# Financial Globalization and the Rising of Public Debt*

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**Abstract**

During the last three decades the stock of government debt has increased in most developed countries. During the same period we also observe a significant liberalization of international financial markets. In this paper we propose a multi-country political economy model with incomplete markets and endogenous government borrowing and show that governments choose higher levels of public debt when financial markets become internationally integrated. We also show that government debt increases with the volatility of uninsurable idiosyncratic income (risk). To the extent that the increase in income inequality observed in some industrialized countries during the last three decades has been associated with higher idiosyncratic risk, the paper suggests another potential mechanism for the rise in public debt.

*Keywords*: Government debt, financial integration, income inequality.

*JEL classification*: E60, F59.

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# 1 Introduction

During the last three decades, we have observed an increase in the stock of public debt in most developed countries. As shown in the top panel of Figure 1, the stock of public debt in OECD countries has increased from around 30 percent of GDP in the early 1980s to about 50 percent in 2005. Similar increases are observed in the United States and Europe.

Historically, the dynamics of public debt have been closely connected to war financing and business cycle fluctuations, where budget deficits and surpluses are instrumental in minimizing the distortionary effects of taxation. The tax-smoothing theory developed by Barro (1979) provides a rationale for such dynamics. However, when we look at the upward trend in public debt that started in the early 1980s, it becomes difficult to rationalize this trend with the tax-smoothing argument since this period has been characterized by relatively peaceful times and low macroeconomic volatility.

The last three decades have also been characterized by two additional trends: the international liberalization of financial markets and the increase in inequality in several industrialized countries. The second panel of Figure 1 plots the index of financial liberalization constructed by Abiad, Detragiache and Tressel (2008) for the group of OECD countries, United States and Europe. As can be seen from the panel, the world financial markets have become much less regulated starting in the early 1980s. A fact also confirmed by other indicators of international capital mobility as shown in Obstfeld and Taylor (2005). The second trend that took place during the last three decades is the increase in inequality. The last panel of Figure 1 plots the share of income earned by the top 1% of the population as reported by Atkinson, Piketty, and Saez (2011).

In this paper we propose a theory in which government borrowing responds positively to financial liberalization. Furthermore, to the extent that income inequality is associated with higher uninsurable income risk, public debt also responds positively to income inequality. We study a multi-country model where agents face uninsurable idiosyncratic risks and public debt is held by private agents to smooth consumption. To keep tractability, we assume that there are two types of agents: those who face idiosyncratic risks (entrepreneurs) and those who are less exposed to these risks (workers). Government policies are determined through the aggregation of agents' preferences based on probabilistic voting.

Both agents have preferences for some public debt. Agents who face higher idiosyncratic risks (entrepreneurs) benefit from public debt because it provides an additional instrument to smooth consumption. The demand

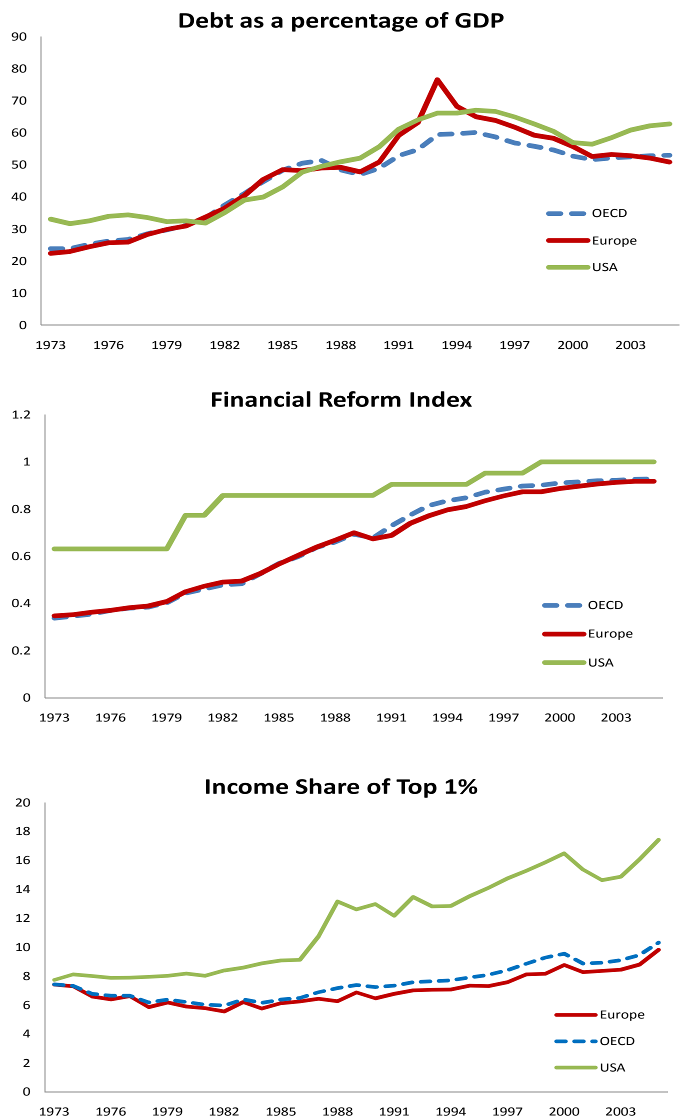


Figure 1: Public debt, financial liberalization, and inequality in advanced economies. Appendix A provides the definition of variables and the data sources.

for safe assets for insurance purposes is reflected in the equilibrium interest rate being lower than the intertemporal discount rate. In this way our model is related to Aiyagari and McGrattan (1998) and Shin (2006) where public debt improves welfare. Agents who face lower risks, the workers, can also benefit from government borrowing. Because the equilibrium interest rate is lower than the intertemporal discount rate, workers would like to borrow. However, if they are constrained, public borrowing becomes a desired substitute. Effectively, the government borrows on behalf of workers. The benefits from public debt, however, fade away as the stock of debt increases. Once the debt has reached a certain level, further increases require higher interest rates since entrepreneurs are already well insured. Although this may be still optimal for entrepreneurs, it becomes suboptimal for workers because they internalize that raising the stock of debt increases its cost. Thus, once the debt has reached a certain level, workers do not support further increases; the internalization of the increasing cost of debt serves as a limit to its growth.

How does financial integration affect the government incentive to issue debt? The central mechanism is the elasticity of the interest rate to the supply of debt. In a globalized world, both the demand and supply of government debt come not only from domestic agents (investors and governments) but also from their foreign counterparts. Therefore, when governments do not coordinate their actions, each country faces a lower elasticity of the interest rate to the supply of 'their own' government debt. Since the interest rate is less responsive to a country's debt, governments have more incentives to increase borrowing provided that workers have sufficient political influence. Thus, we have a mechanism through which capital liberalization increases government debt.

How does income risk affect preferences for public debt? When entrepreneurs face higher risks, they increase the demand of safe assets (government bonds). Then the issuance of more debt is beneficial for both entrepreneurs and workers. For entrepreneurs, it is beneficial because additional public bonds provide safe assets available for consumption smoothing and contrast the decline in the interest rate induced by the increased demand for bonds. For workers, it is beneficial because, through government debt, they can borrow at a lower interest rate.

An increase in uninsurable income risk leads to higher government borrowing independently of the international regime of capital markets. However, if financial markets are integrated, our model could generate an increase in the government debt of all countries even if the increase in income risk arises in only a subset of countries. This is an important property of our model because, although the increase in public debt has been observed in most of the developed countries, the increase in income inequality took place only in a subset of countries. Provided that the observed income inequality is associated to increased uninsurable risk and financial markets are internationally integrated, we do not need that inequality rises in all countries to generate higher worldwide government borrowing. This is another way of seeing the important of financial globalization.

The organization of the paper is as follows. We first describe how the paper relates to various contributions in the literature. After the literature review, Section 2 describes the model and defines the equilibrium. Section 3 explores a simplified version of the model with only two periods, providing simple analytical intuition for the key results of the paper. Section 4.2 performs a quantitative analysis with the infinite horizon model. Section 5 conducts the empirical analysis and Section 6 concludes. All technical proofs are relegated to the Appendix.

## 1.1 Literature review

An influential theoretical literature studies the optimal choice of public debt over the business cycle with contributions by Barro (1979), Lucas and Stokey (1983), Aiyagari, Marcet, Sargent, and Seppala (2002), Angeletos (2002), Chari, Christiano, and Kehoe (1994), and Marcet and Scott (2009). We depart from the tax-smoothing mechanism because we abstract from aggregate fluctuations and distortionary taxation. Instead, we focus on the role of heterogeneity within a country that is assumed away in these papers.

The structure of our model is closer to models studied in Aiyagari and McGrattan (1998) and Shin (2006). In these papers the role of government debt is to partially complete the assets' market when agents are subject to uninsurable idiosyncratic risks. The government accumulates debt in order to crowd out private capital, which is inefficiently high due to precautionary savings. In our model, however, we abstract from capital accumulation. Therefore, the government choice of debt is independent of production efficiency considerations and it is based on redistributive concerns. Because of this, our paper is also related to the literature on optimal redistributive policy in heterogeneous agent economies such as Krusell and Rios-Rull (1999), Golosov, Kocherlakota, and Tsyvinski (2003), Albanesi and Sleet (2006), Farhi and Werning (2008), and Corbae, D'Erasmo, and Kuruscu (2009).

The paper is also related to the literature on the political economy of debt initiated by Alesina and Tabellini (1990), Persson and Svensson (1989), and further developed by Song, Storesletten, and Zilibotti (2012), Battaglini and Coate (2008), Caballero and Yared (2008), Ilzetzki (2011), and Aguiar and Amador (2011). A common feature of these papers is the strategic use of public debt in economies where the interest rate is exogenous and governments with different preferences alternate in power. We abstract from political turnover and consider instead how the supply of government bonds endogenously affects interest rates and redistribution. The 'interest rate manipulation' channel is also present in Krusell, Martin, and Rios-Rull (2006) and in Azzimonti, de Francisco, and Krusell (2008), but it relies on the use of distortionary taxation which we assume away here.

Another difference with many of the papers that study the optimal public debt is that we consider an open economy environment with large countries. An exception is Chang (1990) who also studies how the international liberalization of capital markets affects government borrowing in an economy with overlapping generations. Although the structure of the model is different, the mechanism through which capital liberalization leads to higher government borrowing is similar. The analysis of Chang (1990), however, abstracts from risk and does not investigate how the change in risk affects government borrowing. Kehoe (1989), Mendoza and Tesar (2005), and Quadrini (2005) also study equilibrium government policies with capital mobility, but in models without public debt or with public debt that is not chosen optimally. Cooper, Kempf, and Peled (2008) study the role of debt limits on governments within a federation. Our paper shows that even in the absence of the free rider problem present in fiscal federations, a country's participation in the international bond market can lead to higher sovereign debt.

The paper is also related to the literature that explores the importance of market incompleteness for international financial flows. Caballero, Farhi, and Gourinchas (2008), Mendoza, Quadrini, and Rios-Rull (2009), and Angeletos and Panousi (2010) have all emphasized the importance of cross-country heterogeneity in financial markets for global imbalances. Our study differs from these contributions in two dimensions. First, our focus is on public debt while the above contributions have focused on private debt. There is an important difference between public and private debt that is crucial for our results: while in private borrowing atomistic agents do not internalize the impact that the issuance of debt has on the interest rate, governments do. Part of our results are driven by the fact that governments do not take the interest rate as given, as individual agents do. The second difference is that the goal of our study is to explain the gross volumes of (public) debt, while the contributions mentioned above focus on net volumes. In these models financial liberalization leads to higher liabilities in one country but lower liabilities in others, with the difference defining the imbalance. The global volume of credit, however, does not change significantly. In contrast, in our model capital liberalization (and income risk) generates an increase in the global stock of debt even if countries are symmetric and liberalization (and income risk) does not generate international imbalances.

## 2 Theoretical environment

In this section we describe the model and characterize the competitive equilibrium for *given* government policies. The derivation of the optimal policies (public debt) will be described in the following sections.

### 2.1 The model

Consider an economy composed of $N$ symmetric countries indexed by $j \in \{1, .., N\}$ that lasts for $T$ periods. The infinite horizon case is obtained as a special case with $T \to \infty$. Agents face uninsurable idiosyncratic risks, but some agents are more exposed to risk than others.

To model heterogeneous exposure to risk in a tractable manner, we assume that there are two types of agents: a measure $\Phi$ of workers and a measure 1 of entrepreneurs. Workers do not face any idiosyncratic uncertainty, while entrepreneurs are subject to investment risks. In modeling entrepreneurs, we adopt the approach proposed by Angeletos (2007), which allows for linear aggregation. We can then analyze the general equilibrium by focusing on a representative worker and a representative entrepreneur, without keeping track of the wealth distribution among entrepreneurs.

Although we focus on heterogeneity between workers and entrepreneurs and make the extreme assumption that workers do not face any risk, the model should be interpreted more generally as an environment in which some agents face more risk than others. Because of the different exposure to risk, preferences over government debt differ between workers and entrepreneurs. Thus, the level of public debt chosen by the government will depend on the relative political power (or size) of these two groups.

Both types of agents maximize the expected lifetime utility

$$\mathbb{E}_0 \sum_{t=1}^{T} \beta^t \ln(c_t), \tag{1}$$

where $c_t$ is consumption and $\beta \in (0, 1)$ denotes the discount factor.

In each country $j$ there is a unit supply of land, an international immobile asset traded at price $p_{j,t}$. Entrepreneurs are individual owners of firms, each operating the production function $F(z, k, l)$, where $k$ is the input of land, $l$ the input of labor supplied by workers, and $z$ is an idiosyncratic productivity shock that is observed after the input of land but before the input of labor. The productivity shock is independently and identically distributed among agents and over time, and takes values in the set $\{z_1, .., z_m\}$, with probabilities $\{\mu_1, .., \mu_m\}$. This is the only source of risk in the model. The function $F(z, k, l)$ is strictly increasing in $z$, $k$, $l$ and homogeneous of degree 1 in $k$ and $l$ (constant returns).

Entrepreneurs hire workers in a competitive labor market at wage $w$. The hiring decision is static because it affects only current profits. Given productivity $z$ and land $k$, the marginal product of labor is equalized to the wage rate, that is, $F_l(z, k, l) = w$. Because the production function is homogeneous of degree 1, the demand for labor is linear in the input of land and can be expressed as $l = l(z, w)\, k$. The entrepreneurial profits are also linear in the input of land and can be expressed as

$$F(z, k, l) - wl = A(z, w)k. \tag{2}$$

Entrepreneur $i$ in country $j$ enters period $t$ with risk-free bonds $b^i_{j,t}$, land $k^i_{j,t}$ and productivity $z^i_{j,t}$ and receives lump-sum transfers $\tau_{j,t}$ from the government (or pays taxes if $\tau_{j,t} < 0$). The budget constraint is

$$c^i_{j,t} + p_{j,t} k^i_{j,t+1} + \frac{b^i_{j,t+1}}{R_{j,t}} = A(z^i_{j,t}, w_{j,t}) k^i_{j,t} + p_{j,t} k^i_{j,t} + b^i_{j,t} + \tau_{j,t}. \tag{3}$$

Entrepreneurs also face the terminal condition $b^i_{j,T+1} \geq 0$. This condition imposes that any outstanding debt needs to be fully repaid in period $T$. In the limiting case with $T \to \infty$, this is replaced by a transversality condition.

