital accumulation. Similarly,

an increase in A' generates an increase in next period production but it does not have any impact on production in period 1. Again, this is because of the absence of capital accumulation. As we will see, this feature of the model will change with financial frictions.

5. COSTLY STATE VERIFICATION MODEL

In the costly state verification model frictions derive from information asymmetry. This is the centerpiece of the financial accelerator model proposed by Bernanke and Gertler (1989). The model has been further embedded in more complex macroeconomic models with infinitely lived agents by Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999).

To illustrate the key elements of the financial accelerator, I specialize the analysis to the case in which frictions are only in the production of capital goods and, in the analysis of this section, I assume that $E\omega = 1$. This guarantees that capital goods are produced and there is capital accumulation in the model.

The frictions derive from the assumption that ω is freely observable only by entrepreneurs. Other agents could observe ω but only at the cost μi. This limits the feasibility of financial contracts that are contingent on ω. As it is well known from the work of Townsend (1979), the optimal contract takes the form of a standard debt contract in which the entrepreneur promises to repay an amount that is independent of the realization of ω. If the entrepreneur does not repay, the lender incurs the verification cost and confiscates the residual assets.

Optimal Contract with Costly State Verification

The central element of this model is the net worth of entrepreneurs. Before starting the production of new capital goods, entrepreneurs' net worth is $n = qK + Y^e - B$, where Y^e is defined in (11). Therefore, if the entrepreneur purchases i units of intermediate goods, he or she has to borrow $i - n$ units of intermediate goods on the promise to pay back $(i - n)(1 + r^k)$ units of capital goods. Notice that the interest rate r^k is denominated in capital goods, which explains the different denomination of the loan (denominated in intermediate goods) and the repayment (in capital goods). The particular choice of the denomination is a simple convention that is inconsequential for the properties of the model.

After the realization of the idiosyncratic shock ω, the entrepreneur defaults only if the production of new capital goods is smaller than the debt repayment, that is, $\omega i \leq (1 + r^k)(i - n)$. We can then define $\bar{\omega}$ as the shock below which

the entrepreneur defaults, which is equal to

$$\bar{\omega} = (1 + r^k) \left(\frac{i - n}{i} \right).$$

This equation makes clear that the default threshold is increasing in the leverage ratio $\frac{i-n}{i}$ and in the interest rate. Assuming competition in financial markets, the interest rate charged by the lender must satisfy the zero-profit condition

$$q \left[\int_0^{\bar{\omega}(n,i,r^k)} (\omega - \mu) i \, \Phi(d\omega) + \int_{\bar{\omega}(n,i,r^k)}^\infty (1 + r^k)(i - n) \Phi(d\omega) \right] = i - n.$$

Notice that there is no interest in the cost of funds on the right-hand side of the equation since the loan is intraperiod, that is, issued and repaid in the same period. This is different from the intertemporal debt b'. The equation defines implicitly the interest rate charged by the bank as a function of n, i, q, which I denote as $r^k(n, i, q)$. The default threshold can also be expressed as a function of the same variables, that is, $\bar{\omega}(n, i, q)$.

Since entrepreneurs are risk neutral, the production choice is independent of the consumption/saving decision. More specifically, the optimal choice of i maximizes the expected entrepreneur's net worth, that is,

$$\max_i q \int_{\bar{\omega}(n,i,q)}^\infty \left[\omega i - (1 + r^k(n, i, q))(i - n) \right] \Phi(d\omega).$$

Notice that the integral starts at $\bar{\omega}$ because the entrepreneur defaults for values of $\omega < \bar{\omega}$ and the ex-post net worth is zero in the event of default.

Let $i(n, q)$ be the optimal scale chosen by the entrepreneur in the production of capital goods. We can define the net worth after production as

$$\pi(n, q, \omega) = \max \left\{ 0, \, q \left[\omega i(n, q) - (1 + r^k(n, i(n, q), q))(i(n, q) - n) \right] \right\}.$$

Using this function, the consumption/saving problem solved by the entrepreneur can be written as

$$\max_{c,c',k',b'} \quad \left\{ c + \beta c' \right\}$$

subject to:

$$\pi(n, q, \omega) = c + qk' - \frac{b'}{R}$$

$$c' = A'k' - b'$$

$$c \geq 0, \quad c' \geq 0.$$

Equilibrium and Response to Productivity Shocks

There are two possible equilibria depending on the net worth of entrepreneurs. In the first equilibrium, the net worth of entrepreneurs is sufficiently large that

the whole production of intermediate goods is used in the production of new capital goods. This case is similar to the baseline model without financial frictions characterized in Section 4.

The second type of equilibrium arises when the net worth of entrepreneurs is not large enough to use the whole production of intermediate goods to produce new capital goods. We have defined above $i(n, q)$ the production scale of entrepreneurs, that is, the demand of intermediate goods used in the production of new capital goods. Since there is a unit mass of entrepreneurs that are initially homogeneous, $i(n, q)$ is also aggregate investment. If $i(n, q) < AK^\theta h^{1-\theta}$, then only part of the production of intermediate goods is used in the production of capital goods. This implies that the consumption in period 1 of workers and/or entrepreneurs is positive. Thus, the multiplier associated with the non-negativity of consumption is $\gamma = 0$ and the equilibrium satisfies the first-order conditions

$$
\begin{aligned}
q &= \beta A' \\
1 &= \beta R.
\end{aligned}
$$

Thus, the price of capital is equal to $\beta A'$, which is bigger than one by Assumption 1. I will focus on this particular equilibrium since this is when financial frictions matter.

I can now study the response of the economy to productivity shocks, that is, changes in A and A'.

- *Increase in A.* The increase in A raises the net worth of entrepreneurs $n = qK + Y^e - B$, where Y^e is defined in (11). Since $q = \beta A'$, the price of capital q does not change if A' does not change. Therefore, the increase in net worth is only determined by the increase in capital income Y^e earned in the first stage of production.

  The next step is to see what happens to investment in response to the higher net worth. We have already seen that investment i increases with n. Therefore, the productivity improvement generates an investment boom and increases next period production. In this way the model generates a persistent impact of productivity shocks. This effect, however, is not necessarily bigger than the effects of a productivity shock in the baseline model without frictions characterized in Section 4. For this to be the case, the net worth n has to increase proportionally more than the increase in output. This requires $qK - B < 0$, which is unlikely to be an empirically relevant condition. Therefore, the model with financial frictions could generate a lower response to nonpersistent productivity shocks.

  If the shock is persistent, that is, a higher A implies a higher value of A', then the model would generate an increase in net worth also through

the market value of owned capital (as we will see next). The impact on investment could then be bigger.

- *Increase in A'.* An anticipated increase in A' generates an increase in the price of capital today since $q = \beta A'$. The price increase has two effects. First, since entrepreneurs own the capital K, the higher q generates an increase in the entrepreneur's net worth $n = qK + Y^e - B$. Notice that the initial leverage is higher, that is, the debt B relative to the owned capital K, and the (proportional) effect on the net worth is bigger. The increase in net worth affects investment similarly to the increase in current productivity. This first channel induces an increase in the production scale i without changing the probability of default if we assume that the leverage does not change.

The second effect derives from the impact on the intraperiod leverage. Since a higher q implies higher profits from producing capital goods, entrepreneurs have an incentive to expand production proportionally more than the increase in net worth, even if this increases the cost of external financing. As a result, the probability of default, or bankruptcy rate, increases in response to an anticipated productivity shock. Thus, the model generates pro-cyclical bankruptcy rates and pro-cyclical interest rate premiums—a point emphasized, among others, by Gomes, Yaron, and Zhang (2003).