Workers are endowed with $1/\Phi$ units of labor supplied inelastically in the domestic market for the wage $w_{j,t}$. Labor is internationally immobile.[1] Workers also receive lump-sum transfers $\tau_{j,t}$ from the government but they do not hold assets or borrow. Thus, workers' consumption is equal to

$$c^w_{j,t} = w_{j,t} \left(\frac{1}{\Phi}\right) + \tau_{j,t}. \tag{4}$$

The assumption that workers do not hold assets is without loss of generality. As we will see, the equilibrium interest rate is smaller than the intertemporal discount rate, that is, $R_{j,t} < 1/\beta$. Since workers do not face any risk, they will not hold bonds in the long run. The inability to borrow can be rationalized by limited enforcement, leading to an upper bound in the amount of borrowing. Her we set the upper bound to zero but later we will consider less tight borrowing constraints.

[1] The assumption that the individual labor supply is $1/\Phi$ is simply a normalization that keeps the ratio of total land over the aggregate supply of labor equal to 1.

Governments raise revenues by issuing one-period bonds. The proceeds are used to pay lump-sum transfers to workers and entrepreneurs and to repay the outstanding debt. Thus, the government budget constraint is

$$(1+\Phi)\tau_{j,t} + B_{j,t} = \frac{B_{j,t+1}}{R_{j,t}}, \tag{5}$$

where $B_{j,t}$ are the bonds issued at time $t-1$ and due in period $t$, and $B_{j,t+1}$ are the new bonds. Governments face the terminal condition $B_{j,T+1} = 0$.

**Discussion.** There are two forms of market incompleteness. The first is the absence of markets for claims that are conditional on the realization of idiosyncratic productivity $z^i_{j,t}$. The second is that workers are not allowed to borrow, that is, they cannot issue bonds. Therefore, the only way for entrepreneurs to insure the idiosyncratic risk is by purchasing government bonds. As we will see, relaxing the second form of market incompleteness by allowing workers to borrow, does not make government debt irrelevant.

## 2.2 Competitive equilibrium for given policies

We start by characterizing the competitive equilibrium, taking as given government policies. This is the necessary first step to characterize the *optimal* government policies. We consider two capital regimes. In the first regime each country is under financial autarky and, therefore, government bonds cannot be traded in international markets. In the second regime countries are financially integrated so governments can sell bonds to (borrow from) domestic and foreign entrepreneurs.

The decision problem of workers is trivial because transfers are taken as given and the supply of labor is inelastic. They simply consume their income. The decision problem of entrepreneurs is more complex. Given the initial holdings of land and bonds, they choose labor input, consumption and new holdings of land and bonds that maximize their lifetime utility. These choices are functions of their individual history $\mathbf{z}^i_{j,t} = \{z^i_{j,1}, ..., z^i_{j,t}\}$.

**Definition 2.1 (Autarkic Equilibrium)** *Given a sequence of government debt $\{B_{j,t}\}_{t=1}^T$ and $B_{j,T+1} = 0$, a competitive equilibrium without mobility of capital is defined as a sequence of prices $\{w_{j,t}, p_{j,t}, R_{j,t}\}_{t=1}^T$, entrepreneurs' decisions $\{c^i_{j,t}(\mathbf{z}^i_{j,t}), l^i_{j,t}(\mathbf{z}^i_{j,t}), k^i_{j,t+1}(\mathbf{z}^i_{j,t}), b^i_{j,t+1}(\mathbf{z}^i_{j,t})\}_{t=1}^T$, consumption of workers $\{c^w_{j,t}\}_{t=1}^T$, and transfers $\{\tau_{j,t}\}_{t=1}^T$ for $j \in \{1, .., N\}$ such that:*

*i. Entrepreneurs' decisions maximize (1) subject to the budget constraint (3) and the terminal condition $b^i_{j,T+1} \geq 0$. Workers' consumption satisfies the budget constraint (4).*

*ii. Prices clear domestic markets for labor, $\int_i l^i_{j,t}(\mathbf{z}^i_{j,t})di = 1$, for land, $\int_i k^i_{j,t+1}(\mathbf{z}^i_{j,t})di = 1$, and for bonds, $\int_i b^i_{j,t+1}(\mathbf{z}^i_{j,t})di = B_{j,t+1}$.*

*iii. Domestic bonds and transfers satisfy the government's budget (5).*

The definition of a competitive equilibrium with integrated capital markets is similar. The only difference is that the bond market clears internationally instead of country by country, that is, $\sum_{j=1}^{N} \int_i b^i_{j,t+1}(\mathbf{z}^i_{j,t})\mathrm{d}i = \sum_{j=1}^{N} B_{j,t+1}$, and interest rates are equalized worldwide, $R_{1,t} = ... = R_{N,t}$.

For the analysis that follows, it will be convenient to define

$$\tilde{b}^i_{j,t} = b^i_{j,t} - \frac{B_{j,t}}{1+\Phi},$$

which is the difference between the demand of bonds for an individual entrepreneur, $b^i_{j,t}$, and the economy wide per-capital debt issued by the government, $B_{j,t}/(1+\Phi)$. We refer to this variable as "excess demand for bonds". The entrepreneurs' aggregate excess demand in country $j$ is $\tilde{b}_{j,t} = \int_i \tilde{b}^i_{j,t}$.

Using $\tilde{b}^i_{j,t}$ together with the government budget (5), we can re-write the entrepreneurs' budget constraint as

$$c^i_{j,t} + p_{j,t}k^i_{j,t+1} + \frac{\tilde{b}^i_{j,t+1}}{R_{j,t}} = A(z^i_{j,t}, w_{j,t})k^i_{j,t} + p_{j,t}k^i_{j,t} + \tilde{b}^i_{j,t}. \tag{6}$$

Given the linearity of the profit function, we can show that entrepreneurs' decision rules are linear in wealth $a^i_{j,t} = A(z^i_{j,t}, w_{j,t})k^i_{j,t} + p_{j,t}k^i_{j,t} + \tilde{b}^i_{j,t}$. This generalizes the findings of Angeletos (2007) to an economy with fiscal policy.

**Lemma 2.1** *Let $\eta_t = \frac{\beta(1-\beta^{T-t})}{1-\beta^{T-t+1}}$. Given the sequence of prices $\{w_{j,t}, p_{j,t}, R_{j,t}\}_{t=1}^{T}$, entrepreneur's policies are*

$$\begin{aligned} c^i_{j,t} &= (1-\eta_t)a^i_{j,t}, \\ p_{j,t}k^i_{j,t+1} &= \phi_{j,t}\eta_t a^i_{j,t}, \\ \frac{\tilde{b}^i_{j,t+1}}{R_{j,t}} &= (1-\phi_{j,t})\eta_t a^i_{j,t}, \end{aligned}$$

*where* $\phi_{j,t}$ *satisfies* $\mathbb{E}_t\left[\frac{R_{j,t}}{\left(\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t}+R_{j,t}(1-\phi_{j,t})}\right]=1.$

**Proof 2.1** *Appendix B.*

Aggregating agents' decisions using Lemma 2.1 and imposing market clearing, we establish the following proposition, similar to Angeletos (2007).

**Proposition 2.1** *Given the sequence of public debt* $\{B_{1,t},..,B_{N,t}\}_{t=1}^{T}$, $B_{1,T+1}=,..,=B_{N,T+1}=0$, *the equilibrium wage is constant,* $w_{j,t}=\bar{w}$, *and the remaining prices and aggregate allocations are*

$$\phi_{j,t} = \mathbb{E}_t\left[\frac{A(z^i_{j,t+1})+p_{j,t+1}}{A(z^i_{j,t+1})+p_{j,t+1}+\tilde{b}_{j,t+1}}\right], \tag{7}$$

$$p_{j,t} = \frac{\eta_t\phi_{j,t}(\bar{A}+\tilde{b}_{j,t})}{(1-\eta_t\phi_{j,t})}, \tag{8}$$

$$R_{j,t} = \frac{(1-\eta_t\phi_{j,t})\tilde{b}_{j,t+1}}{\eta_t(1-\phi_{j,t})(\bar{A}+\tilde{b}_{j,t})}, \tag{9}$$

$$c^e_{j,t} = \bar{A}+\tilde{b}_{j,t}-\frac{\tilde{b}_{j,t+1}}{R_{j,t}}, \tag{10}$$

$$c^w_{j,t} = \bar{w}+\nu\left(\frac{B_{j,t+1}}{R_{j,t}}-B_{j,t}\right), \tag{11}$$

*where* $\nu=\Phi/(1+\Phi)$ *is the share of workers in the population,* $A(z^i_{j,t})\equiv A(z^i_{j,t},\bar{w})$, $\bar{A}=\sum_\ell A(z_\ell)\mu_\ell$. *The variable* $c^e_{j,t}=\int_i c^i_{j,t}$ *is the aggregate consumption of entrepreneurs and* $c^w_{j,t}$ *is the aggregate consumption of workers.*

**Proof 2.1** *Appendix C.*

The above proposition holds with and without capital mobility. Without mobility of capital (autarky), the bond holdings of residents must be equal to the bonds issued by the domestic government, that is, $\int_i b^i_{j,t}=B_{j,t}$. In terms of excess demand for bonds, $\tilde{b}_{j,t}=\int_i\tilde{b}^i_{j,t}=\nu B$. When financial markets are integrated, however, the bond holdings of residents may differ from the bonds issued by their domestic governments. A corollary to Proposition 2.1 characterizes bond holdings with capital mobility.

**Corollary 2.1** *Consider the environment with capital mobility. If* $\tilde{b}_{j,1} = \tilde{b}_1$*, then* $\tilde{b}_{j,t} = \nu \left( \frac{\sum_{j=1}^{N} B_{j,t}}{N} \right)$ *for all* $t > 1$*.*

**Proof 2.1** *Appendix D.*

If the initial aggregate excess holdings of bonds is equal across countries, then future excess holdings are also equalized across countries. This is a consequence of the assumption that countries are homogenous in endowments and technology and, with integrated financial markets, interest rates are equalized across countries. Since the excess demand $\tilde{b}_{j,t}$ is the difference between the bonds purchased by entrepreneurs and the outstanding government liabilities, in countries where governments issue more liabilities, entrepreneurs save more because they anticipate higher payment of future taxes (negative transfers). Notice that this result does not apply if entrepreneurs face different investment risk in different countries.

### 2.3 Choice of policies

We now briefly describe the political process. Government policies are implemented by representatives who are selected through democratic elections. Consider a political race between two opportunistic candidates who only care about gaining power and have commitment to some platforms. Under standard assumptions made in the probabilistic voting literature, political competition leads to convergence in policy proposals. As shown in Persson and Tabellini (2000), government policies maximize a weighted sum of agents' welfare. Thus, the government's objective is a weighted sum of the welfare of workers and entrepreneurs. The government behaves, de-facto, as a benevolent planner but without commitment to future policies.

## 3 Politico-equilibrium in the two-period model

We start analyzing the optimal government policies in a special version of the model with only two periods, $T = 2$. This allows us to characterize several properties of the model analytically. Section 4 will generalize the analysis to an economy with a large number of period $T$, which we interpret as an approximation to the infinite horizon version of the model.

To further simplify the analysis of the two-period model, we assume that in period 1 governments have zero debt, that is, $B_{j,1} = 0$. Furthermore, all entrepreneurs start period 1 with one unit of land, $k^i_{j,1} = 1$, zero bonds, $b^i_{j,1} = 0$, and they have the same productivity $z^i_{j,1} = \bar{z}$. Under these conditions, initial entrepreneurs' wealth, including current profits, is $a^i_{j,1} = \bar{A} + p_{j,1}$. Wealth in period 1 is allocated between consumption and savings in the form of bonds, $b^i_{j,2}$, and land, $k^i_{j,2}$. Thus, wealth in period 2 is $a^i_{j,2} = A(z^i_{j,2}) + \tilde{b}^i_{j,2}$, which is stochastic because profits depend on the realization of the idiosyncratic shock $z^i_{j,2}$. Remember that $\tilde{b}^i_{j,2} = b^i_{j,2} - B_{j,2}/(1+\Phi)$ is the excess demand of bonds. Since period 2 is the terminal period, land has no value after production.

## 3.1 Financial autarky

We first characterize the equilibrium with financial autarky. Since in period 1 entrepreneurs are homogeneous, we drop the individual superscript $i$. We also ignore country and time subscripts and let $k$ and $b$ denote the individual land and bonds purchased at time 1. Furthermore, we use $p$, $R$, and $B$, without subscripts, to denote the price of land, the gross interest rate and the bonds issued in period 1. The idiosyncratic shock realized in period 2 is denoted by $z$. Total government transfers paid in period 1 equal government borrowing $B/R$, while total government transfers paid in period 2 equal the repayment of debt $-B$. Given that the total population is $1 + \Phi$, the per-capital transfers are $\tau_1 = (B/R)/(1 + \Phi) = \nu(B/R)/\Phi$ in period 1 and $\tau_2 = -B/(1 + \Phi) = -\nu B/\Phi$ in period 2.

Workers earn the wage $\bar{w}$ in both periods on labor endowment $1/\Phi$ and receive transfers $\tau_1$ and $\tau_2$. Thus workers' consumption is $(\bar{w} + \nu B/R)/\Phi$ in period 1 and $(\bar{w} - \nu B)/\Phi$ in period 2, and their lifetime utility is

$$W(B) = \chi + \ln\left(\bar{w} + \nu\frac{B}{R}\right) + \beta \ln\Big(\bar{w} - \nu B\Big), \tag{12}$$

where $\chi = -(1 + \beta)\ln\Phi$ is a constant.

Entrepreneurs start period 1 with wealth $a = \bar{A} + p$ and consume $c_1 = a - \tilde{b}/R - pk$. Since entrepreneurs start with the same wealth, they all choose the same land and bonds to carry to next period. Thus, $k = 1$ and $b = B$, which implies $\tilde{b} = \nu B$. This also implies that $c_1 = \bar{A} - \nu B/R$. Next period consumption depends on the realization of the idiosyncratic shock and it is equal to $c_2 = A(z) + \nu B$. Therefore, entrepreneurs' lifetime utility is

$$V(B) = \ln\left(\bar{A} - \nu\frac{B}{R}\right) + \beta\mathbb{E}\ln\Big(A(z) + \nu B\Big). \tag{13}$$

Apart from the effects that the issuance of debt has on prices $R$ and $p$, equations (12) and (13) make clear that public debt redistributes consumption inter-temporally between workers and entrepreneurs. The following lemma establishes some properties of the lifetime utilities.

**Lemma 3.1** *In the autarky equilibrium, the indirect utility of workers (12) is strictly concave in $B$ with a unique maximum in the interval $\left(0, \frac{\bar{w}}{\nu}\right)$. The indirect utility of entrepreneurs (13) is strictly increasing in $B$.*

**Proof 3.1** *Appendix E.*

Workers would like to borrow initially, since the interest rate is lower than the intertemporal discount rate. In fact, as $B$ converges to zero, the interest rate converges to $R < 1/\beta$. However, as the government borrows more, it reaches a point in which workers' welfare starts to decrease. This happens for two reasons. First, keeping the interest rate fixed, the marginal utility of consumption in the next period becomes bigger than the marginal utility of consumption in the current period. Second, as the government borrows more, the interest rate increases, raising the cost of borrowing. Entrepreneurs, on the other hand, always prefer higher debt because it increases the interest rate and, therefore, the return on their financial wealth.