One reason the model generates a pro-cyclical interest rate premium is because investment is very sensitive to the asset price q. The addition of adjustment costs as in Bernanke, Gertler, and Gilchrist (1999) could revert this property. In this case, the higher price of capital improves the net worth position of the entrepreneur, but the adjustment cost contains the expansion of the production scale. As a result, entrepreneurs could end up with a lower leverage and lower probability of default. See also Covas and Den Haan (2010).

Quantitative Performance

In general, it is not easy for the model to generate large amplification effects in response to productivity changes. In fact, as observed above, financial frictions could dampen the impact of productivity shocks. Because of the higher profitability in the production of capital goods, entrepreneurs would like to expand the production scale. However, as they produce more, the cost of external financing increases. In a frictionless economy, instead, the cost of external finance does not increase with individual production. So the initial impact on investment is larger. In essence, financial frictions act like adjustment costs in investment, which could dampen aggregate volatility. Wang

and Wen (forthcoming) provide a formal analysis of the similarity between financial frictions and adjustment cost at the aggregate level.

Even though the model has difficulties generating large amplifications, it has the potential to generate greater persistence. In fact, higher profits earned by entrepreneurs allow them to enter the next period with higher net worth. This cannot be shown explicitly with the current model since there are only two periods. However, suppose that entrepreneurs enter period 1 with a higher K made possible by the higher profits earned in the previous periods. This will reduce the external cost of financing, allowing entrepreneurs to produce more capital goods, which in turn increases production in future periods. The model could then generate a hump-shape response of output as shown in Carlstrom and Fuerst (1997).

Although quantitative applications of the financial accelerator do not find large amplification effects of productivity shocks, it could still amplify the macroeconomic response to other types of shocks. For example, Bernanke, Gertler, and Gilchrist (1999) add adjustment costs in the production of capital goods in order to generate larger fluctuations in q and find that the financial accelerator could generate sizable amplifications of monetary policy shocks.

6.   COLLATERAL CONSTRAINT MODEL

Here I illustrate the main idea of models with collateral constraints as the one studied in Kiyotaki and Moore (1997). An alternative to models with collateral constraints is the consideration of optimal contracts subject to enforcement constraints as in Kehoe and Levine (1993) and Cooley, Marimon, and Quadrini (2004). However, the business cycle implications of these two modeling approaches are similar.

To illustrate the idea of the collateral model, I assume that the frictions are not in the production of capital goods, as in the costly state verification model. Instead they derive from the ability of borrowers to repudiate their intertemporal debt. In some models, like in Kiyotaki and Moore (1997), it is even assumed that physical capital is not reproducible. Therefore, in this section I assume that $E\omega = 0$ and all intermediate goods are transformed one to one into consumption goods. I denote by $\overline{K}$ the aggregate fixed stock of capital. Since capital is not reproducible, its price fluctuates endogenously in response to changing market conditions. The price fluctuation plays a central role in the model. An alternative way to generating price fluctuations is to relax the assumption that capital is not reproducible but with the addition of adjustment costs in investment and/or risk aversion.

Frictions on the Intertemporal Margin

From an efficiency point of view, the stock of capital should be allocated between entrepreneurs and workers to equalize their marginal product in period 2. More specifically, given $K^{e'}$, the capital allocated to the entrepreneurial sector (that is, capital purchased by entrepreneurs), efficiency requires $A' = A'G'(\overline{K} - K^{e'})$. The first term is the expected marginal productivity in the entrepreneurial sector and the second is the marginal productivity in the residual sector. Since $G'(.)$ is strictly decreasing and $G'(0) = 1 < A'$, the equalization of marginal productivities requires $K^{e'} = \overline{K}$, that is, all the capital should be allocated to the entrepreneurial sector in period 2.

The problem is that entrepreneurs may be unable to purchase $K^{e'} = \overline{K}$ in period 1. Because of limited enforceability of debt contracts, entrepreneurs are subject to the collateral constraint

$$b' \le \xi q' k'.$$

Here b' is the new debt, k' is the capital purchased by an individual entrepreneur, q' is the expected price of capital in period 2, and $\xi < 1$ is a parameter that captures possible losses associated with the reallocation of capital in case of default.

The theory underlying this constraint is developed in Hart and Moore (1994) and it is based on the idea that entrepreneurs cannot be forced to produce once they renege on the debt. Thus, in case of default the lender can only recover a fraction ξ of the capital that can be resold at price q'. Since this is the last period in the model, the price of capital would be zero in the second period. In an infinite horizon model, however, the price would not be zero because the capital can still be used in production in future periods. In our two-period model we can achieve the same outcome by assuming that a fraction ξ of the liquidated capital can be reallocated to the residual sector. Therefore, the liquidation price of capital in period 2 is equal to $q' = \xi A'G'(\overline{K} - K^{e'})$. Since $G'(.) \le 1$ and only a fraction ξ can be resold, the value of capital for lenders is smaller than for entrepreneurs. This is what limits the entrepreneurs' ability to borrow.

Before continuing I should observe that, in absence of capital accumulation, period 1 consumption cannot be zero for both workers and entrepreneurs. This is because period 1 production can only be used for consumption. Thus, the first-order conditions for workers are given by (2)–(4) but with $\lambda = 0$ and the supply of labor is $h = w$.

The problem solved by entrepreneurs is

$$\max_{h,k',b'} \quad \left\{ c + \beta c' \right\}$$

subject to:

$$c = q\overline{K} + A\overline{K}^{\theta} h^{1-\theta} - wh - B + \frac{b'}{R} - qk'$$

$$\xi q'k' \geq b'$$

$$c' = A'k' - b',$$

$$c \geq 0, \quad c' \geq 0, \tag{12}$$

which is deterministic since there is no capital production ($\omega = 0$) and A' is perfectly anticipated.

The first-order condition for the input of labor is still given by (5), that is, the entrepreneur equalizes the marginal product of labor to the wage rate. At the center stage of the model are the choices of next period capital and debt. The first-order conditions for k' and b' in problem (12) are

$$(1+\gamma)q = \beta A' + \mu\xi q' \tag{13}$$

$$1+\gamma = (\beta + \mu)R, \tag{14}$$

where μ and γ are, respectively, the Lagrange multipliers associated with the collateral constraint and the non-negativity of consumption in period 1.

I can now use equations (13)–(14) together with (3)–(4) to derive an expression for μ. Using the fact that the liquidation price of capital in period 2 is $q' = A'G'(\overline{K} - K^{e'})$, we derive

$$\mu = \frac{\beta[1 - G'(\overline{K} - K^{e'})]}{(1 - \xi)G'(\overline{K} - K^{e'})}. \tag{15}$$

This equation relates the multiplier μ to the capital accumulated by entrepreneurs $K^{e'}$. Since the function $G(.)$ is concave, $G'(\overline{K} - K^{e'})$ is increasing in $K^{e'}$. Therefore, if the capital accumulated by entrepreneurs is higher, μ is lower.

The equilibrium can take two configurations.

- *All the capital is accumulated by entrepreneurs.* In the first equilibrium entrepreneurs have sufficient net worth to purchase all the capital, that is, $K^{e'} = \overline{K}$. Equation (15) then implies that $\mu = 0$ since $G'(0) = 1$. In this case, entrepreneurs' consumption is positive ($\gamma = 0$) and the price of capital is $q = \beta A'$.

  This is possible only if entrepreneurs start with sufficiently high net worth, that is, small B. To see this, consider an entrepreneur's budget constraint when the entrepreneur borrows up to the limit and chooses zero consumption. Substituting $c = 0$ and $b' = \xi q'k'$, the budget constraint becomes $q\overline{K} + Y^e + \xi q'k'/R = B + qk'$, which can be

rearranged to

$$\left(q - \frac{\xi q'}{R}\right) k' = q\overline{K} + Y^e - B. \tag{16}$$

The term $Y^e = A\overline{K}^\theta h^{1-\theta} - wh$ is the entrepreneur's income earned in period 1.

Equations (3)–(4) imply $A'G'(\overline{K} - K^{e'}) = qR$. Furthermore, using $q' = A'G'(\overline{K} - K^{e'})$, the above condition can be written as

$$k'_{max} = \left(\frac{1}{1-\xi}\right)\left(\overline{K} - \frac{B - Y^e}{q}\right). \tag{17}$$

This is the maximum capital that entrepreneurs can buy given the capital price $q = \beta A'$, which I made explicit by adding the subscript. It depends negatively on B. Therefore, if the initial net worth is not sufficiently high, entrepreneurs will be unable to purchase $\overline{K}$ and some of the capital will be inefficiently allocated to the residual sector. In this case, $K^{e'} = k'_{max} < \overline{K}$. We are then in the second type of equilibrium configuration.

- *Only part of the capital is accumulated by entrepreneurs.* In the second equilibrium, entrepreneurs choose zero consumption and the collateral constraint is binding. Since entrepreneurs cannot purchase enough capital, $G'(\overline{K} - K^{e'}) < 1$. Then equation (15) tells us that $\mu > 0$ and equation (14) implies that $\gamma > 0$ since $\beta R = 1$ (from [4] if workers' consumption is positive, implying $\lambda = 0$). Therefore, the entrepreneur borrows up to the limit and the non-negativity constraint on consumption is binding.

Using the binding collateral constraint and zero consumption, the budget constraint can be rewritten again as in (16). This expression provides a simple intuition for the key mechanism of the model. The cost of one unit of capital, q, can be financed with $\frac{\xi q'}{R}$ units of debt and the rest must be financed with owned wealth. Therefore, $q - \frac{\xi q'}{R}$ is the minimum down payment required on each unit of capital. Multiplied by k' we get the total down payment necessary to purchase k' units of capital. In order to make the down payment, the entrepreneur needs to have enough net worth, which is the term on the right-hand side of (16). Therefore, the lower is the entrepreneurs' net worth, the lower is the amount of capital allocated to entrepreneurs. Since entrepreneurs are more productive than producers in the residual sector of the economy, lower net worth in period 1 implies lower production in period 2.

As equation (16) makes clear, the capital allocated to the entrepreneurial sector depends crucially on the equilibrium prices R, q, and q'. Although all three prices contribute to the equilibrium outcome, it will

be helpful to focus on q and q' to see the importance of asset prices. There are several effects induced by changes in these prices.

– *Current price*: An increase in the current price, q, has two effects. On the one hand, it increases the entrepreneur's net worth $q\overline{K} + Y^e - B$. On the other hand, it increases the cost of purchasing new capital. The first effect has a positive impact on k', while the impact of the second effect is negative.

– *Next period price*: An increase in the (expected) next period price, q', allows entrepreneurs to issue more debt. Therefore, for a given net worth, more capital can be purchased.

Following Kiyotaki and Moore (1997), suppose that q and q' both increase by the same proportion. For example they both increase by 1 percent.[2] Provided that $B > Y^e$, this generates an increase in the capital purchased by entrepreneurs, which, in the next period, increases output. The condition $B > Y^e$ is a leverage condition. Therefore, if entrepreneurs enter the period with a high leverage, a persistent increase in prices generates an output boom.

How would the response change if contracts were enforceable? This is equivalent to the equilibrium in which the collateral constraint is not binding. In particular, all the capital is purchased by entrepreneurs since they can borrow without limit. Then a change in price would not affect the allocation of $\overline{K}$ and would not have any additional impact on aggregate production beyond the direct impact of the factors that cause the price change.

Response to Productivity Shocks

I will now focus on the equilibrium in which the collateral constraint is binding, that is, the equilibrium that prevails if entrepreneurs are highly leveraged. In a general model with infinitely lived agents this would arise in the long run if entrepreneurs have some incentives to take on more debt. As discussed in Section 2, there are different assumptions made in the literature to have this property. For example, a common assumption is that entrepreneurs (borrowers) are more impatient than workers (lenders). In the simple two-period model considered here, however, we can simply take the initial leverage to be sufficiently high.

_____

[2] To facilitate the intuition, I take a partial equilibrium approach here and assume that the prices change exogenously.

If the collateral constraint is binding, the capital acquired by entrepreneurs is given by equation (17), which for convenience I rewrite here:

$$K^{e'} = \left(\frac{1}{1-\xi} \right) \left(\overline{K} - \frac{b - Y^e}{q} \right). \tag{18}$$

We now consider the impact of an increase in current and (anticipated) future productivity.

- *Increase in A.* The higher value of A increases entrepreneurs' income Y^e in period 1 (see equation [11]). We see from equation (18) that this induces an increase in $K^{e'}$. Essentially, entrepreneurs earn higher capital income in period 1 and this allows them to purchase more capital for period 2.

  In addition to this direct effect, there is an indirect effect induced by the price of capital. Since $K^{e'}$ increases, equation (3) implies that the current price of capital q also increases. As long as $B > Y^e$, that is, entrepreneurs are sufficiently leveraged, the increase in q induces a further increase in $K^{e'}$. Since entrepreneurs are more productive, that is, $G'(.) < 1$ for $K^{e'} < \overline{K}$, the reallocation of productive capital to the entrepreneurial sector generates an output boom in period 2. This second effect comes from the endogeneity of the collateral constraint, which depends on the market price q. Since the value of capital depends on q while the value of debt is fixed, the change in price has a large impact on the net worth if entrepreneurs are highly leveraged. This is the celebrated "amplification" effect of productivity shocks induced by endogenous asset prices.

- *Increase in A′.* Suppose that A' increases, that is, in period 1 we expect a higher productivity in period 2. We can think of this as a "news" shock. In this way it relates to the recent literature that investigates the impact of anticipated future productivity changes on the macroeconomy. See, for example, Beaudry and Portier (2006) and Jiamovich and Rebelo (2009). Here I show that financial markets could be an important transmission of these news shocks.

  From equation (2) we see that an increase in A' generates an increase in the price of capital q. Then, equation (18) shows that the increase in q induces a reallocation of capital to the entrepreneurial sector, further increasing q. This implies that production in period 2 increases more than the increase in productivity. We thus have an "amplification" effect. As far as current production is concerned, however, output does not change. We will see in the next section that, with the addition of working capital, the anticipated news can also affect employment in the current period. Therefore, in addition to generating an immediate

asset price boom, the news shock also generates an immediate macro-economic boom. This mechanism has been explored in Jermann and Quadrini (2007) and Chen and Song (2009).

Although we have considered only the case of nonreproducible capital, similar results apply when there is capital accumulation together with adjust-ment costs on investment. With investment adjustment costs, the price of capital is not always one. An increase in future productivity raises the demand of capital, inducing an asset price boom, which in turn amplifies the impact of the initial productivity improvement. Sometimes the adjustment costs can be in the form of capital irreversibility as in Caggese (2007).

Quantitative Performance

There are many quantitative applications of the collateral model. Some-times the borrowers are households engaged in real estate investments as in Iacoviello (2005). Other studies consider firms to be in need of funds for productive investments. However, the quantitative amplification induced by collateral constraints is often weak. This point has been emphasized in Cordoba and Ripoll (2004).