Based on probabilistic voting, the debt is chosen to maximize the weighted sum of workers' and entrepreneurs' utilities, that is,

$$\max_{B} \Big\{ \Phi W(B) + V(B) \Big\}. \tag{14}$$

Although we cannot establish the global concavity of the objective function, we know that there is an optimal $B$ in the interval $\left[0, \frac{\bar{w}}{\nu}\right]$. This must be the case because the objective function is continuous and converges to minus infinity as $B$ converges to $\frac{\bar{w}}{\nu}$. Since the objective function is differentiable, its derivative must be zero at the optimum. Differentiating (14) we obtain

$$\Phi \cdot \left[ \frac{\partial \left(\frac{B}{R}\right)}{\partial B} \left(\frac{1}{c_1^w}\right) - \beta \left(\frac{1}{c_2^w}\right) \right] = \left[ \frac{\partial \left(\frac{B}{R}\right)}{\partial B} \left(\frac{1}{c_1^e}\right) - \beta \mathbb{E} \left(\frac{1}{c_2^e(z)}\right) \right], \tag{15}$$

where $c_1^w$ and $c_2^w$ are the aggregate consumptions of workers (per-capita consumption multiplied by the mass of workers $\Phi$); $c_1^e$ and $c_2^e(z)$ are individual consumptions of entrepreneurs. Entrepreneurs's consumption in period 2 is stochastic because it depends on the idiosyncratic shock $z$.

A marginal unit of debt issued by the government in period 1 transfers consumption from entrepreneurs (who save, net of transfers, to buy bonds) to workers (who receive transfers financed by government debt). In period

2 the government pays back the debt by taxing agents (negative transfers). This reduces worker's consumption, $c_2^w$, and increases the consumption of entrepreneurs, $c_2^e(z)$. As the size of workers $\Phi$ increases, the left-hand-side of (15) receives more weight, meaning that the effect of public borrowing on workers' welfare becomes more important in the government's objective.

Because the government is a monopolist in the supply of bonds, it takes into account that its debt affects the interest rate. Remember that the total transfers made by the government in period 1 are $B/R$. When the government increases $B$ marginally by one unit, the increase in the current transfers is not $1/R$ because the interest rate $R$ will also change. More specifically, the marginal change in period 1 transfers is

$$\frac{\partial\left(\frac{B}{R}\right)}{\partial B} = \frac{1}{R}\Big(1 - \epsilon^A(B)\Big),$$

where $\epsilon^A(B) = \frac{\partial R}{\partial B}\frac{B}{R}$ is the elasticity of the interest rate $R$ to the supply of bonds in autarky. Clearly, higher values of the elasticity imply smaller transfers allowed by higher borrowing.

The internalization of the interest rate elasticity in the decision of governments, is the key difference between public and private borrowing. With private borrowing, atomistic agents take the interest rate as given, and $\epsilon^A(B)$ is zero in their individual optimality condition. In this case the perceived increase in consumption in period 1 from private borrowing would be $1/R$.

Figure 2 plots the welfare of workers and entrepreneurs in the domestic country, for a parameterized version of the model. The production function is specified as $F(z,k,l) = z^\theta k^\theta l^{1-\theta}$ and the parameters values are reported at the bottom of the figure. With this specification of the production function, the wage is $\bar{w} = (1-\theta)\bar{z}^\theta$ and $A(z) = \theta z/\bar{z}^{1-\theta}$.

The continuous lines, denoted by $V^A$ and $W^A$, are for the autarky regime. The dashed lines are for the regime with capital mobility when there are $N$ symmetric countries. We will come back to the case of capital mobility in the next section. The debt chosen by the government depends on the size of workers $\Phi$. Although the indirect utility of workers $W(B)$ is strictly concave, the indirect utility of entrepreneurs $V(B)$ is not. As a result, the government's objective is not necessarily concave. We can establish concavity only for large values of $\Phi$.

**Proposition 3.1** *If $\Phi > \frac{(1+\beta)\bar{w}}{\bar{A}} + \beta$, the government's objective is strictly concave, and there is a unique maximum in the interval $\left(0, \frac{\bar{w}}{\nu}\right)$.*

**Proof 3.1** *Appendix F.*

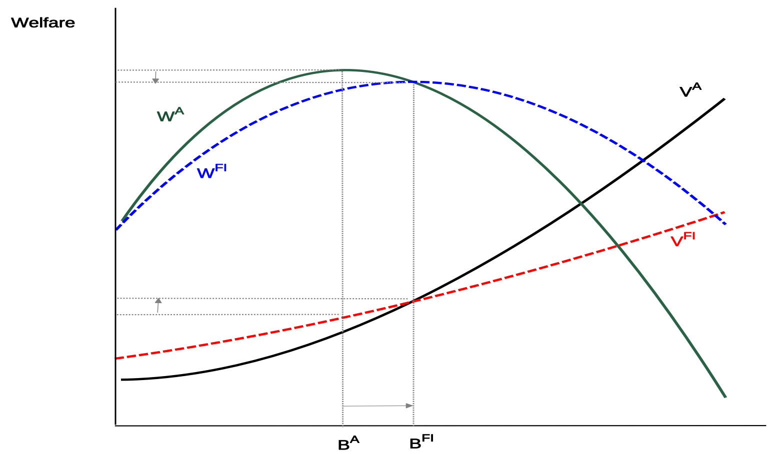


Figure 2: Indirect utilities with and without capital mobility. The parameter values are $\beta = 0.95$, $\theta = 0.36$, $z \in \{1, 3\}$, $\text{Prob}(z) \in \{0.5, 0.5\}$.

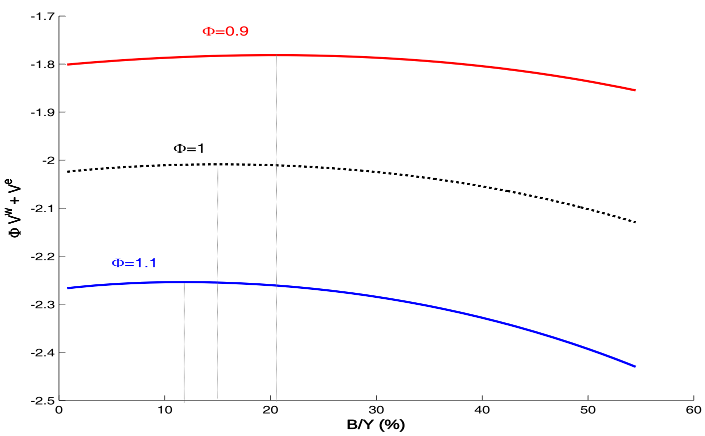


Figure 3: Government's objective function in autarky. The parameter values are $\beta = 0.95$, $\theta = 0.36$, $z \in \{1, 3\}$, $\text{Prob}(z) \in \{0.5, 0.5\}$.

Two remarks are in order here. First, the condition on $\Phi$ is sufficient but not necessary. Second, even if the government objective is not strictly concave, the maximum is still interior, although we can not establish uniqueness. For the simple model considered here, however, we can always check concavity numerically as we do in Figure 3. This figure plots the government objective for different values of $\Phi$ and shows that the optimal $B$ decreases with the relative population of workers.

### 3.2 The effects of financial integration

We now consider the case in which the financial markets of $N$ countries are integrated. We focus on Nash equilibria where governments choose the supply of bonds independently and simultaneously.

Entrepreneurs' consumption depends on the choice of the excess demand of bonds $\tilde{b} = b - B/(1+\Phi)$. In the autarky equilibrium we have that $b = B$, and therefore, $\tilde{b} = \nu B$. When countries are financially integrated, however, entrepreneurs can purchase both domestic and foreign bonds, while transfers are only a function of domestic debt. Thus, $b$ is not necessarily equal to $B$. However, corollary 2.1 has established that the excess holdings of bonds will be equalized across countries. Therefore, $\tilde{b} = \nu \sum_{j=1}^{N} B_j/N$. Effectively, in countries where governments make larger transfers in period 1, entrepreneurs save more because they anticipate the higher payment of taxes in period 2. Using this result, the indirect utility of entrepreneurs in country $j$ is

$$V_j(\mathbf{B}) = \ln\left(\bar{A} - \frac{\tilde{b}}{R}\right) + \beta \mathbb{E} \ln\left(A(z) + \tilde{b}\right), \tag{16}$$

where $\mathbf{B} = (B_1, .., B_N)$ is the vector of government debts chosen by the $N$ countries. Since $\tilde{b} = \nu \sum_{j=1}^{N} B_j/N$ (see Corollary 2.1), entrepreneurs' welfare depends on the debt issued by all countries, $\mathbf{B}$.

The properties of $V_j(\mathbf{B})$ are similar to the autarky case. Keeping the debts in all other countries constant, entrepreneurs still prefer higher $B_j$ since this increases the equilibrium interest rate and, therefore, the return on the risk-free bonds held to hedge the idiosyncratic risk.

The indirect utility of workers can be written as

$$W_j(\mathbf{B}) = \chi + \ln\left(\bar{w} + \nu \frac{B_j}{R}\right) + \beta \ln\left(\bar{w} - \nu B_j\right), \tag{17}$$

which is similar to equation (12) for the autarky case.

The interest rate $R$ is now determined in the world market. Using equation (9), this can be expressed as

$$R = \nu \Big( \frac{\sum_{j=1}^{N} B_j}{N} \Big) \left[ \frac{1 + \beta(1-\phi)}{\beta(1-\phi)\bar{A}} \right], \tag{18}$$

where $\phi = \mathbb{E}\left(\frac{A(z)}{A(z)+\nu(\sum_{j=1}^{N} B_j)/N}\right)$. This expression makes clear that it is the worldwide debt that determines the interest rate. Thus, the debt of an individual country affects the interest rate in proportion to the share of the worldwide debt. This implies that the debt issued by a very small country will have a small impact on the world interest rate.

In a Nash equilibrium, each government chooses its own debt taking as given the debts issued by all other countries, that is,

$$\max_{B_j} \left\{ \Phi W_j(\mathbf{B}) + V_j(\mathbf{B}) \right\} \tag{19}$$

The optimal choice of debt is denoted by $B_j = \varphi_j(\mathbf{B}_{-j})$, where $\mathbf{B}_{-j}$ is the vector of public debts chosen by all other countries, except country $j$ (rest of the world). This is the response function to other countries policies. A Nash policy equilibrium is a vector $\mathbf{B}^* = (B_1^*, .., B_N^*)$ that satisfies

$$B_j^* = \varphi_j(\mathbf{B}_{-j}^*), \qquad \text{for all } j = 1, .., N.$$

We can now characterize the properties of the Nash policy equilibrium. For each country $j$, the optimal debt $B_j$ satisfies the first order condition

$$\Phi \cdot \left[ \frac{\partial(\frac{B_j}{R})}{\partial B_j} \left(\frac{1}{c_1^w}\right) - \beta \left(\frac{1}{c_2^w}\right) \right] =$$
$$\left[ \frac{\partial \left( \frac{\sum_{j=1}^{N} B_j}{NR} \right)}{\partial B} \left(\frac{1}{c_1^e}\right) - \beta \frac{1}{N} \mathbb{E}\left(\frac{1}{c_2^e(z)}\right) \right], \tag{20}$$

which is derived by differentiating the government objective (19). This condition is necessary but not sufficient as in the autarky regime.

While the government still faces the trade-off between the benefits and costs of transferring consumption from entrepreneurs to workers in the first period, this expression differs from equation (15) in several respects. First, workers' transfers depend only on the domestic supply of government bonds $B_j$, while entrepreneurs' utility depends on both domestic and foreign bonds. Hence, an extra unit of $B_j$ increases $c_1^w$ by $\frac{\partial(B_j/R)}{\partial B_j}$ but decreases $c_1^e$ by only $\frac{\partial\left(\sum_{j=1}^{N} B_j/(NR)\right)}{\partial B_j} < \frac{\partial(\nu B_j/R)}{\partial B_j}$. This is because part of the extra bonds issued by the government are absorbed by entrepreneurs in the rest of the world. In the second period, the government repays $B_j$ by taxing agents (with negative transfers), which reduces $c_2^w$ in the same amount as before. The increase in $c_2^e(z)$, however, is smaller than in the autarky case because the increase in bonds held by domestic entrepreneurs is smaller than the increase in domestic debt. This is because the increase in domestic government bonds is shared with foreign entrepreneurs.

Another difference between equations (15) and (20) comes from the fact that the effect of a unilateral change in $B$ on the world-wide interest rate is smaller when financial markets are integrated (see equation (18)). In a symmetric equilibrium, $B_j = \sum_{j=1}^{N} B_j/N = B$ and condition (20) becomes

$$\Phi \cdot \left[\frac{1}{c_1^w}\frac{1}{R}\left(1-\frac{\epsilon^A(B)}{N}\right)-\frac{\beta}{c_2^w}\right]=\frac{1}{N}\left[\frac{1}{c_1^e}\frac{1}{R}\Big(1-\epsilon^A(B)\Big)-\mathbb{E}\left(\frac{\beta}{c_2^e(z)}\right)\right], \tag{21}$$

where $\epsilon^A(B)$ is the elasticity of the interest rate in autarky.

Relative to the autarky case, the cost of increasing the debt unilaterally by one country is smaller, since the perceived elasticity of the interest rate is $\epsilon^A(B)/N$. The costs and benefits for entrepreneurs are also different, since they are split between domestic and foreign residents. More specifically, the marginal effects on $V(\mathbf{B})$ are reduced when the economy is financially integrated. Thus, whether financial integration leads to more or less public debt depends on the relative size of workers and entrepreneurs.

**Proposition 3.2** *Suppose that $\Phi/(1+\Phi) \simeq 1$. Per-capita debt is strictly increasing in the number of countries $N$. As $N \to \infty$, there exists a unique symmetric equilibrium where debt is bounded and $\beta R < 1$. Financial integration generates welfare losses for workers and welfare gains for entrepreneurs.*

**Proof 3.2** *Appendix G.*

When the size of entrepreneurs is small, the government objective is approximately equal to the utility of workers. Since the interest rate is less elastic to domestic debt in an integrated world, workers would like the government to borrow more (see Figure 2). Notice that the equilibrium must be symmetric, that is, it is not possible to have one country choosing a level of debt different from other countries (see the proof of the proposition).

This channel, also emphasized in Chang (1990), derives from the non-atomistic nature of governments and it is essential to differentiate the equilibrium with public borrowing from the equilibrium with private borrowing. This is because private issuers do not internalize the impact of their own borrowing on the equilibrium interest rate since each agent is too small to affect aggregate prices. Therefore, with only private borrowers, the autarky equilibrium would not be different from the equilibrium with capital mobility. In our framework, on the contrary, when governments issue debt, they fully internalize the effect of higher borrowing on the interest rate. Since the effect on the interest rate depends on the international capital market regime, the equilibrium debt differs in the economy with and without mobility of capital. As a result, the model predicts that financial integration affects the equilibrium outcome even if countries are homogeneous. This property differentiates our study from the recent literature on global imbalances where liberalization affects the equilibrium because countries are heterogeneous in some important dimension.[2]

**Size heterogeneity:** The effects of financial integration on the debt issued by the integrating countries depend on their relative size. Suppose that there are only two countries, $N = 2$. The population and land endowment of country 1 is a proportion $\alpha$ of the worldwide endowment. If $\alpha = 0.5$, we revert to the symmetric case studied in the previous section.