There are two reasons for the weak amplification. Similar to the simple model described above, for a group of models proposed in the literature, the "direct" effect of the frictions is on investment, not on the input of labor. Although this has the potential to generate large fluctuations in investments, the production inputs—capital and labor—are only marginally affected by this mechanism. As a result, output fluctuations are not affected in important ways by the financial frictions. I would also like to point out that the consideration of risk-averse agents will further reduce the amplification effects since savings, and therefore investments, will become more stable (see Kocherlakota [2000] and Cordoba and Ripoll [2004]). For the financial frictions to generate large output fluctuations that are in line with the data, they need to have a direct impact on labor. This point will be further developed in the next section.

The second reason for the weak amplification is that typical macromodels do not generate large asset price fluctuations even with the addition of binding marginal requirements (see Coen-Pirani [2005]). The centerpiece of the am-plification mechanism induced by the collateral constraint model is the fact that the availability of credit, and therefore investment, depends on the price of assets, that is,

$$b' \leq \xi q' k'.$$

In economic expansions q' increases and this allows for more capital invest-ment thanks to the relaxation of the borrowing constraint. However, for this mechanism to be quantitatively important, the model should generate sizable fluctuations in q', which is typically not the case in standard macromodels. In

this regard, the inability of the model to generate large amplification effects is more a consequence of the poor asset price performance of macromodels (which generate much lower asset price fluctuations than in the data) than the weakness of the collateral or financial accelerator mechanisms.

This suggests that an improvement in the asset price performance of macromodels could also enhance the amplification effect induced by financial frictions. In making this conjecture, however, we should use some caution. If the model generates large asset price fluctuations, borrowing up to the limit becomes riskier. Thus, agents may choose to stay away from the limit, that is, they will act in a precautionary manner. As a result, it is not obvious whether large asset price fluctuations will generate large macroeconomic fluctuations since, as shown in the simple model studied above, this requires the collateral constraint to be binding. But with precautionary behavior, the borrowing limit is only occasionally binding.

Unfortunately, exploring the quantitative importance of occasionally binding constraints cannot be done with local approximation techniques, which is the dominant approach used to study quantitative general equilibrium models. It is only recently that the importance of occasionally binding constraints for business cycle fluctuations has been fully recognized. Mendoza (2010) is one of the first articles that explores this issue quantitatively. I will return to the issue of occasionally binding collateral constraints later.

Working Capital Model

The financial mechanisms presented so far affect the transmission of productivity shocks through the investment channel. For example, in the costly state verification model, the entrepreneur's net worth affects the production of new capital goods, which in turn affects next period production. In the model with collateral constraints, the net worth of entrepreneurs also plays a central role. Higher net worth allows entrepreneurs to purchase more capital. As a result, a larger fraction of productive assets are used in the more productive entrepreneurial sector enhancing aggregate output. In both models the price of capital q plays a central role. However, this mechanism has a limited impact on labor.

The intuition for the weak impact on labor is simple. If we use a Cobb-Douglas production function $y = AK^\theta h^{1-\theta}$, an increase in the input of capital increases the demand of labor because h is complementary to K. However, even though investment is highly volatile, the volatility of capital is small. Thus, changes in investment that are quantitatively plausible are unlikely to generate large fluctuations in labor. Empirically, however, labor input fluctuations are an important driver of output volatility. So in general, having financial frictions that primarily affect investment may not be enough for the frictions to play a central role in labor and output fluctuations. A more direct impact can be obtained if financial frictions directly affect the demand of labor.

One way to achieve this is by assuming that employers need working capital, which is complementary to labor.

The idea of working capital is not new in macroeconomics. For example, the limited participation models of monetary policy are based on the idea that producers need to finance working capital. See, for example, Christiano and Eichenbaum (1992); Fuerst (1992); Christiano, Eichenbaum, and Evans (1997); and Cooley and Quadrini (1999, 2004). See also Neumeyer and Perri (2005) for the modeling of working capital in a nonmonetary model. On one hand, besides the need of working capital, there are not other financial frictions in these models. On the other hand, business cycle models with financial frictions have mostly focused on investment, posing little importance on working capital. Jermann and Quadrini (2006), Mendoza (2010), and Jermann and Quadrini (forthcoming) are attempts at merging the two ideas: working capital needs with financially constrained borrowers.

To show how working capital interacts with financial constraints, I consider again the collateral model studied in the previous section. The only additional assumption is that entrepreneurs also need working capital in the first period of production. Specifically, they need to pay wages before the realization of revenues. To make these payments, entrepreneurs must borrow wh. This is an intraperiod loan, and therefore, there are no interest payments. The collateral constraint becomes

$$b' + wh \leq \xi q' k'. \tag{19}$$

The left-hand side is the total debt: intertemporal debt that will be paid back next period and the intraperiod debt that needs to be repaid at the end of period 1. The right-hand side is the collateral value of assets.

The problem solved by the entrepreneur is similar to (12) but with the new collateral constraint, that is,

$$\max_{h,k',b'} \quad \left\{ c + \beta c' \right\}$$

subject to:

$$c = q\overline{K} + A\overline{K}^\theta h^{1-\theta} - wh - B + \frac{b'}{R} - qk'$$

$$\xi q' k' \geq b' + wh$$

$$c' = A'k' - b'$$

$$c \geq 0, \quad c' \geq 0. \tag{20}$$

The first-order conditions are also similar with the exception of the optimality condition for the input of labor, which becomes

$$(1 - \theta)A\overline{K}^\theta h^{-\theta} = w(1 + \mu). \tag{21}$$

The variable μ is the Lagrange multiplier associated with the collateral constraint as in the model without working capital. The multiplier creates a

wedge in the demand for labor.[3] When the collateral constraint is tighter, μ increases and the demand for labor declines.

Using the supply of labor, $h = w$, the wage rate is

$$w(\mu) = \left(\frac{1-\theta}{1+\mu}\right)^{\frac{1}{1+\theta}} A^{\frac{1}{1+\theta}} \overline{K}^{\frac{\theta}{1+\theta}}.$$

We can see that the wage depends negatively on the multiplier μ, which I made explicit in the notation. This also implies that the entrepreneur's income, $Y^e(\mu) = A\overline{K}^\theta h^{1-\theta} - wh$, depends on μ.

The budget constraint for the entrepreneur under a binding collateral constraint (and zero consumption) is

$$\left(q - \frac{\xi q'}{R}\right) k' = q\overline{K} + Y^e(\mu) - B. \tag{22}$$

From this equation I can derive the maximum capital that entrepreneurs can acquire as

$$k'_{max} = \left(\frac{1}{1-\xi}\right) \left(k - \frac{B - Y^e(\mu)}{q}\right). \tag{23}$$

The actual capital acquired in equilibrium by entrepreneurs is $K^{e'} = \min\left\{k'_{max}, \overline{K}\right\}$.

Response to Productivity Shocks

I now consider the impact of changes in current and future productivity.

- *Increase in A.* Keeping constant μ, the higher productivity induces an increase in entrepreneurial income $Y^e(\mu)$. This implies that the net worth of entrepreneurs increases and, as we can see in (23), more capital will be allocated to the entrepreneurial sector.

  The next step is to see what happens to the price of capital, q, and to the multiplier μ. From equation (3) we see that the higher $K^{e'}$ (smaller capital k' accumulated by workers) must be associated with an increase in the price of capital q. As long as $B > Y^e(\mu)$, that is, entrepreneurs are sufficiently leveraged, the increase in q further increases $K^{e'}$.

  We can now see what happens to the Lagrange multiplier μ. According to equation (15), an increase in $K^{e'}$ must be associated with a decline in μ. Going back to the first-order condition for labor—equation (21)—we observe that this reduces the labor wedge and generates an increase in the demand for labor, busting current production.

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[3] It is common in the literature to use the phrase "labor wedge" to refer to terms that modify the optimality condition for the input of labor that we would have without frictions. Later I will discuss in more detail the issue of the labor wedge and provide a more precise definition.