**Proposition 3.3** *Suppose that $\Phi/(1+\Phi) \simeq 1$. If $\alpha < 0.5$, in the regime with capital mobility, country 1 issues higher per-capita debt than country 2, that is, $B_1 > B_2$.*

**Proof 3.3** *Appendix H.*

Since small countries face a larger world market relative to their own economy, they perceive the world interest rate as less sensitive to their own per-capita debt. As a result, they issue more debt. For this result to hold, however, the relative size of workers, which determines their political power, must be sufficiently high. Otherwise, the government objective is dominated by the benefit of providing safe assets to entrepreneurs and, since in an open economy these benefits are shared with foreign entrepreneurs, the government my have lower incentives to borrow.

Figure 4 plots the equilibrium debt for different sizes of country 1. When $\alpha = 0$, country 1 is a small open economy and country 2 is effectively in autarky. Thus, the debt chosen by country 2 does not change from the autarky level. When $\alpha = 0.5$, we are back to the symmetric case, so both countries choose the same level of debt.

[2] Examples are Fogli and Perri (2006), Caballero, Farhi, and Gourinchas (2008), Mendoza, Quadrini, and Rios-Rull (2009), and Angeletos and Panousi (2010).

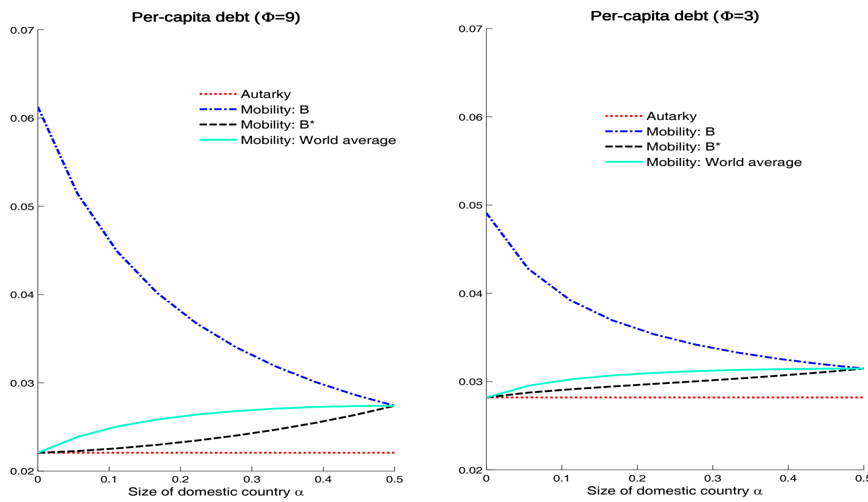


Figure 4: Country size and equilibrium government debt with capital mobility. The parameter values are $\beta = 0.95$, $\theta = 0.36$, $z \in \{1, 3\}$, $\text{Prob}(z) \in \{0.5, 0.5\}$.

### 3.3 The effects of rising income risk

The fact that entrepreneurs face idiosyncratic investment risks implies that their incomes become unequal in period 2. The goal of this section is to analyze how the change in income risk affects the choice of public debt.

**Proposition 3.4** *Consider the autarky regime and suppose that* $\Phi/(1+\Phi) \simeq 1$. *If an increase in the mean preserving spread of the distribution of* $z$ *raises the term* $(1-\epsilon(B))/(\bar{w}R(B)+B)$, *then* $B$ *increases.*

**Proof 3.4** *Appendix I.*

In general, an increase in the volatility of the idiosyncratic shock implies that entrepreneurs face higher risk. This strengthens the demand for safe assets (government bonds) and reduces the interest rate. Because of the lower interest rate, workers would like to increase borrowing. The government, however, takes into account not only the level of the interest rate but also the elasticity of the interest rate to public debt. At the same time, the government also finds it optimal to increase public debt to improve entrepreneurs' welfare. In general, we cannot establish unambiguously whether government debt increases in response to an increase in risk. However, as long as the term $(1-\epsilon(B))/(\bar{w}R(B)+B)$ increases, public debt does rises as we show in the proof of the proposition. The dependence of public debt from risk is shown in Figure 5 which plots the equilibrium debt in autarky as a function of the volatility of the idiosyncratic shock.

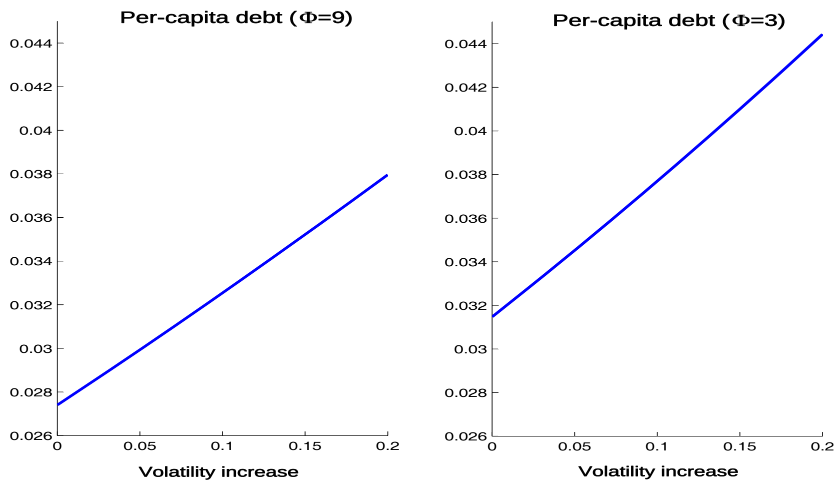


Figure 5: Inequality and government debt in autarky. The parameter values are $\beta = 0.95$, $\theta = 0.36$, $\text{Prob}(z) \in \{0.5, 0.5\}$. Starting from $z \in \{1, 3\}$, we change the two values of $z$ by the volatility increase reported in the graph.

Next, we show what happens to government borrowing in the regime with capital mobility when inequality increases only in one country. Figure 6 plots the stock of debt in the two-country economy when the volatility of the idiosyncratic shock increases only in country 1. Even if income inequality changes only in country 1, the stock of debt increases in both countries. This happens because the higher risk faced by domestic entrepreneurs increases their demand for bonds and reduces the world interest rate. If the government's weight assigned to workers (their relative size) is sizable—as assumed in the numerical example—the lower interest rate makes public debt more attractive for the governments of both countries.

The response of debt, however, differs in the two countries. In the left panel of Figure 6, we see that country 2 is more responsive to inequality than country 1 (note that $B_2$ is always above $B_1$). As the weight given by governments to entrepreneurs increases (i.e., as $\Phi$ decreases), the government of country 1 has more incentive to increase debt because of the higher risk faced by domestic entrepreneurs. Since this decreases the interest rate also for entrepreneurs in country 2, the government of country 2 increases debt

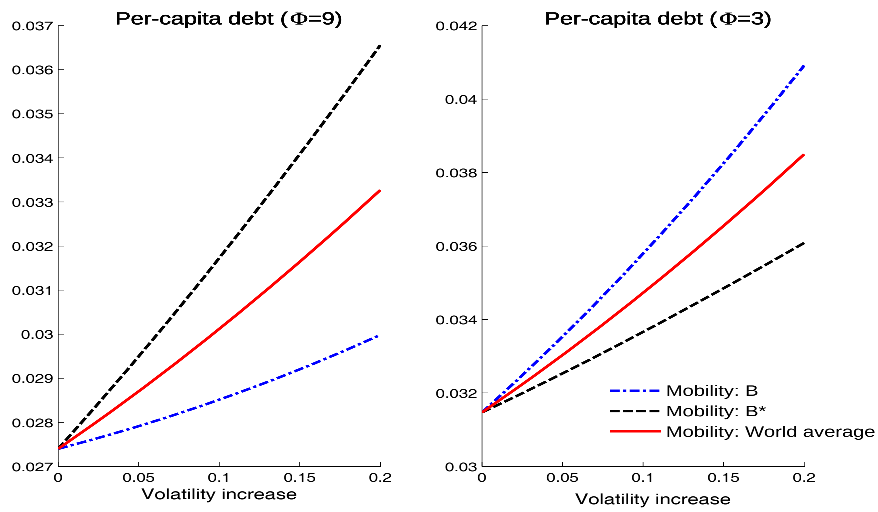


Figure 6: Inequality and government debt with capital mobility. The parameter values are $\beta = 0.95$, $\theta = 0.36$, $\mathrm{Prob}(z) \in \{0.5, 0.5\}$. Starting from $z \in \{1, 3\}$ in both countries, we change the values of $z$ only in the domestic country by the volatility increase reported in the graph.

less to compensate for the higher debt issued by country 1. This is depicted in the right panel of Figure 6 where, in contrast to the case with larger $\Phi$, the increase in the supply of bonds in country 2 is smaller than the supply in country 1 ($B_2$ lies below $B_1$).

The finding that the increase in risk in few countries may trigger an increase in government borrowing in other countries is important to reconcile the theory with the data. In fact, the increase in income inequality since the early 1980s was observed only in a few countries (see Atkinson, Piketty and Saez (2011)), while the cross-country increase in public debt was more general. The fact that in the 1980s capital markets were liberalized may explain why the increase in inequality in a few countries may have triggered the increase in government borrowing in other countries, provided that the higher inequality was associated with higher risk and in large economies such as the United States.

# 4 Infinite horizon model

Since entrepreneurs face idiosyncratic shocks, the model generates a complex distribution of income and wealth. By virtue of the linearity of the production function, the model admits aggregation. An implication of this property is that income and wealth follow random walk processes and their economy-wide distributions are not stationary. This property becomes problematic if we want to compare the inequality generated by the model with the inequality observed in the data.

To have stationary distributions of income and wealth in the infinite horizon model, we now assume that agents survive with some probability $\omega < 1$ and they are replaced by the same number of newborn agents. The discount factor then results from the product of two terms: the intertemporal discount factor in preferences, $\hat{\beta} \in (0, 1)$, and the survival probability, $\omega \in (0, 1]$. Thus, $\beta = \hat{\beta}\omega$. The assets left by exiting entrepreneurs are redistributed equally (lump-sum) to the newborn entrepreneurs.

All the properties of the competitive equilibrium derived earlier apply to the model with stochastic mortality. We only need to reinterpret the discount factor as $\beta = \hat{\beta}\omega$. The distributions of income and wealth, however, now converge to a steady state if public debt stays constant over time.

## 4.1 Politico-economic equilibrium

We think of the infinite horizon model as the limit of $T \to \infty$. It will then be convenient to define first the politico-economic equilibrium for a finite $T$. Policies are chosen in every period and they are functions of the relevant aggregate states. Suppose that at $t = 1$, countries start with the same aggregate excess holdings of bonds, that is, $\tilde{b}_{j,1} = \nu \sum_{j=1}^{N} B_{j,1}$. Using corollary 2.1, the sufficient set of aggregate states are the debts issued by the $N$ countries, $\mathbf{B}_1 = (B_{1,1}, .., B_{N,1})$.

To characterize the strategic interaction between governments, we restrict attention to Nash equilibria where public borrowing decisions are made simultaneously and independently (i.e., there is no coordination among governments). The politico-economic equilibrium is characterized by a sequence of policy functions $\mathbf{B}_{t+1} = \mathcal{B}_t(\mathbf{B}_t)$ for $t = 1, .., T$. These functions are determined by solving the model backward, starting at $t = T$. Let's define first the governments' objectives at any $t$.

**Proposition 4.1** *Given current states $\mathbf{B}_t$ and policy function $\mathcal{B}_{t+1}(\mathbf{B}_{t+1})$ for next period policies, the problem solved by government $j$ at time $t$ is*

$$\max_{B_{j,t+1}} \left\{ \Phi W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) \right\}, \tag{22}$$

*where the functions $W_{j,t}$ and $V_{j,t}$ are defined by the backward recursion*

$$\begin{aligned} W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) &= \ln\left(\bar{w} + \frac{\nu B_{j,t+1}}{R_{j,t}} - \nu B_{j,t}\right) + \beta W_{j,t+1}\Big(\mathbf{B}_{t+1}; \mathcal{B}_{t+1}(\mathbf{B}_{t+1})\Big), \\ V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) &= \ln(1-\eta_t) + \left(\frac{1}{1-\eta_t}\right)\Bigg[\mathbb{E}\ln\Big(A(z^i_{j,t}) + \nu B_{j,t} + p_{j,t}\Big) + \\ &\qquad \eta_t \ln\left(\frac{\eta_{j,t}\phi_{j,t}}{p_{j,t}}\right)\Bigg] + \beta\mathbb{E}V_{j,t+1}\Big(\mathbf{B}_{t+1}, \mathcal{B}_{t+1}(\mathbf{B}_{t+1})\Big). \end{aligned}$$

**Proof 4.1** *Appendix J.*

In solving problem (22), the government of country $j$ takes as given the debts chosen by all other countries, the vector $\mathbf{B}_{-j,t+1}$. The optimal (response) policy is $B_{j,t+1} = \varphi_{j,t}(\mathbf{B}_t, \mathbf{B}_{-j,t+1})$.

**Definition 4.1 (Nash policy game)** *For given states $\mathbf{B}_t$, the solution to the Nash policy game at time $t$ is the vector $\mathbf{B}^*_{t+1}$ that satisfies $B^*_{j,t+1} = \varphi_{j,t}(\mathbf{B}_t, \mathbf{B}^*_{-j,t+1})$, for all $j = 1, .., N$.*

The solution to the policy game depends the initial states $\mathbf{B}_t$. This provides us with the policy function $\mathbf{B}_{t+1} = \mathcal{B}_t(\mathbf{B}_t)$ at time $t$. Therefore, starting from $t = T$ and taking into account the terminal condition $\mathcal{B}_T(\mathbf{B}_T) = 0$, we can construct the whole sequence of policy functions backward.

The infinite horizon is obtained as $T \to \infty$. Assuming convergence, the politico-equilibrium is characterized by an invariant policy function $\mathcal{B}(\mathbf{B}) = \lim_{T\to\infty} \mathcal{B}_1(\mathbf{B}_1)$. Because of the complexity of the model, we are unable to find a closed-form solution and to establish the existence of an invariant policy function. Thus, we will only provide a numerical characterization by solving the model for a large but finite number of period $T$. We interpret the solution for a large $T$ as an approximation to the infinite horizon model.

## 4.2 Quantitative analysis

In this section we solve the model numerically and shows the dynamics induced by financial liberalization and rising income risk for public debt. To show the impact of financial liberalization, we start from a steady state equilibrium without mobility of capital and compute the transition dynamics following financial integration. To show the impact of rising income risk, we start from a steady-state equilibrium with low income risk and compute the transition dynamics following the increase in volatility of the idiosyncratic shock. We solve the model numerically using a global approach based on the discretization of the state space (the stock of public debt in the two countries) and maximization over a grid search. The detailed description of the numerical procedure is available in electronic form at `http://www-bcf.usc.edu/~quadrini/papers/PDpapApp.pdf`.

**Parametrization:** Although the numerical simulation is not meant to provide a rigorous quantitative exercise but to illustrate the qualitative dynamic features of the model, we try as much as possible to choose the parameter values according to observed empirical targets. More specifically, we choose variables observed pre-1980s as the initial calibration targets. This is motivated by the view that the process of international financial liberalization started in the 1980s. The pre-1980s period can then be considered as closer to a regime of financial autarky. Also, as can be seen from Figure 1, the average income inequality in industrialized countries started to increase toward the end of the 1970s and early 1980s. This motivates our choice to calibrate the autarky version of the model to the early 1980s. In particular, we focus on two targets: a ratio of public debt over income of 30 percent and a share of income earned by the top 1 percent of the population equal to 6 percent. These are the approximate numbers reported in Figure 1 for the OECD countries in the 1970s. We now describe in detail how the initial calibration targets can be used to pin down the parameters of the model.

A period in the model is one year and the discount factor is set to $\beta = 0.9466$, which results from an intertemporal discount rate of 3 percent and a survival probability $\omega = 0.975$. The value of $\omega$ implies an average (active) life of 40 years.[3]

For the production function we would like to use a Cobb-Douglas specification, that is, $F(z, k, l) = z^\theta k^\theta l^{1-\theta}$, with $\bar{z} = \mathbb{E}z$ normalized to 1. However, the amount of idiosyncratic risk generated by this specification is bounded by the restriction that $z$ cannot be negative. In order to have more flexibility in choosing the amount of risk faced by entrepreneurs, we assume that the shock $z$ also affects the effective quantity of land after production. Thus, the total income generated by the entrepreneur is $F(z, k, l) = zk^\theta l^{1-\theta} + (z - \bar{z})kp$. The first component is pure production while the second component can be interpreted as capital gains (or losses if negative). Notice that in aggregate the capital gains or losses are zero. Thus, the aggregate production is ex-

[3]Recall that we assumed that agents die with probability $1 - \omega$ and that the assets of exiting entrepreneurs are redistributed equally to newborn entrepreneurs.

actly the same in the two cases. Thus, the parameter $\theta$ represents the capital income share which we set to 0.2. This is lower than the typical number used in the literature because there is no depreciation in our model.

Productivity is uniformly distributed in the domain $1 \pm \Delta$, where $\Delta$ is chosen so that the share of income earned by the top 1 percent is equal to 6 percent in the autarky steady state.[4] However, this also depends on $\Phi$, which in turn is chosen to have a steady state of public debt over income of 30 percent in the autarky steady state. These are the approximate numbers for income concentration and public debt in the OECD countries in pre-1980s reported in Figure 1. To reach these two targets, the values of $\Delta$ and $\Phi$ are chosen simultaneously through an iterative procedure. The resulting values are $\Delta = 0.14$ and $\Phi = 3.902$. These values imply that the standard deviation of entrepreneurial income is about 15% the value of land used in production, $pk$, and the population share of workers is slightly below 80%.

**Results:** Figure 7 plots the transition dynamics for government debt induced by international capital market liberalization and increased income inequality. The increase in income inequality is generated by a higher volatility of the idiosyncratic risk, which changes from $\Delta = 0.14$ to $\Delta = 0.1725$. As described above, $\Delta = 0.14$ was chosen to generate the 6% concentration of income at the top 1% in the autarky steady state. The new value is chosen to have a share of 7.5% for the top income earners in the steady state with capital mobility. As shown in Figure 1, this is about half the increase in concentration for the OECD countries during the sample period: the top 1% share is about 9% toward the end of the sample. Since we do not know which part of the increase in inequality is driven by income risk, as opposed to cross-sectional inequality that is predictable at the individual level, we have assumed that the increase in risk contributed only 50%. The targeted number has been replicated in the steady state with mobility because the 2000s are characterized by a high degree of financial integration among industrialized countries.

Before continuing, we would like to explain why we make the assumption that inequality increases in both countries even if in the data the increase is observed only in some countries (see Atkinson, Piketty, and Saez (2011)). Our choice is motivated by computational considerations. When countries have different $\Delta$, Corollary 2.1 no longer holds. Thus, we cannot impose

[4]Entrepreneurial income is equal to $A(z_t^i)k_t^i+(z_t^i-\bar{z})k_t^ip_t+b_t^i-b_t^i/R_t$, that is, profits plus the interest earned on bonds. The income of an individual worker is equal to $(w_t+\tau_t)/\Phi$, that is, the labor income plus the government transfer $\tau_t = (B_{t+1}/R_t - B_t)/(1+\Phi)$.

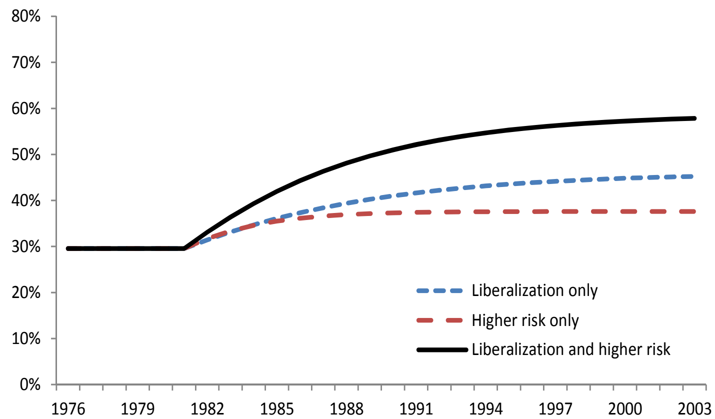


Figure 7: Dynamics of public debt in response to financial liberalization and increased income risk.

that domestic and foreign entrepreneurs hold the same $\tilde{b}$. This implies that to compute the equilibrium we have to add another state variable: $\tilde{b}_1$ or $\tilde{b}_2$. With capital mobility this increases significantly the computational complexity. However, limiting the analysis to the symmetric case is not a major shortcoming because, as shown in Section 3 with the two-period model, the change in inequality in only one country also affects the debt chosen by the other country when financial markets are integrated. Thus, using the average change in inequality as the target for all countries provides a reasonable approximation to the response of public debt in all integrated economies when the change is asymmetric.

As can been seen in Figure 7, capital liberalization (ignoring higher risk) increases long-term debt from 30% of income to about 46% of income. If we focus instead on the change in risk alone (keeping the economies in autarky), long-term debt increases to 38% of income. When the two changes are considered together, long-term debt increases to 59%.

To compare the dynamics of the model to the empirical series, Figure 8 plots the data generated by the model (with both liberalization and increased risk) and the empirical data for the average of the OECD countries, Europe, and the United States. The response of the interest rate is also plotted. The dynamic path of public debt generated by the model (continuous line) resembles the dynamics observed in the data (dashed lines). The dynamics of the interest rates are also qualitatively similar, particularly for Europe and OECD countries where we see hikes in the real rates in the first half of the 1980s, with subsequent decline later in the sample.

The initial jump in the interest rate generated by the model is necessary to make bonds attractive to entrepreneurs who are the buyers of the additional bonds. The increase in the holding of bonds requires entrepreneurs to reduce current consumption in compensation for higher future consumption, which in turn requires higher interest rates. Since the government continues to increase the debt after the first period, the interest rate remains high. However, since the increase in government debt slows down over time, the interest rate declines gradually after the initial jump. In the long run, $R$ is higher than in the autarky steady state, but the difference is small.

*(a) Public debt* *(b) Interest rate*

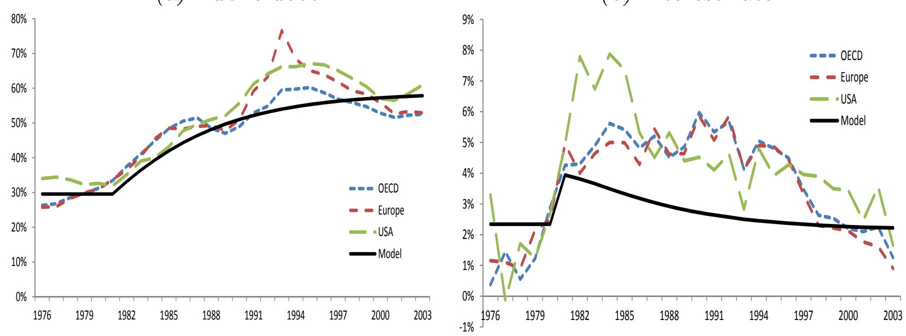


Figure 8: Dynamics of public debt and real interest rates in response to liberalization and increase in income risk.

We would like to emphasize that the comparison of the dynamics of the interest rate generated by the model with the empirical series is not meant to show that the interest rate dynamics can be fully explained by capital markets liberalization and increased income risk. Of course, there are many other factors that contributed to the interest rate dynamics, especially the hike in the early 1980s. We only want to show that the pattern predicted by the model is not inconsistent with the pattern observed in the data.

# 5 Empirical analysis

The analysis conducted in the previous sections has shown that greater mobility of capital and higher inequality raises government borrowing. In this section we conduct a simple empirical investigation of this prediction using cross-country data for the OECD countries. The main objective is to check whether there are statistically significant links between indices of capital market liberalization, income inequality, and government borrowing.

To do so we regress the growth rate of real government debt on two main variables: (i) an index that captures the change in capital mobility, and (ii) changes in the share of income earned by the top 1% of the population. We estimate the following fixed effect regression equation:

$$\begin{aligned}\mathrm{dDEBT}_{j,t} \;&=\; \alpha_D \cdot \mathrm{DEBT}_{j,t-1} + \alpha_G \cdot \mathrm{dGDP}_{j,t-1} + \alpha_M \cdot \mathrm{dMOB}_t \\ &\qquad + \alpha_I \cdot dINEQ_t + \alpha_X \cdot X_{j,t} + u_{j,t}.\end{aligned}$$

- $\mathrm{dDEBT}_{j,t}$: Log-change in real public debt of country $j$ in year $t$.
- $\mathrm{DEBT}_{j,t-1}$: Ratio of public debt to the GDP of country $j$ in year $t-1$.
- $\mathrm{dGDP}_{j,t}$: Log-change in the GDP of country $j$ in year $t$.
- $\mathrm{dMOB}_t$: Change in the index of capital mobility in year $t$ or $t-1$.
- $\mathrm{dINEQ}_t$: Log-change in top 1% of income shares in year $t$.
- $X_{j,t}$: Set of control variables for country $j$.
- $u_{j,t}$: Residuals containing country and year fixed effects.

A few remarks are in order. First, we relate the *change* in public debt to the *change* in the liberalization index, instead of the *level* of the index. This better captures the dynamics predicted by the model. In fact, in the long run, there is no relation between the degree of capital mobility and the change in debt, since the stock of debt converges to the steady state.

The second remark pertains to the construction of the index of financial liberalization. This index is not country-specific as can be noticed from the absence of the country subscript $j$. Instead, we construct the index as the average of country-specific indices for all countries included in the sample, weighted by their size (measured by total GDP). The motivation for adopting this measure of capital liberalization can be explained as follows.

Indicators of financial liberalization refer to the private sector, not the public sector. Thus, the fact that one country has very strict international capital controls does not mean that the government is restrained from borrowing abroad. What is relevant for the government ability to borrow abroad is the openness of other countries. Therefore, to determine the easiness with which the government can sell its debt to foreign (private) investors, we have to look at the capital controls imposed by other countries. This is done by computing an average index for all countries included in the sample.[5]

[5] Another way of showing the irrelevance of the country's own indicator is with the following example. Suppose that country A liberalizes its capital markets, allowing free international mobility of capital. However, all other countries maintain strict controls. Obviously, the government of country A does not have access to the foreign market even if it had liberalized its own market.

A related issue is whether in computing the weighted average of the liberalization index we should exclude the country of reference. For example, to evaluate the importance of capital mobility for the U.S. public debt, we should perhaps average the indices of the OECD countries excluding the U.S. We have chosen not to do so for the following reason. Although the liberalization of other countries is what defines the foreign market for government bonds, the domestic liberalization can still affect domestic issuance through an indirect channel. However, we also tried the alternative index and the results (not reported) are robust.

Regarding the data for the liberalization variable, we use two indices, both based on de-jure measures. The first is the liberalization index constructed by Abiad, Detragiache, and Tressel (2008). The results based on this index are reported in Table 1. The second index uses the capital account openness indicator constructed by Chinn and Ito (2008), with results reported in Table 2. Income inequality is proxied by the share of income earned by the top 1% of the population, compiled by Atkinson, Piketty, and Saez (2011). The data sources are described in the tables.

We estimate the regression equation on a sample that includes 22 OECD countries. The selection of countries in the first set of regressions is based on data availability for government debt and financial index, which restrict the sample to 26 countries. From this selected group, we exclude four countries: Hungary, Poland, Mexico, and Turkey. The first two countries are excluded since the available data start in the 1990s, when they became market oriented economies. Mexico and Turkey are excluded because they were at a lower stage of economic development compared to the other countries in the sample and they experienced various degrees of market turbulence during the sample period. For robustness, however, we also repeated the estimations for the whole sample with 26 countries, and the results are consistent with those obtained with the restricted sample, including 22 countries. The results for the extended sample are available upon request from the authors.

We start by analyzing the effects of financial integration on debt accumulation, but initially excluding inequality $\mathrm{dINEQ}_t$. By doing so we can use a larger sample since the inequality variable is unavailable for Austria, Belgium, Switzerland, Germany, Greece, and Korea. The sample size consists of 677 observations. In the simplest specification, we also abstract from any controls $X_{j,t}$. In the second specification we include a dummy for the countries that joined the European Monetary System. Since the membership was conditional on fulfilling certain requirements in terms of public debt (Maastricht Treaty), it is possible that the government debt of certain European countries has been affected by joining the EMU.

Table 1: Country fixed-effect regression. The dependent variable is real public debt growth. The financial index is based on Abiad, Detragiache, and Tressel (2008).

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| *Lag debt to GDP ratio* | $-0.149^{***}$ | $-0.146^{***}$ | $-0.149^{***}$ | $-0.170^{***}$ | $-0.162^{***}$ |
| | (0.0374) | (0.0375) | (0.0378) | (0.0383) | (0.0253) |
| *Lag real GDP growth* | $-1.235^{***}$ | $-1.210^{**}$ | $-1.216^{***}$ | $-1.159^{**}$ | $-1.381^{**}$ |
| | (0.433) | (0.430) | (0.429) | (0.413) | (0.571) |
| *Lag change in financial index* | $0.688^{**}$ | $0.697^{**}$ | $0.966^{***}$ | $1.180^{***}$ | $1.555^{***}$ |
| | (0.269) | (0.270) | (0.281) | (0.278) | (0.331) |
| *Lag EMU dummy* | | $-0.0478^{**}$ | $-0.0474^{**}$ | $-0.0521^{**}$ | $-0.084^{***}$ |
| | | (0.0189) | (0.0190) | (0.0185) | (0.0259) |
| *Size × Lag change in FI* | | | $-6.136$ | $-6.602^{*}$ | $-7.883^{*}$ |
| | | | (3.818) | (3.554) | (3.932) |
| *Change in dependency ratio* | | | | $0.0695^{**}$ | $0.0636^{**}$ |
| | | | | (0.0256) | (0.0223) |
| *Log change in inequality* | | | | | $0.128^{**}$ |
| | | | | | (0.0536) |
| Observations | 677 | 677 | 677 | 677 | 435 |
| R-squared | 0.130 | 0.132 | 0.137 | 0.150 | 0.199 |
| Number of countries | 22 | 22 | 22 | 22 | 16 |

Notes: The variable *Financial Index* (FI) is constructed using the liberalization index of Abiad, Detragiache, and Tressel (2008). We compute the financial index for a year as a weighted average of all the country indexes where weights are given by their relative GDP shares. The ratio of debt to GDP is from Reinhart and Rogoff (2011), and real GDP and population data are from the World Development Indicators (World Bank). Real debt is constructed by multiplying the ratio of debt to GDP by real GDP. Size is the lagged logarithm of real GDP. The EMU dummy is equal to 1 in the year the country joined the European Monetary Union and 0 otherwise. The old dependency ratio is the population 65 and above divided by the population in the age group 15-64. Inequality index is measured by the top 1% income share calculated by Atkinson, Piketty, and Saez (2011). The sample period is 1973-2005 and includes the following countries for specifications (1) to (4): Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States. Austria, Belgium, Germany, Greece, and Korea are excluded in specification (5) due to data availability. Robust standard errors are in parenthesis.