To summarize, the model with working capital can generate an amplification of productivity shocks also in the current period, in addition to next period output. A key element of the amplification mechanism is the endogeneity of the asset price q. Because of the asset price boom, the borrowing constraint is relaxed and firms can borrow more. They will use the higher borrowing to increase both current employment and next period capital.

- *Increase in A'.* Let's consider now the impact of an anticipated productivity improvement (news shock). From equation (3) we see that an increase in A' generates an increase in the price of capital q. Then, from equation (23) we observe that the increase in q must be associated with a reallocation of capital to the entrepreneurial sector, further increasing q (again from equation [23]). This implies that the increase in next period production is bigger than the increase in next period productivity (amplification).

As we have seen earlier, the amplification result for period 2 is also obtained in the model without working capital. With working capital, however, the news shock also generates an output boom in the current period. Therefore, news shocks affect current employment and production even if there is no productivity change in the current period. This mechanism has been studied in Jermann and Quadrini (2007) and Chen and Song (2009) and it is consistent with the findings of Beaudry and Portier (2006) based on the estimation of structural vector autoregressions.

Labor Wedge

Financial frictions have the ability to generate a labor wedge if wages or other costs that are complementary to labor require advance financing (working capital). Since there is an extensive literature studying the importance of the labor wedge for business cycle fluctuations, it will be helpful to relate the properties of the wedge generated by financial frictions with the labor wedge discussed in the literature.

The labor wedge is defined in the literature as a deviation from the optimality condition for the supply of labor we would have in an economy without frictions. Without frictions the optimality condition equalizes two terms: (i) the marginal rate of substitution between consumption and leisure; and (ii) the marginal product of labor. Thus, the labor wedge is defined as the difference between these two terms. If the difference is zero, we have the same optimality condition as in the frictionless model and, therefore, there is no wedge. If the difference is not zero, we have a labor wedge since we are deviating from the optimality condition without frictions.

Using a constant elasticity of substitution utility and a Cobb-Douglas production function, the wedge can be written as

$$Wedge \equiv mrs - mpl = \frac{\phi C}{1 - H} - (1 - \theta)\frac{Y}{H}, \qquad (24)$$

where C is consumption, H is hours worked, Y is output, and ϕ and θ are, respectively, preferences and technology parameters. With the special utility function for workers used here, the wedge is

$$Wedge \equiv mrs - mpl = H - (1 - \theta)\frac{Y}{H}.$$

Besides the fact that consumption does not enter the equation, the wedge generated by the model is very similar to the wedge derived from a more standard model. Since the labor supply is $H = w$ and the demand of labor satisfies $(1 - \theta)\frac{Y}{H} = w(1 + \mu)$, the wedge is equal to $-w\mu$.

Gali, Gertler, and López-Salido (2007) conduct a decomposition of the labor wedge in two components. The first component is the wedge between the marginal rate of substitution (mrs) and the wage rate (w). The second component is the wedge between the wage rate (w) and the marginal product of labor (mpl). More specifically,

$$Wedge \equiv mrs - w + w - mpl \equiv Wedge_1 + Wedge_2.$$

Using postwar data for the United States (although excluding the period of the recent crisis), Gali, Gertler, and López-Salido (2007) show that the first component of the wedge ($Wedge_1$) has played a predominant role in the dynamics of the whole wedge. In the version of the model studied here, however, the opposite is true since financial frictions generate only a wedge between the wage rate and the marginal product of labor ($Wedge_2$). In the model presented here, wages are fully flexible and the mrs is always equal to the wage rate. Therefore, $Wedge_1 = 0$.

At first, this finding may seem to cast doubts on the empirical relevance of financial frictions for the dynamics of labor. However, it is important to recognize that the problem arises because wages are assumed to be fully flexible. To make this point, suppose that there is some wage rigidity. For example, we could assume that workers update their wages only with some probability (like in Calvo pricing). Then a change in the labor demand would lead to a change in the labor supply but with a small change in the wage. As a result, $Wedge_1$ is no longer zero.

To show this point more clearly, suppose that the wage is fixed at $\bar{w}$. The first component of the wedge is equal to $Wedge_1 = H - \bar{w}$. Eliminating H using the first-order condition of firms $(1 - \theta)A\overline{K}^{\theta}H^{-\theta} = (1 + \mu)\bar{w}$, we get

$$Wedge_1 = \frac{(1 - \theta)A\overline{K}^{\theta}}{(1 + \mu)\bar{w}} - \bar{w}.$$

Therefore, the first component of the wedge is now dependent on μ, which in turn depends on the shock. So, in principle, by adding wage rigidities the model could capture some of the movements in the two components of the wedge.

Quantitative Performance

As discussed above, the addition of working capital gives an extra kick to the amplification potential of the model. As far as productivity shocks are concerned, the amplification effect remains weak. As for the collateral model without working capital, large amplification effects require sizable fluctuations in asset prices q'. However, I have already observed that standard macroeconomic models, even with the addition of financial frictions, find it difficult to generate large fluctuations in asset prices. As a result, the amplification effect remains weak.

The analysis of the amplification of other shocks, besides productivity, has not received much attention in the literature. An exception is Bernanke, Gertler, and Gilchrist (1999). They embed the financial accelerator in a New Keynesian monetary model and find that the amplification effects on monetary policy shocks could be sizable: Based on their calibration, the impulse response of output to a monetary policy shock is about 50 percent larger with financial frictions.

7.   MODEL WITH CREDIT SHOCKS

In the analysis conducted in the previous sections, I have focused on the propagation of productivity shocks, that is, shocks that arise in the real sector of the economy. Although the analysis of real shocks is clearly important for business cycle fluctuations, less attention has been devoted to studying the macroeconomic impact of shocks that arise in the financial sector of the economy—in particular, shocks that directly impact the ability of entrepreneurs or other borrowers to raise debt. Of course, we would like to have a theory of why the ability to borrow could change independently of changes that arise in the real sector of the economy. I will describe one possible theory later. For the moment, however, I start with a reduced form approach where the shocks are exogenous.

Model with Exogenous Credit Shocks

Consider the model with working capital analyzed in the previous section where entrepreneurs face the collateral constraint (19). Now, however, I assume that the constraint factor ξ is stochastic. I will call the stochastic changes in ξ "credit" shocks since they affect the borrowing capability of entrepreneurs.

Given the analysis conducted in the previous section, it is easy to see how the economy responds to these shocks. The impact is similar to the response to an asset price boom induced by a productivity improvement: By changing the tightness of the collateral constraint, the shock has an immediate impact on the multiplier μ, and, therefore, on the labor wedge. The change in ξ also affects the price of capital and this interacts with the exogenous change in the borrowing limit. Thus, the price mechanism described in the previous section also acts as an amplification mechanism for the credit shock.

To show this in more detail, consider again equation (23) derived from the budget constraint of entrepreneurs when the collateral constraint is binding (and consumption is zero). For simplicity I rewrite the equation here,

$$K^{e'} = \left(\frac{1}{1-\xi} \right) \left(k - \frac{b - Y^e(\mu)}{q} \right). \tag{25}$$

This equation makes clear that, keeping constant the multiplier μ, an increase in ξ (positive credit shock) increases the capital allocated to the entrepreneurial sector. As a result of this reallocation, we can see from equation (3) that the price of capital q increases. As long as $B > Y^e(\mu)$, that is, entrepreneurs are sufficiently leveraged, the increase in q further increases $K^{e'}$. Thus, the positive credit shock generates a reallocation of capital to the entrepreneurial sector, which in turn increases next period output.

The reallocation of capital also affects the multiplier μ and, therefore, the labor wedge. From equation (15) we can see that an increase in $K^{e'}$ generates a decline in μ. The multiplier also depends positively on ξ. However, if the negative effect from the increase in $K^{e'}$ dominates the positive effect from ξ, the multiplier μ and the labor wedge both decline in response to the positive credit shock. Therefore, credit shocks also have a positive impact on current employment and production. This is the channel explored in Jermann and Quadrini (forthcoming).

More on Credit Shocks

There are several articles that consider shocks to collateral or enforcement constraints. Some examples are Kiyotaki and Moore (2008), Del Negro et al. (2010), and Gertler and Karadi (2011). In the latter article, the shock arises in the financial intermediation sector. Mendoza and Quadrini (2010) also consider a financial shock to the intermediation sector but in the form of losses on outstanding loans. The impact of these shocks is very similar to a change in ξ.

Christiano, Motto, and Rostano (2008) propose a different way of modeling a credit shock. They use a version of the costly state verification model described in Section 5 and assume that the "volatility" of the idiosyncratic shock ω is time-variant. Thus, the financial shock is associated with greater investment risks. Since the risk is idiosyncratic and entrepreneurs are risk

neutral, the transmission mechanism is similar to shocks that affect the verification cost. Furthermore, once we recognize that a higher verification cost reduces the liquidation value of assets, this is not that different from the collateral model in which ξ falls because a lower portion of the capital can be recovered. The importance of time-varying risk when there are financial frictions is also studied in Arellano, Bai, and Kehoe (2010) and Gilchrist, Sim, and Zakrajsek (2010).

Alterative Specification of the Collateral Constraint

From a quantitative point of view and abstracting from changes in ξ, the collateral constraint (19) may have an undesirable quantitative property. In particular, if this constraint is binding, the model generates a volatility of debt that is similar or higher than the volatility of the market price of capital q. The reason is because k' co-moves positively with q' in the model. Then, the linear relation between $q'k'$ and the debt b' implies that the volatility of b' is bigger than the volatility of q'. In the data, however, asset prices are much more volatile than debt. Thus, if the model can generate plausible fluctuations in asset prices, it also generates excessive fluctuations in the stock of debt.

This problem does not arise if we use an enforcement constraint in which the liquidation value of capital is related to the book value, that is,

$$b' + wh \leq \xi k'. \tag{26}$$

Conceptually, this could derive from the fact that, once the firm goes in the liquidation stage, the capital ends up being reallocated to alternative uses and the price is different from q'.

With this specification the model could generate plausible fluctuations in both asset prices and debt. Recognizing this, Perri and Quadrini (2011) and Jermann and Quadrini (forthcoming) use a specification of the collateral constraint where the liquidation value of capital does not depend on q'. Of course, by eliminating q' in the collateral constraint we no longer have the amplification mechanism generated by the price of capital. However, once we focus on credit shocks, the amplification mechanism becomes secondary since these shocks can already generate significant macroeconomic volatility.

Asset Price Bubbles and Financial Frictions

In various versions of the model presented so far, we have seen that the price of assets plays an important role when there are financial market frictions. Whatever makes the price of assets move, it can affect the real sector of the economy by changing the tightness of the borrowing constraint. One factor that could generate movement in asset prices is bubbles. Traditionally we think of bubbles as situations in which the price of assets keeps growing over time even if nothing "fundamental" changes in the economy. Independent of

what can generate and sustain a bubble, it is easy to see the macroeconomic implications in the context of the simple model studied here.

Consider the version of the collateral model with working capital studied in Section 6. In this model, the fundamental price of capital in period 2 is $A'G'(\overline{K} - K^{e'})$. In the presence of a bubble, the price of capital would be higher. Without going into the details of whether the bubble is rational or not, the asset price with a bubble will be $A'G'(\overline{K} - K^{e'}) + \mathcal{B}'$, where $\mathcal{B}'$ is the bubble component. The macroeconomic effects are similar to the ones we have already examined when the change in asset prices was driven by productivity.

The modeling of rational bubbles is often challenging, especially in models with infinitely lived agents. To avoid this problem, Jermann and Quadrini (2007) design a mechanism that looks like a bubble, that is, it generates asset price movements, but it is based on fundamentals. The idea is that the economy can experience different rates of growth and switches from one growth regime to the other with some probability. When the "believed" probability of switching to a higher growth regime increases, current asset prices increase and the model generates a macroeconomic expansion. Even if the mechanism is not technically a bubble, it generates similar macroeconomic effects.

An alternative approach is to work with models where agents have limited life spans. These models allow for rational bubbles if certain conditions about discounting and population growth are met. Examples of these studies are Farhi and Tirole (2011) and Martin and Ventura (2011).

A third approach is based on the idea of multiple equilibria as in Kocherlakota (2009). This study is inspired by the study of Kiyotaki and Moore (2008), who develop a model with two monetary equilibria. In the first equilibrium, money is valued because there is the expectation that agents are willing to accept money, while in the second equilibrium money has no value because agents are not willing to accept it. Kocherlakota (2009) reinterprets money more generally as a nonproductive asset that could be used as a collateral. For example, housing. He then considers sunspot equilibria in which the economy switches stochastically from one equilibrium to the other. The switch is associated with asset price fluctuations, which have an impact on the real sector of the economy.

Quantitative Performance

The study of the quantitative implications of credit shocks is relatively recent but the findings suggest that these shocks play an important role for the business cycle. This is especially true if they directly affect the demand of labor.

An important issue in conducting a quantitative exploration of these shocks is their identification. Jermann and Quadrini (forthcoming) propose two approaches. The first approach uses a strategy that is reminiscent of the Solow

Figure 2 Responses to Financial Shocks from Jermann and Quadrini (Forthcoming)
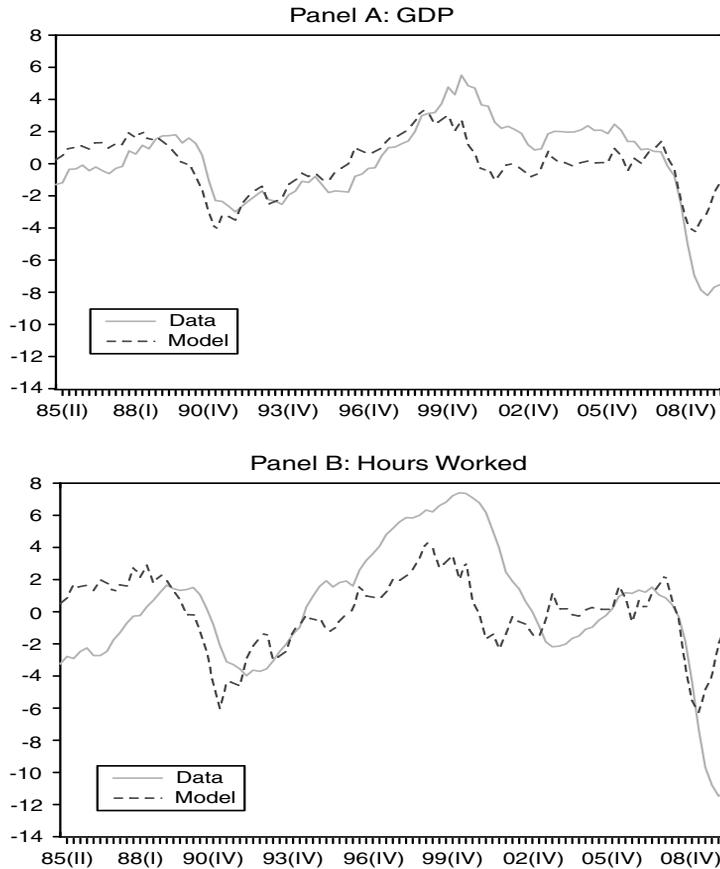
residual procedure to construct productivity shocks. Consider the enforcement constraint specified in equation (26). If this constraint is always binding, we can use empirical time series for debt, b', wages, wh, and capital, k', to construct time series for the credit variable ξ as residuals from this equation. Once we have the time series for ξ we can feed the constructed series into the (calibrated) model and study the response of the variables of interest.