* Significant at 10%. ** Significant at 5%. *** Significant at 1%.

As can be seen in the first two columns of Tables 1 and 2, the coefficient on the financial index is positive and highly significant, meaning that the change in capital market integration is positively correlated with the change in public debt. Although we do not claim that this proves causation, there is a strong conditional correlation between these two variables. As far as the EMU dummy is concerned, the coefficient is negative, consistent with the

Table 2: Country fixed-effect regression. The dependent variable is real public debt growth. The financial index is based on Chinn and Ito (2008).

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| *Lag debt to GDP ratio* | $-0.150^{***}$ | $-0.147^{***}$ | $-0.148^{***}$ | $-0.166^{***}$ | $-0.157^{***}$ |
| | (0.0366) | (0.0366) | (0.0368) | (0.0380) | (0.0267) |
| *Lag real GDP growth* | $-1.262^{***}$ | $-1.235^{***}$ | $-1.230^{***}$ | $-1.189^{***}$ | $-1.400^{**}$ |
| | (0.428) | (0.425) | (0.423) | (0.410) | (0.585) |
| *Change in financial index* | $0.113^{**}$ | $0.116^{**}$ | $0.177^{***}$ | $0.205^{***}$ | $0.253^{***}$ |
| | (0.0539) | (0.0539) | (0.0575) | (0.0630) | (0.0606) |
| *Lag EMU dummy* | | $-0.0485^{**}$ | $-0.0487^{**}$ | $-0.0528^{**}$ | $-0.0854^{***}$ |
| | | (0.0189) | (0.0192) | (0.0187) | (0.0264) |
| *Size × Change in fin index* | | | $-1.375^{**}$ | $-1.428^{**}$ | $-1.437^{**}$ |
| | | | (0.728) | (0.680) | (0.617) |
| *Change in dependency ratio* | | | | $0.0594^{**}$ | $0.0535^{**}$ |
| | | | | (0.0259) | (0.0250) |
| *Change in top 1% share* | | | | | $0.106^{*}$ |
| | | | | | (0.0599) |
| Observations | 677 | 677 | 677 | 677 | 435 |
| R-squared | 0.130 | 0.132 | 0.137 | 0.150 | 0.199 |
| Number of countries | 22 | 22 | 22 | 22 | 16 |

Notes: The variable *Financial Index* is constructed using the capital account openness index of Chinn and Ito (2008). For the other variables, see notes in Table 1

view that EMU countries were forced to adjust their public finances before becoming full members.

Next, we add the interaction term between the financial index and the size of the country, measured by real GDP. The motivation to include this term is dictated by the theory. We have seen in Section 3 that the effect of capital liberalization is stronger for smaller countries. Since small countries have a lower ability to affect the world interest rate, their governments have a higher incentive to borrow once they have access to the world financial market. The third column of Tables 1 and 2 show that the coefficient on the interaction term between the financial index and the country size is negative, as expected from the theory, and statistically significant in some cases.

The fourth specification adds a demographic variable. This is the *Old Dependency Ratio* between the population in the age group 65 and higher and the population in the age group 15-64. Although our model abstracts from demographic considerations, there is a widespread belief that aging in industrialized countries is an important force for the rising public debt.

This is because the political weight shifts toward older generations that may prefer higher debt. As can be seen from the fourth column of Tables 1 and 2, the coefficient associated with the change in this variable is positive. However, the inclusion of the old dependency ratio does not affect the sign and significance of the financial index, confirming the importance of capital market liberalization for government borrowing.

The final specification introduces income inequality. With the inclusion of the inequality index we lose some observations, since the index is not available for all countries. As a result, the sample shrinks to 435 observations. The coefficient is positive and statistically significant, indicating that rising income inequality is associated with higher borrowing.

As far as the other variables are concerned, we find that the lagged stock of debt is negatively correlated with its change. This is what we expect if the debt tends to converge to a long-term level. The change in GDP is meant to capture business cycle effects, and it has the expected negative sign: when the economy does well, government revenues increase and automatic expenditures decline so that government debt increases less.

## 6 Conclusion

The stock of public debt has increased in most advanced economies during the last 30 years, a period also characterized by extensive liberalization of international capital markets and a sustained increase in income inequality. In this paper we study a multi-country politico-economic model where the incentives of governments to borrow increase both when financial markets become internationally integrated and when inequality rises. We propose this mechanism as one of the possible explanations for the growing stocks of government debt observed in most of the advanced economies since the early 1980s. We have also conducted a cross-country empirical analysis using OECD data, and the results are consistent with the theoretical predictions.

Although we have focused on government debt, it is natural to ask whether public debt is simply a substitute for private debt. Since the issuance of government debt could be Pareto improving relative to an economy where governments' budgets have to be balanced in every period, it is natural to ask whether the welfare gains can also be achieved with private debt once we allow workers to borrow from entrepreneurs. Although under certain conditions the economy with public debt can be replicated by an economy with private debt—a point also made by Kocherlakota (2007)—there are two potential limitations.

First, in our economy the competitive equilibrium with private debt is different from the equilibrium with public debt. As emphasized throughout the paper, governments internalize the effect of issuing bonds on interest rates while individual agents take prices as given when they choose their bond holdings. This implies that, if workers were allowed to borrow, the equilibrium private debt would be very different from the debt chosen by the government. Therefore, from the point of view of a positive analysis—that is, explaining the actual level of borrowing that would arise in equilibrium—the consideration of public debt is not a substitute for private debt. Of course, we can consider an environment in which the government intervenes with policies insuring that private agents choose the same amount of debt as the one chosen by the government (see Yared (2011) for an example in which public debt can serve as a substitute for private credit if private borrowing is limited). However, in absence of these policies, the equilibrium with private borrowing will be different from the equilibrium with public borrowing.[6]

The second limitation to the application of the equivalence result is that private agents may face tighter constraints than governments. In our framework private debt arises if workers are allowed to borrow. But in the presence of limited enforcement of private contracts, workers may not be able to borrow or their borrowing capacity may be limited. If governments have higher credit capacity than workers, then the economy with public debt will not be equivalent to the economy with private debt since the latter will have zero or insufficient private debt.

The final remark relates to the relevance of the analysis conducted in this paper for understanding the recent difficulties in sovereign borrowing. If debt crises are more likely to arise when the stock of public debt is higher, then the growth in government borrowing induced by capital markets liberalization and increased income inequality may contribute to trigger a sovereign debt crisis. An extension that explicitly studies the possibility of default on sovereign debt is, however, left for future research.

[6]In particular, if we allow workers to borrow privately, the equilibrium debt will grow until it reaches some borrowing limit. Without a limit the debt will converge to infinity. On the other hand, the debt chosen endogenously by the government is bounded even in absence of a very tight borrowing limit. This is an important feature of our model where the imposition of a borrowing limit for the government may not be necessary other than, of course, the imposition of some transversality condition.

# A Data appendix for Figure 1 and Figure 8

*Variables and Sources*

1) *Debt/GDP Ratio* is total (domestic plus external) gross central government debt over GDP, from Reinhart and Rogoff (2011). The sample period is 1973-2005.

2) *Financial Liberalization Index* is from Abiad, Detragiache, and Tressel (2008). The sample period is 1973-2005.

3) *Income Share of Top 1%* is from Alvaredo, Atkinson, Piketty, and Saez (2011).

4) *Deflator, p*, is the GDP deflator from Kaminsky, Reinhart, and Végh (2004). The sample period is 1970-2003.

5) *Inflation,* $\pi$, is computed as $\pi_t = p_t/p_{t-1} - 1$.

6) *Expected Inflation,* $\pi^e$, is computed as the fitted values from the regression

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \alpha_4\pi_{t-4} + \epsilon_t.$$

7) *Nominal Interest Rate, i*, is the long-term (10 years) interest rates on government bonds from OECD Statistics. Generally the yield is calculated at the pre-tax level and before deductions for brokerage costs and commissions and is derived from the relationship between the present market value of the bond and at maturity, also taking into account interest payments paid through to maturity.

8) *Real Interest Rate, r*, is computed as $r_t = (1 + i_t)/(1 + \pi^e_{t+1}) - 1$, where $i$ is the nominal interest rate and $\pi^e$ is expected inflation.

*Countries*

*OECD*: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States. *EUROPE*: Austria, Belgium, Bulgaria, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Netherlands, Norway, Poland, Portugal, Russia, Spain, Sweden, Switzerland, Turkey, and United Kingdom.

# B Proof of Lemma 2.1

Terminal conditions imply $k^i_{j,T+1} = \tilde{b}^i_{j,T+1} = 0$. For $t < T$, guess that $k^i_{j,t+1}$ and $\tilde{b}^i_{j,t+1}$ are linear in wealth $a^i_{j,t}$: $k^i_{j,t+1} = \frac{\eta_t\phi_{j,t}}{p_{j,t}}a^i_{j,t}$ and $\tilde{b}^i_{j,t+1} = R_{j,t}\eta_t(1-\phi_{j,t})a^i_{j,t}$, where $\eta_t$ is an unknown time-varying parameter. Thus, consumption follows $c^i_{j,t} = (1-\eta_t)a^i_{j,t}$ and $a^i_{j,t+1}$ satisfies

$$a^i_{j,t+1} = \eta_t\left[\left(\frac{A(z^i_{j,t+1}, w_{j,t+1}) + p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t} + R_{j,t}(1-\phi_{j,t})\right]a^i_{j,t}.$$

The first order conditions with respect to land and bond holdings for $t < T$ become

$$\frac{\eta_t}{1-\eta_t} = \beta\mathbb{E}\left\{\frac{\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}}{(1-\eta_{t+1})\left[\left(\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t}+R_{j,t}(1-\phi_{j,t})\right]}\right\} \tag{23}$$

$$\frac{\eta_t}{1-\eta_t} = \beta\mathbb{E}\left\{\frac{R_{j,t}}{(1-\eta_{t+1})\left[\left(\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t}+R_{j,t}(1-\phi_{j,t})\right]}\right\} \tag{24}$$

Multiply the two conditions by $\phi_{j,t}$ and $1-\phi_{j,t}$, respectively, and add them to get

$$\frac{\eta_t}{1-\eta_t} = \beta\mathbb{E}\left(\frac{1}{1-\eta_{t+1}}\right).$$

Hence, $\eta_T = 0$ and

$$\eta_t = \beta\frac{1}{1+\beta^{T-t}\left(\sum_{s=1}^{T-t}\beta^{s-1}\right)^{-1}} \qquad \forall t < T$$

verify the guess, and the first optimality condition becomes

$$\mathbb{E}\left[\frac{R_{j,t}}{\left(\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t}+R_{j,t}(1-\phi_{j,t})}\right] = 1. \tag{25}$$

*Q.E.D.*

# C Proof of Proposition 2.1

We first show that the wage rate does not depend on the distribution and it is constant. The optimality condition for the input of labor is $F_l(z^i_{j,t}, k^i_{j,t}, l^i_{j,t}) = w_{j,t}$. Because the production function is homogeneous of degree 1, the demand of labor is linear in land, that is, $l^i_{j,t} = l(z^i_{j,t}, w_{j,t})k^i_{j,t}$. If we integrate over all $i$ and average over $z$, we obtain the aggregate demand of labor

$$\int_i \sum_\ell l(z_\ell, w_{j,t})k^i_{j,t}\mu_\ell = \sum_\ell l(z_\ell, w_{j,t})\mu_\ell \int_i k^i_{j,t},$$

where the expression on the right-hand-side uses the law of large numbers. Since in equilibrium the demand of labor must be equal to the supply, which is 1, and total land is also 1, the above condition can be rewritten as $1 = \sum_\ell l(z_\ell, w_{j,t})\mu_\ell$. This defines implicitly the wage which does depend on endogenous variables. Therefore, the wage is constant. Since the distribution of $z$ is the same across countries, the wage rate must also be equal across countries, that is, $w_{j,t} = \bar{w}$.

Equation (11) follows from replacing the government's budget constraint (5) into the worker's budget constraint (equation (4)). Equation (7) is obtained from equation (23) after replacing $R_{j,t}(1-\phi_{j,t}) = \phi_{j,t}\tilde{b}_{j,t+1}/p_{j,t}$. This expression is derived from Lemma 2.1. To obtain equation (8), combine aggregate assets holdings $\bar{a}_{j,t} = \sum_\ell A(z_\ell, \bar{w})\mu_\ell + p_{j,t} + \tilde{b}_{j,t}$ with the aggregated choice of land, $p_{j,t}\bar{k} = \beta\phi_{j,t}\bar{a}_{j,t}$. Taking into account that the wage is $\bar{w}$, $\bar{k}=1$, and defining $\sum_\ell A(z_\ell, \bar{w})\mu_\ell = \bar{A}$, we obtain equation (8).

To derive equation (9), consider the aggregate entrepreneurs' budget constraint $c^e_{j,t} + \frac{\tilde{b}_{j,t+1}}{R_{j,t}} = \bar{A} + \tilde{b}_{j,t}$. We can now use the aggregate policy $c^e_{j,t} = (1-\beta)\bar{a}_{j,t}$ to eliminate consumption and use equation (8) to eliminate $p_{j,t}$ and solve for $R_{j,t}$.

To derive equation (10), aggregate consumption across entrepreneurs $c^e_{j,t} = (1-\beta)\bar{a}_{j,t}$ and use their (aggregate) budget constraint $\bar{a}_{j,t} = c^e_{j,t} + p_{j,t} + \tilde{b}_{j,t+1}/R_{j,t}$ to eliminate $\bar{a}_{j,t}$. *Q.E.D.*

# D Proof of Corollary 2.1

Proof of Proposition 2.1 established that $\eta_T = 0$, which, together with eq. (8), implies that $p_{j,T} = 0 \ \forall j$. Replacing this in eq. (7), we obtain

$$\phi_{j,T-1} = \mathbb{E}_{T-1}\left[\frac{A(z^i_{j,T})}{A(z^i_{j,T}) + \tilde{b}_{j,T}}\right]. \tag{26}$$

Re-writing eq. (9) at date $t = T$, and using the fact that $R_{j,t} = R_t$, $\forall j$ in an integrated equilibrium delivers

$$\tilde{b}_{j,T} = R_{T-1}(\bar{A} + \tilde{b}_{j,T-1})\frac{\eta_{T-1}(1-\phi_{j,T-1})}{(1-\eta_t\phi_{j,T-1})}. \tag{27}$$

Replacing eq. (27) into eq. (26) results in $\phi_{j,T-1}$ being a function of $\tilde{b}_{j,T-1}$. Notice that this holds because we have assumed a common shock structure across countries. Using the function $\phi_{j,T-1}(\tilde{b}_{j,T-1})$ in eq. (27) delivers $\tilde{b}_{j,T}(\tilde{b}_{j,T-1})$. Substituting these two functions in eq. (8), evaluated at $t = T-1$, yields $p_{T-1}(\tilde{b}_{j,T-1})$.