Figure 2 shows the empirical and simulated series of output and labor generated by the model studied in Jermann and Quadrini (forthcoming). According to the simulation, credit shocks have played an important role in capturing the dynamics of labor and output in the U.S. economy during the last two and a half decades.

Another approach used to evaluate the importance of the credit shocks is to conduct a structural estimation of the model. This, however, requires the consideration of many more shocks because, effectively, a structural estimation has the flavor of a horse race among the shocks included in the model. For that reason Jermann and Quadrini (forthcoming) extend the basic model by adding more frictions and shocks. The estimated model is similar to Smets and Wouters (2007) but with financial frictions and financial shocks. Through the structural estimation they find that credit shocks contributed at least one-third to the variance of U.S. output and labor. Christiano, Motto, and Rostagno (2008) and Liu, Wang, and Zha (2011) also conduct a structural estimation of a model with financial frictions and financial shocks and they find that these shocks contributed significantly to the volatility of aggregate output.

Model with Endogenous Liquidity and Multiple Equilibria

So far the analysis has focused on equilibria in which entrepreneurs face binding collateral constraints. This is typically the case when there is no uncertainty. However, in the presence of uncertainty and especially with credit shocks, the enforcement constraint may not be binding in some contingencies. The possibility of "occasionally" binding constraints allows us to think about the issue of liquidity and the emergence of multiple equilibria.[4]

I continue to use the collateral constraint specified in (19) but with further assumptions about the liquidation value of capital.

Following Perri and Quadrini (2011), I assume that in the event of debt repudiation, the liquidated capital can be sold not only to the residual sector (as in the previous model) but also to other nondefaulting entrepreneurs. However, if the capital is sold to the residual sector, only a fraction ξ is usable. Instead, if the capital is sold to other entrepreneurs, there is no loss of capital. Since the marginal productivity of capital for entrepreneurs in the next period is A', this is also the price that nondefaulting entrepreneurs would be willing to pay for the liquidated capital. The price obtained by selling capital to the residual sector, instead, is $A'G'(\overline{K} - K^{e'})$. Because $\xi < 1$ and $G'(\overline{K} - K^{e'}) \le 1$, the resale to the entrepreneurial sector is the preferred option.

Notice that the default decision is made after all entrepreneurs have decided their borrowing b'. If there were no limits to the ability of nondefaulting entrepreneurs to purchase liquidated capital, then the residual sector would

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[4] Occasionally binding constraints is a feature of models studied in Brunnermeier and Sannikov (2010) and Mendoza (2010), although they abstract from credit shocks and there are not multiple equilibria. Boz and Mendoza (2010) also consider occasionally binding constraints with credit shocks but not multiple equilibria. See also Guerrieri and Lorenzoni (2009).

be irrelevant. Thus, I now introduce an additional assumption that in some contingencies limits the ability of the entrepreneurial sector to purchase the liquidated capital.

The assumption is that entrepreneurs can purchase the capital of liquidated firms only if they have the liquidity to do so. In this context, the liquidity is determined by the credit ability of entrepreneurs, which in turn depends on their borrowing decision. More specifically, if the collateral constraint binds, entrepreneurs will not be able to purchase the capital of liquidated firms since they no longer have access to additional credit. In this case, the only available option for the lender is to sell the liquidated capital to the residual sector at a lower price. However, if entrepreneurs do not borrow up to the limit, they still have access to credit that can be used in the event of an investment opportunity. In this case the capital of a liquidated firm can be sold to entrepreneurs at a higher price.

We now have all the elements to show that the model has the potential to generate multiple equilibria. Suppose that the initial state B is such that the enforcement constraint (19) is binding if the residual sector is the only option for the liquidated capital but it is not binding if the liquidated capital can be sold to entrepreneurs. In the first case the collateral value is $\xi q' k' = \xi A' G'(\overline{K} - K^{e'})$, while in the second it is $\xi q' k' = \xi A'$. Since the second is bigger than the first, it is possible that the collateral constraint is binding in the first case but not in the second. Under these conditions the model admits multiple self-fulfilling equilibria.

- *Bad equilibrium.* Suppose that agents expect that the unit value of the liquidated capital is $\xi q' = \xi A' G'(\overline{K} - K^{e'})$. This imposes a tight constraint on entrepreneurs and, as a result, they borrow up to the limit. But then, if an entrepreneur defaults, the lender is unable to sell the liquidated capital to other entrepreneurs since there are no entrepreneurs capable of purchasing the capital. The recovery value is $\xi A' G'(\overline{K} - K^{e'})$ per each unit of capital. Therefore, the expectation of a lower liquidation price is ex-post validated by the lack of "liquidity" available to entrepreneurs.

- *Good equilibrium.* Suppose that agents expect that the unit value of the liquidated capital is $q' = A'$. This relaxes the borrowing constraint on entrepreneurs and allows them to borrow more than required to purchase $k' = \overline{K}$. Thus, the collateral constraint is not binding. But then, if an entrepreneur defaults, the lender is able to sell the liquidated capital to other entrepreneurs and the recovery value is A'. Therefore, the expectation of high liquidation prices is ex-post validated by the "liquidity" available to entrepreneurs.

The possibility of multiple equilibria introduces an endogenous mechanism for fluctuations in ξ. More specifically, the value of ξ is low if the

enforcement constraint is binding, which in turn generates a low value of ξ. Instead, if the value of ξ is high, the enforcement constraint is not binding, which in turn generates a high value of ξ. In this way the credit shock ξ becomes endogenous and could fluctuate in response to the states of the economy. This provides a concept of liquidity-driven crisis: Expectations of high prices increase liquidity, which in turn sustains high prices. Instead, expectations of low prices generate a contraction in liquidity, which in turn induces a downfall in the liquidation price. The transmission of "endogenous" credit shocks to the real sector of the economy in a closed economy is similar to the model with "exogenous" credit shocks already described in the previous section.

The International Transmission of Credit Shocks

The 2007–2009 crisis has been characterized by a high degree of international synchronization in which most of the industrialized countries experienced large macroeconomic contractions. There are two main explanations for the synchronization. The first explanation is that country-specific shocks are internationally correlated. The second explanation is that shocks that arise in one or few countries are transmitted to other countries because of economic and financial integration.

The first hypothesis is not truly an explanation: If shocks are correlated across countries, we would like to understand why they are correlated. Although this is obvious for certain shocks, think for example to oil shocks, it is less intuitive for others. For instance, if we think that shocks to the labor wedge are important drivers of the business cycle, it is not obvious why they should be correlated across countries. The second hypothesis—international transmission of country-specific shocks—is a more interesting line of research.

In this section I show that "credit" shocks that hit one or few countries could generate large macroeconomic spillovers to other countries if they are financially integrated. Therefore, these shocks are possible candidates to account for the international co-movement in macroeconomic aggregates.

To show this, I will consider a two-country version of the collateral model described earlier. The only additional feature I need to specify is the meaning of financial integration. One obvious implication of financial integration is that borrowing and lending can be done internationally. This also implies that the interest rate is equalized across countries (law of one price). In the simple model studied here, however, this is inconsequential because agents are risk neutral and the interest rates are constant and equal across countries even if they operate in a regime of financial autarky. Therefore, this is not the important dimension of international integration that matters here.

Another possible implication of financial integration is that investors (in our case entrepreneurs) hold domestic and foreign firms. Effectively, it is as

if each firm has two units: one operating in country 1 and the other operating in country 2. The problem solved by the entrepreneur can be written as

$$\max_{\{h_j, k'_j, b'_j\}_{j=1}^2} \left\{ c + \beta c' \right\}$$

subject to:

$$c = \sum_{j=1}^2 \left[q_j \overline{K} + A_j \overline{K}^\theta h_j^{1-\theta} - w_j h_j - B + \frac{b'_j}{R} - q_j k'_j \right]$$

$$\xi_j q'_j k'_j \geq b'_j + w_j h_j, \qquad j = 1, 2$$

$$c' = \sum_{j=1}^2 (A'_j k'_j - b'_j)$$

$$c \geq 0, \quad c' \geq 0, \tag{27}$$

where the index $j = 1, 2$ identifies the country. Since entrepreneurs have operations at home and abroad, they make production and investment decisions in both countries. Notice, however, that they face a consolidated budget constraint. Also notice that the variable ξ is indexed by j since credit shocks could be country-specific. This would be the case, for example, if there are financial problems in the banking system of one country but not in the other.

I now show how a credit shock in country 1 (changes in ξ_1) affects the economies of both countries. This can easily be seen from the first-order conditions with respect to k'_1, b'_1, k'_2, and b'_2,

$$(1 + \gamma)q_1 = \beta A'_1 + \mu_1 \xi_1 E q'_1 \tag{28}$$
$$1 + \gamma = (\beta + \mu_1)R \tag{29}$$
$$(1 + \gamma)q_2 = \beta A'_2 + \mu_2 \xi_2 E q'_2 \tag{30}$$
$$1 + \gamma = (\beta + \mu_2)R. \tag{31}$$

Equations (29) and (31) imply that the Lagrange multipliers are equalized across countries, that is, $\mu_1 = \mu_2$.

Now consider the first-order conditions with respect to labor,

$$(1 - \theta)A_1 \overline{K}^\theta h_1^{-\theta} = w_1(1 + \mu_1)$$
$$(1 - \theta)A_2 \overline{K}^\theta h_2^{-\theta} = w_2(1 + \mu_2).$$

Since $\mu_1 = \mu_2$, a credit shock in country 1 (change in ξ_1) has the same impact on the demand of labor of both countries. Therefore, a country-specific credit shock gets propagated to other countries through the labor wedge. This mechanism is emphasized in Perri and Quadrini (2008, 2011).

The impact on the accumulation of capital is not perfectly symmetric in the two countries, as we can see from equations (28) and (30). However, k'_1 and k'_2 move in the same direction.

Endogenous Credit Shocks

Perri and Quadrini (2011) go beyond "exogenous" credit shocks and, adopting a framework with occasionally binding constraints similar to the model described in the previous subsection, they study the implications of endogenous ξ_j in an international environment.

The emergence of multiple equilibria characterized by different degrees of liquidity also arises in the two-country model. What is interesting is that, if countries are financially integrated, then *bad* and *good* equilibria outcomes become perfectly correlated across countries. Thus, the model provides not only a mechanism for the international transmission of country-specific credit shocks, but also a mechanism in which "endogenous" credit shocks are internationally correlated. It is important to emphasize that the international correlation of ξ_j is not an assumption but an equilibrium property.

To see this, consider again the two-country model studied in the previous section. The first-order conditions with respect to k_1', b_1', k_2', and b_2' are still given by (28)–(30). Therefore, $\mu_1 = \mu_2$. This means that, if the collateral constraint is binding in one country, it must also be binding in the other country. But then we cannot have that in one country the liquidation price of capital is determined by the marginal product in the entrepreneurial sector while in the other country the price is determined by the marginal product in the residual sector. If the collateral constraints are binding in both countries, entrepreneurs lack the liquidity to purchase the capital of liquidated firms and the collateral value will be low in both countries. This makes the collateral constraints tighter and entrepreneurs borrow up to the limit (bad equilibrium outcome). However, if the collateral constraints are not binding in both countries, then entrepreneurs have the liquidity to purchase the liquidated capital in both countries. The collateral value is high in both countries and firms do not borrow up to the limit.

To summarize, either both countries end up in a bad equilibrium or both countries end up in a good equilibrium. In this way self-fulfilling equilibria (endogenous shocks) become perfectly correlated across countries.

Quantitative Performance

To the best of my knowledge, the quantitative properties of the international model with endogenous credit shocks have been explored only in Perri and Quadrini (2011). This article emphasizes four properties. First, the response to credit shocks is highly asymmetric. Negative credit shocks generate large- and short-lived macroeconomic contractions while credit expansions generate gradual and long-lasting macroeconomic booms.

The second finding is that credit contractions (negative credit shocks) have larger macroeconomic effects if they arise after long periods of credit expansions. Therefore, long credit expansions create the conditions for highly

disrupting financial crises. A similar prediction is obtained in Gorton and Ordoñez (2011) but with a mechanism that is based on the information quality of collateral assets.

The third finding relates to the difference between exogenous versus endogenous credit shocks. While exogenous credit shocks can generate macroeconomic co-movement, they do not generate cross-country co-movement in financial flows or leverages, which is a strong empirical regularity. The model with endogenous credit shocks, however, is also capable of generating co-movement in financial flows since ξ_j are endogenously correlated across countries.

The last quantitative feature of the model I would like to emphasize is that it can generate sizable fluctuations in asset prices. For this feature, however, the risk aversion of entrepreneurs becomes important, which we have abstracted from in the simple version of the model presented here. Assuming that there is market segmentation and firms cannot be purchased by workers, a negative credit shock induces firms to pay lower dividends, which in turn reduces the consumption of entrepreneurs. This implies that the discount rate of entrepreneurs, $\beta U'(c_{t+1})/U'(c_t)$, falls. As a result, their valuation of future dividends falls, leading to an immediate drop in the market value of firms. Since the impact of the credit shocks on entrepreneurs' consumption is large, the model generates sizable drops in asset prices.

8. CONCLUSION

The key principles for adding financial market frictions in general equilibrium models are not new in the macroliterature. However, it is only with the recent crisis that the profession has fully recognized the importance of financial markets for business cycle fluctuations. Thus, more effort has been devoted to the construction of models that can capture the role of financial markets for macroeconomic dynamics.

This article has reviewed the most common and popular ideas proposed in the literature. Using a stylized model with only two periods and two types of agents, I have shown that the modeling of financial market frictions is useful for understanding several dynamic features of the macroeconomy in general and of the business cycle in particular.

The ideas reviewed in this article are all based on the transmission of shocks through the "credit channel," that is, conditions that limit the availability of funds or increase the cost of funds needed to make investment and hiring decisions. Some authors have also proposed models in which the credit channel and adverse selection in credit markets could generate economic fluctuations even in absence of exogenous shocks—an example is Suarez and Sussman (1997). Less attention has been devoted in the literature to studying alternative mechanisms through which financial frictions have an impact on

the macroeconomic dynamics. One of these mechanisms is studied in Monacelli, Quadrini, and Trigari (2010), who embed financial market frictions in a matching model of the labor market with wage bargaining. In this article, collateral constraints affect employment not because they limit the amount of funds available to firms for hiring workers, but because they affect the bargaining of wages. One interesting feature of this mechanism is that the impact of credit shocks on employment is much more *persistent* than the impact generated by the typical credit channel reviewed in this article.[5]

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[5] Other contributions that embed financial market frictions in models with searching and matching frictions are Weil and Wasmer (2004), Chugh (2009), Petrosky-Nadeau (2009), and Petrosky-Nadeau and Wasmer (2010). In these articles, however, the main transmission mechanism is still based on the "credit channel."

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