Evaluating eqs. (7) and (9) at $t = T-2$, and using the functions derived above, we can show that $\tilde{b}_{j,T-1}(\tilde{b}_{j,T-2})$. By repeated substitution over time (following the same steps) we can obtain an expression for $\tilde{b}_{j,2}$ which only depends on $\tilde{b}_{j,1}$. Since $\tilde{b}_{j,1} = \tilde{b}_1$, $\forall j$, then $\tilde{b}_{j,2} = \tilde{b}_2$, $\forall j$. Substituting forward, we can easily show that $\tilde{b}_{j,t+1} = \tilde{b}_{t+1} \ \forall j$.

Adding up across countries, and using the bond-market equilibrium condition,

$$\sum_{j=1}^{N}\tilde{b}_{j,t+1} = N\tilde{b}_{t+1} = \nu\sum_{j=1}^{N}\tilde{B}_{j,t+1},$$

which completes the proof. *Q.E.D.*

# E Proof of Lemma 3.1

Follow the steps in the proof of Proposition 2.1 (see section J) to derive

$$\frac{B}{R} = \frac{\beta \bar{A}(1-\phi(B))}{\nu[1+\beta(1-\phi(B))]} > 0, \tag{28}$$

where $\phi(B)$ satisfies

$$\phi(B) = \mathbb{E}\left(\frac{A(z)}{A(z)+\nu B}\right) < 1. \tag{29}$$

*i.* Let $B^A$ satisfy the FOC $\frac{\partial W(B)}{\partial B} = 0$, with

$$\frac{\partial W(B)}{\partial B} = \frac{\nu}{c_1^w}\frac{\partial (B/R)}{\partial B} - \beta \frac{\nu}{c_2^w},$$

where $c_1^w = \bar{w} + \nu B/R$ and $c_2^w = \bar{w} - \nu B$ are aggregate workers' consumption. Since $\frac{\partial W(B)}{\partial B} > 0|_{B=0}$ and $\frac{\partial W(B)}{\partial B} \to -\infty$ as $B \to \frac{\bar{w}}{v}$, then $B^A \in \left[0, \frac{\bar{w}}{\nu}\right]$. Uniqueness follows from the fact that $W(B)$ is strictly concave in this interval. Differentiating equation (12) yields

$$\frac{\partial^2 W(B)}{\partial B^2} = -\frac{\nu^2}{(c_1^w)^2}\left[\frac{\partial (B/R)}{\partial B}\right]^2 + \frac{\nu}{c_1^w}\frac{\partial^2 (B/R)}{\partial B^2} - \frac{\beta \nu^2}{(c_2^w)^2}. \tag{30}$$

Since

$$\frac{\partial^2 \phi(B)}{\partial B^2} = 2\mathbb{E}\left[\frac{A(z)\nu^2}{(A(z)+\nu B)^3}\right] > 0, \tag{31}$$

we have that

$$\frac{\partial^2 (B/R)}{\partial B^2} = -\frac{\beta \bar{A}}{\nu(1+\beta[1-\phi(B)])^3}\left[\frac{\partial^2 \phi(B)}{\partial B^2}(1+\beta[1-\phi(B)]) + 2\beta\left(\frac{\partial \phi(B)}{\partial B}\right)^2\right] < 0,$$

establishing concavity.

*ii.* Replace equation (28) into the representative entrepreneur's consumption and obtain $c_1^e = \frac{\bar{A}}{1+\beta[1-\phi(B)]}$. Then, differentiate the resulting indirect utility

$$\frac{\partial V(B)}{\partial B} = \frac{\beta}{1+\beta(1-\phi(B))}\frac{\partial \phi(B)}{\partial B} + \beta \mathbb{E}\left(\frac{\nu}{A(z)+\nu B}\right).$$

Substitute

$$\frac{\partial \phi(B)}{\partial B} = -\mathbb{E}\left[\frac{\nu A(z)}{(A(z)+\nu B)^2}\right] \tag{32}$$

in the expression above and collect terms to show

$$\frac{\partial V(B)}{\partial B} = \beta \nu \mathbb{E}\left[\frac{\nu B + \beta[1-\phi(B)]\,(A(z)+\nu B)}{(A(z)+\nu B)^2\,(1+\beta[1-\phi(B)])}\right] > 0.$$

*Q.E.D.*

# F Proof of Proposition 3.1

Suppose that $\Phi > \frac{(1+\beta)\bar{w}}{\bar{A}} + \beta$, and let the government's objective be defined by

$$G(B) \equiv \Phi W(B) + V(B)$$

where $W(B)$ and $V(B)$ are given by equations (12) and (13). To prove concavity, differentiate $G(B)$ twice, where $\frac{\partial^2 W(B)}{\partial B^2}$ is defined in equation (30) and

$$\frac{\partial^2 V(B)}{\partial B^2} = -\frac{\nu^2}{(c_1^e)^2}\left[\frac{\partial(B/R)}{\partial B}\right]^2 - \frac{\nu}{c_1^e}\frac{\partial^2(B/R)}{\partial B^2} - \beta\mathbb{E}\frac{\nu^2}{(c_2^e)^2}.$$

After some manipulations, we can show that

$$\begin{aligned}\frac{\partial^2 G(B)}{\partial B^2} &= -\left[\frac{\partial(B/R)}{\partial B}\right]^2 \nu^2\left[\frac{\Phi}{(c_1^w)^2} + \frac{1}{(c_1^e)^2}\right] - \beta\nu^2\left[\frac{\Phi}{(c_2^w)^2} + \mathbb{E}\frac{1}{(c_2^e)^2}\right] \\ &+ \frac{\partial^2(B/R)}{\partial B^2}\nu\left[\frac{\Phi}{c_1^w} - \frac{1}{c_1^e}\right].\end{aligned}$$

The first row is negative for all $B$. Hence, a sufficient condition for $\frac{\partial^2 G(B)}{\partial B^2} < 0$ is that the second row is non positive. We established that $\frac{\partial^2(B/R)}{\partial B^2} < 0$ in Section E (Part *i.*). In addition, we need that

$$\frac{\Phi}{c_1^w} - \frac{1}{c_1^e} = \frac{\Phi c_1^e - c_1^w}{c_1^w c_1^e} > 0,$$

since $c_1^e = \frac{\bar{A}}{1+\beta[1-\phi(B)]}$ and $c_1^w = \bar{w} + \nu B/R$. Substituting for $R$ we get that

$$\begin{aligned}c_1^e - c_1^w/\Phi &= \frac{1}{1+\beta(1-\phi(B))}\left[\bar{A} - \frac{1}{\Phi}\left([1+\beta(1-\phi(B))]\bar{w} + \beta\bar{A}(1-\phi(B))\right)\right] \\ &\geq \frac{1}{\Phi}\frac{1}{1+\beta(1-\phi(B)}[\bar{A}(\Phi-\beta) - \bar{w}(1+\beta)].\end{aligned}$$

Since $0 \leq \phi(B) \leq 1$, the denominator of the above equation is positive. Moreover, the assumption that $\Phi > \frac{(1+\beta)\bar{w}}{\bar{A}} + \beta$ is a sufficient condition for the numerator of the above equation to be positive as well. This establishes concavity.

Let $B^A$ satisfy $\frac{\partial G(B)}{\partial B} = 0$. From Lemma 3.1, $V(B)$ is increasing in $B$ $\forall B \in \left[0, \frac{\bar{w}}{\nu}\right]$ and $\frac{\partial W(B)}{\partial B}|_{B=0} > 0 \Rightarrow \frac{\partial G(B)}{\partial B}|_{B=0} > 0$. Additionally, $\frac{\partial V(B)}{\partial B}$ is finite at $\frac{\bar{w}}{\nu}$ and $\frac{\partial W(B)}{\partial B} \to -\infty$ as $B \to \frac{\bar{w}}{\nu}$, so $\frac{\partial G(B)}{\partial B} \to -\infty$. Hence $B^A \in \left[0, \frac{\bar{w}}{\nu}\right]$. Because $G(B)$ is strictly concave, $B^A$ must be unique. *Q.E.D.*

# G Proof of Proposition 3.2

Let the relative size of workers $\nu = 1$. To show that debt is increasing in $N$, replace $\Phi/(1+\Phi) = 1$ in equation (21) to obtain

$$G(B,N) \equiv \Phi\left[\frac{\partial(B/R)}{\partial B}\left(\frac{1}{c_1^w}\right) - \beta\left(\frac{1}{c_2^w}\right)\right] = 0,$$

where

$$\frac{\partial(B/R)}{\partial B} = \frac{1}{R}\left(1 - \frac{B}{R}\frac{\partial R}{\partial b}\frac{1}{N}\right) \equiv \gamma \text{ and}$$

$$\frac{\partial R}{\partial b} = R\left[\frac{1}{b} + \frac{\partial \phi}{\partial b}\frac{1}{[1+\beta(1-\phi)](1-\phi)}\right]. \tag{33}$$

Recall that $b = \frac{\sum_{j=1}^{N} B_j}{N}$ denotes the demand of bonds.

**Claim G.1:** The interest rate is increasing in $b$, $\frac{\partial R}{\partial b} > 0$.

**Proof:** Re-write eq. (33) as

$$\begin{aligned}\frac{\partial R}{\partial b} &= \frac{R}{b[1+\beta(1-\phi)](1-\phi)}\left[[1+\beta(1-\phi)](1-\phi) + b\frac{\partial \phi}{\partial b}\right] \\ &> \frac{R}{b[1+\beta(1-\phi)](1-\phi)}\left[1-\phi + b\frac{\partial \phi}{\partial b}\right] = 0\end{aligned}$$

The inequality follows from $\beta(1-\phi) < 1$. Replace eqs. (29) and (32) in the bracketed term to show the equality. *Q.E.D.*

**Claim G.2:** *(i.)* $\partial G(B,N)/\partial B < 0$ and *(ii.)* $\partial G(B,N)/\partial N > 0$

**Proof:**

*(i.)* We can show that

$$\frac{\partial G(B,N)}{\partial B} = \Phi\left[\frac{\partial \gamma}{\partial b}\frac{1}{c_1^w} - \gamma^2\frac{\Phi}{(c_1^w)^2} - \beta\frac{\Phi}{(c_2^w)^2}\right]. \tag{34}$$

where

$$\frac{\partial \gamma}{\partial b} = -\left(1 - \frac{B}{Nb}\right)\frac{2}{NR^2}\frac{\partial R}{\partial b} - \frac{B}{Nb}\frac{\beta \bar{A}}{(1+\beta(1-\phi))^2}\left[\frac{\partial^2 \phi}{\partial b^2} + \frac{2\beta\left(\frac{\partial \phi}{\partial b}\right)^2}{1+\beta(1-\phi)}\right].$$

Since $\frac{\partial^2 \phi}{\partial b^2} > 0$ from eq. (31) and $\frac{\partial R}{\partial b} > 0$ from Claim G.1, then $\frac{\partial \gamma}{\partial b} < 0$. Because all terms in equation (34) are negative, the result follows.

*(ii.)* We can show that

$$\frac{\partial G(B,N)}{\partial N} = \Phi\left[\frac{\partial \gamma}{\partial N}\frac{1}{c_1^w} - \gamma\frac{\Phi}{(c_1^w)^2}\frac{\partial(B/R)}{\partial N}\right].$$

The first term is positive. Noting that since $b = B$ then $\frac{\partial b}{\partial N} = \frac{b-B}{N^2} = 0$, and performing some algebraic manipulations, we obtain

$$\frac{\partial \gamma}{\partial N} = \frac{B}{R^2 N^2}\frac{\partial R}{\partial b} > 0$$

from Claim G.1. The second term is zero, since

$$\frac{\partial(B/R)}{\partial N}=-\left[\frac{1-\phi}{b}+\frac{1}{[1+\beta(1-\phi)]}\frac{\partial\phi}{\partial b}\right]\frac{B\beta\bar{A}}{[1-\beta(1-\phi)]b}\frac{\partial b}{\partial N}$$

and $\frac{\partial b}{\partial N}=0$. *Q.E.D.*

Using Claim G.2 and the implicit function theorem, we conclude that domestic debt $B$ is increasing in $N$

$$\frac{\partial B}{\partial N}=-\frac{\partial G(B,N)/\partial N}{\partial G(B,N)/\partial B}>0.$$

For the limiting case, let $N\to\infty$ in equation (21). Substituting $c_1^w$ and $c_2^w$ and rearranging, we obtain

$$\beta R=1-\frac{1+\beta}{\bar{w}}B. \tag{35}$$

This equation determines country 1's supply of debt given $R$. In equilibrium, $B_1=B_2=..B_N=b=b$ where the per-capita demand for debt $b$ satisfies equation (18). The financially integrated equilibrium levels of $b$ and $R$ are thus determined by equations (18) and (35).

Existence and uniqueness follow from: (i) the LHS of equation (35) is decreasing in $b$ and equals 1 at the origin, and (ii) the RHS of equation (35) is increasing in $b$ (since $R_b>0$) and has an intercept at $\left[\mathbb{E}\left(\frac{\bar{z}}{z}\right)\right]^{-1}<1$. Denote the intersection point by $B^{FI}$. From (i) and (ii) it also follows that $B^{FI}$ is bounded and $\beta R<1$ when $b=B^{FI}$.

Under autarky, equation (35) is instead

$$\beta R=1-\frac{1+\beta}{\bar{w}}b-\epsilon(b)\left(1-\frac{b}{\bar{w}}\right). \tag{36}$$

The LHS is the same as before. The RHS is also equal to 1 at the origin because $\epsilon(0)=0$. Since $\epsilon(b)>0$ and $\bar{w}-b=c_2^w>0$ when $b>0$, the new term in the RHS is positive. Hence, the intersection of the two curves in equation (36) occurs at $B^A<B^{FI}$, since the RHS is steeper.

Since debt is larger and $V$ is increasing in $b$, $V(B^A)<V(B^{FI})$. Since $W$ is concave in $b$ and $W(b)$ is decreasing when $b>B^A$, then $W(B^A)>W(B^{FI})$.

What is left to prove is that the equilibrium must be symmetric. This can be shown starting from the first order condition of the government

$$\Phi\cdot\left[\frac{1-\frac{\epsilon(\overline{B})B}{N\overline{B}}}{c_1^w}-\frac{\beta R}{c_2^w}\right]=\left(\frac{1}{N}\right)\cdot\left[\frac{1-\epsilon(\overline{B})}{c_1^e}-\mathbb{E}_t\left(\frac{\beta R}{c_2^e}\right)\right], \tag{37}$$

which must be satisfied for all countries.

An equilibrium is characterized by a worldwide debt $\overline{B}$. Given $\overline{B}$, the elasticity $\epsilon$ and the interest rate $R$ are determined. Also notice that the right-hand side of (37) is the same for all countries, since entrepreneurs choose to hold the same stock of bond in all countries. The left-hand side could differ since governments could choose different $B$. However, since the left-hand side is strictly decreasing in $B$ (keeping $\overline{B}$ constant), the fact that the right-hand side is the same for all countries implies that $B$ must be the same for all countries. Otherwise, the first order condition (37) will not hold for all countries. Notice that this result applies for any value of $\Phi$, not only for the limiting case $\Phi/(1+\Phi) = 1$. *Q.E.D.*

# H Proof of Proposition 3.3

Setting $\Phi/(1+\Phi) = 1$, the first order conditions for the domestic and foreign country become

$$1 - \frac{\alpha B_1}{\overline{B}}\epsilon(\overline{B}) = \beta R(\overline{B}) \left( \frac{c_1^w(B_1)}{c_2^w(B_1)} \right) \tag{38}$$

$$1 - \frac{(1-\alpha)B_2}{\overline{B}}\epsilon(\overline{B}) = \beta R(\overline{B}) \left( \frac{c_1^w(B_2)}{c_2^w(B_2)} \right), \tag{39}$$

where we have made it explicit that the interest rate elasticity, $\epsilon(\overline{B})$, and the interest rate, $R(\overline{B})$, are functions of the average worldwide debt $\overline{B} = \alpha B_1 + (1-\alpha)B_2$.

An equilibrium will be characterized by $B_1$ and $B_2$ (and $\overline{B}$) that satisfy conditions (38) and (39). We want to show that in an integrated economy $B_1 > B_2$ if $\alpha < 1/2$, that is, the per-capita debt of the large country is lower than the per-capita debt of the small country.

Subtracting (39) to (38) and substituting $(1-\alpha)B_2 = \overline{B} - \alpha B_1$ we get

$$\left(1 - \frac{2\alpha B_1}{\overline{B}}\right)\epsilon(\overline{B}) = \beta R(\overline{B}) \left( \frac{c_1^w(B_1)}{c_2^w(B_1)} - \frac{c_1^w(B_2)}{c_2^w(B_2)} \right) \tag{40}$$

For a given $\overline{B}$ that characterizes the equilibrium, the left-hand-side term is decreasing in $B_1$. Since $\overline{B}$ is the equilibrium worldwide debt taken as given in this exercise, an increase in $B_1$ must be associated to a decline in $B_2$. Therefore, it is the ratio $B_1/B_2$ that matters. The right-hand-side term, instead, is increasing in $B_1$. To see this, we can define aggregate workers' consumption using the budget constraints as

$$c_1^w(B_1) = \bar{w} + \frac{B_1}{R(\overline{B})}, \qquad c_2^w(B_1) = \bar{w} - B_1 \tag{41}$$

$$c_1^w(B_2) = \bar{w} + \frac{B_2}{R(\overline{B})}, \qquad c_2^w(B_2) = \bar{w} - B_2 \tag{42}$$

From these equations it is clear that $c_1^w(B_1)/c_2^w(B_1)$ is increasing in $B_1$ and $c_1^w(B_2)/c_2^w(B_2)$ is increasing in $B_2$. Since an increase in $B_1$ must be associated with a decline in $B_2$, then $c_1^w(B_2)/c_2^w(B_2)$ is decreasing in $B_1$. Thus, the right-hand side of equation (40) must be increasing in $B_1$.

So far, we have established that the LHS of equation (40) is decreasing and the RHS is increasing in $B_1$. Next, we observe that, if $\alpha < 1/2$, then the LHS is positive when $B_1 = B_2$. The RHS, instead, is zero. Therefore, to equalize the LHS (which is decreasing in $B_1$) to the RHS (which is increasing in $B_1$) we have to increase $B_1$ (which must be associated with a decrease in $B_2$). Therefore, if $\alpha < 1/2$, $B_1 > B_2$.

Finally, since in the autarky equilibrium both countries had the same debt, the growth in debt following financial liberalization is bigger for the small country. Notice that this does not exclude the possibility of negative growth. *Q.E.D.*

# I Proof of Proposition 3.4

Let $\Phi/(1+\Phi) = 1$, then the autarky equilibrium satisfies the government's first order condition

$$\frac{1-\epsilon(B)}{R(B)c_1^w} = \frac{\beta}{c_2^w},$$

where we made it explicit that the interest elasticity $\epsilon$ and the interest rate $R$ are functions of debt $B$. Since $c_1^w = \bar{w} + B/R(B)$ and $c_2^w = \bar{w} - B$, the first order condition can be rewritten as

$$\frac{1-\epsilon(B)}{\bar{w}R(B)+B} = \frac{\beta}{\bar{w}-B}. \tag{43}$$

The right-hand side of (43) is clearly increasing in $B$. We now show that the left-hand side is decreasing $B$. First let's rewrite the left-hand side as

$$\frac{1-\epsilon(B)}{\bar{w}R(B)+B} = \left(\frac{1-\epsilon(B)}{R(B)}\right)\cdot\left(\frac{1}{\bar{w}+B/R(B)}\right), \tag{44}$$

which is the product of two terms. We want to show that both terms are decreasing in $B$. Let's start with the first term which is equal to

$$\frac{1-\epsilon(B)}{R(B)} = -\frac{\beta\phi'(B)\bar{A}}{[1+\beta(1-\phi(B))]^2}.$$

Since $\phi(B) = \mathbb{E}[A(z)/(A(z)+B)]$ and $-\phi'(B) = \mathbb{E}[A(z)/(A(z)+B)^2]$ are both decreasing in $B$, then the first term in (44) is also decreasing in $B$. The second term in (44) depends negatively on $B/R(B) = \beta(1-\phi(B)\bar{A}/[1+\beta(1-\phi(B))]$. As we have already observed, $\phi(B) = \mathbb{E}A(z)/(A(z)+B)$ depends negatively on $B$ and, therefore, $B/R(B)$ increases in $B$. Thus, the second term in (44) decreases with $B$. This proves that (44) is decreasing in $B$.

To summarize, we have shown that the left-hand side of the first order condition (43) decreases with $B$, while the right-hand side increases with $B$. Therefore, if an increase in the mean preserving spread of $z$ raises the term $(1-\epsilon(B))/[\bar{w}R(B)+B]$, which is the left-hand side of condition (43), to re-establish equality $B$ has to rise. *Q.E.D.*

# J Proof of Proposition 4.1

Let's first derive the value for workers. Individual workers' consumption is equal to the labor income plus the transfers, that is,

$$c_{j,t} = \left(\frac{1}{\Phi}\right)\bar{w} + \tau_{j,t}.$$

Since the total government transfers are equal to $B_{j,T+1}/R_{j,t} - B_{j,t}$ and the population is $1+\Phi$, each worker gets $\tau_{j,t} = (B_{j,T+1}/R_{j,t} - B_{j,t})/(1+\Phi)$. Substituting and collection $1/\Phi$ we get

$$c_{j,t} = \left(\frac{1}{\Phi}\right)\left[\bar{w} + \nu\left(\frac{B_{j,t+1}}{R_{j,t}} - B_{j,t}\right)\right]. \tag{45}$$

To derive the workers' value we start from the terminal period $t = T$ where

$$\widetilde{W}_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln(c_{j,T}).$$

Substituting (45) we can rewrite it as

$$W_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln\left(\bar{w} + \frac{\nu B_{j,T+1}}{R_{j,T}} - \nu B_{j,T}\right),$$

where $W_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \widetilde{W}_{j,t}(\mathbf{B}_T, \mathbf{B}_{T+1}) + \ln(\Phi)$.

We now consider the earlier period $t = T-1$ where the workers value is

$$\widetilde{W}_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(c_{j,T-1}) + \beta\widetilde{W}_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}).$$

Substituting (45), the worker's value can be rearranged as

$$W_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln\left(\bar{w} + \frac{\nu B_{j,T}}{R_{j,T-1}} - \nu B_{j,T-1}\right) + \beta W_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}),$$

where $W_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \widetilde{W}_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) + (1+\beta)\ln(\Phi)$.

Continuing with $t = T-2, .., 1$ we can derive the general expression

$$W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \ln\left(\bar{w} + \frac{\nu B_{j,t+1}}{R_{j,t}} - \nu B_{j,t}\right) + \beta W_{j,t+1}(\mathbf{B}_{t+1}, \mathbf{B}_{t+2}), \tag{46}$$

where

$$W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \widetilde{W}_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + \left(\frac{1}{1-\eta_t}\right)\ln(\Phi). \tag{47}$$

Replacing $\mathbf{B}_{t+2} = \mathcal{B}_{t+1}(\mathbf{B}_{t+1})$, we obtain the expression for $W_{j,t}$ reported in Proposition 4.1.

We now derive the value for entrepreneurs. By Lemma 2.1, entrepreneur's consumption is equal $c^i_{j,t} = (1-\eta_t)a^i_{j,t}$, where $a^i_{j,t} = [A(z^i_{j,t}) + p_{j,t} + \tilde{b}^i_{j,t}/k^i_{j,t}]k^i_{j,t}$.

Since $\tilde{b}^i_{j,t}/k^i_{j,t}$ is the same across entrepreneurs and the aggregate stock of land is 1, we can write $a^i_{j,t} = [A(z^i_{j,t}) + p_{j,t} + \tilde{b}^i_{j,t}]k^i_{j,t} = \hat{a}^i_{j,t}k^i_{j,t}$, with consumption equal to

$$c^i_{j,t} = (1-\eta_t)\hat{a}^i_{j,t}k^i_{j,t}. \tag{48}$$

Using Lemma 2.1 we can also write the individual gross growth rate of land as

$$\frac{k^i_{j,t+1}}{k^i_{j,t}} = \frac{\eta_t \phi_{j,t}\hat{a}^i_{j,t}}{p_{j,t}}. \tag{49}$$

Since we have a finite number of periods, we start with the terminal period $t = T$. The indirect utility of an entrepreneur $i$ at $t = T$ is equal to

$$\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln(c^i_{j,T}).$$

Substituting (48) for $t = T$ we have

$$\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln(1-\eta_T) + \ln(\hat{a}^i_{j,T}) + \ln(k^i_{j,T}).$$

Subtracting $\ln(k^i_{j,T})$ on both sides and integrating over $z^i_{j,t}$, we define the expected average normalized value

$$V_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \mathbb{E}\Big[\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) - \ln(k^i_{j,T})\Big] = \ln(1-\eta_T) + \mathbb{E}\ln(\hat{a}^i_{j,T}).$$

Here the expectation operator $\mathbb{E}$ represents the integration over all individual entrepreneurs indexed by $i$. Once we integrate, the resulting value does not depend on individual characteristics nor the distribution of $k^i_{j,t}$. Thus, we have dropped the superscript $i$.

We move next to the earlier period $t = T-1$. The indirect utility of an entrepreneur $i$ can be written as

$$\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(c^i_{j,T-1}) + \beta\mathbb{E}\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}),$$

was derived above. Substituting (48) for $t = T-1$ we have

$$\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(1-\eta_{T-1}) + \ln(\hat{a}^i_{j,T-1}) + \ln(k^i_{j,T-1}) + \beta\mathbb{E}\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}).$$

Subtracting $(1+\beta)\ln(k^i_{j,T-1})$ from both sides and adding and subtracting $\beta\ln(k^i_{j,T})$ on the right-hand-side we obtain

$$\begin{aligned}\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) - (1+\beta)\ln(k^i_{j,T-1}) \quad = \quad & \ln(1-\eta_{T-1}) + \ln(\hat{a}^i_{j,T-1}) + \beta\ln\left(\frac{k^i_{j,T}}{k^i_{j,T-1}}\right) \\ & + \beta\mathbb{E}\Big[\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) - \ln(k^i_{j,T})\Big].\end{aligned}$$

Using equation (49) to eliminate $k^i_{j,T}/k^i_{j,T-1}$ and integrating over $i$ we get,

$$V_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(1-\eta_{T-1}) + (1+\beta)\mathbb{E}\ln(\hat{a}^i_{j,T-1}) + \beta\ln\left(\frac{\eta_{T-1}\phi_{j,T-1}}{p_{j,T-1}}\right) + \beta V_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}),$$

where we have defined

$$V_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \mathbb{E}\Big[\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) - (1+\beta)\ln(k^i_{j,T-1})\Big]$$

The next step is to consider the period $t = T-2$ and continue backward until $t = 1$. The expression for a generic $t$ is

$$V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \ln(1-\eta_t) + \left(\frac{1}{1-\eta_t}\right)\left[\eta_t \ln\left(\frac{\eta_t \phi_{j,t}}{p_{j,t}}\right) + \mathbb{E}_t \ln \hat{a}^i_{j,t}\right] + \beta V_{j,t+1}(\mathbf{B}_{t+1}, \mathbf{B}_{t+2}), \tag{50}$$

where

$$V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \mathbb{E}\left[\widetilde{V}^i_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) - \left(\frac{1}{1-\eta_t}\right)\ln(k^i_{j,t})\right] \tag{51}$$

Replacing $\mathbf{B}_{t+2} = \mathcal{B}_{t+1}(\mathbf{B}_{t+1})$, we obtain the expression for $V_{j,t}$ reported in Proposition 4.1.

The government objective is

$$\Phi \tilde{W}_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + \mathbb{E}\widetilde{V}^i_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}).$$

Remember that the objective of the government is the integral of the ‘non-normalized’ values for workers and entrepreneurs. Using (47) and (51), the objective can be rewritten as

$$\Phi W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + \left(\frac{1}{1-\eta_t}\right)\Big(\mathbb{E}\ln(k^i_{j,t}) - \Phi\ln(\Phi)\Big)$$

Since the last term enters additively and does not depend on $\mathbf{B}_{t+1}$, the optimal debt is independent of this term. Therefore, we can focus on the first two terms as reported in Proposition 4.1. *Q.E.D.*

## References


Abiad, Abdul, Detragiache, Enrica and Therry Tressel. 2008. "A New Database of Financial Reform." IMF Working Paper No. 08/226.

Aguiar, Mark and Manuel Amador. 2011. "Growth in the Shadow of Expropriation." *Quarterly Journal of Economics*, 126(2): 651-697.

Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent, and Juha Seppala. 2002. "Optimal Taxation Without State-Contingent Debt." *Journal of Political Economy*, 110: 1220-1254.

Aiyagari, S. Rao, and Ellen R. McGrattan. 1998. "The Optimum Quantity of Debt." *Journal of Monetary Economics*, 42(3): 44769.

Albanesi, Stefania and Christopher Sleet. 2006. "Dynamic Optimal Taxation with Private Information." *Review of Economic Studies*, 73(1): 1-30.

Alesina, Alberto and Guido Tabellini. 1990. "A Positive Theory of Fiscal Deficits and Government Debt." *Review of Economic Studies*, 57(3): 403-414.

Angeletos, George-Marios. 2002. "Fiscal Policy with Non-Contingent Debt and the Optimal Maturity Structure." *Quarterly Journal of Economics*, 117(3): 1105-1131.

Angeletos, George-Marios. 2007. "Uninsured Idiosyncratic Investment Risk and Aggregate Saving." *Review of Economic Dynamics*, 10(1): 1-30.

Angeletos, George-Marios, and Vasia Panousi. 2011. "Financial Integration, Entrepreneurial Risk and Global Dynamics." *Journal of Economic Theory*, 146(3): 863-96.

Alvaredo, Facundo, Anthony B. Atkinson, Thomas Piketty and Emmanuel Saez. 2011. "The World Top Incomes Database." http://g-mond.parisschoolofeconomics.eu/topincomes, 19/12/2011.

Atkinson, Anthony B., Thomas Piketty, and Emmanuel Saez. 2011. "Top Incomes in the Long Run of History." *Journal of Economic Literature*, 49(1): 3-71.

Azzimonti, Marina, Eva de Francisco and Per Krusell. 2008. "Production Subsidies and Redistribution." *Journal of Economic Theory*, 142: 77-99.

Barro, R. 1979. "On the Determination of the Public Debt." *Journal of Political Economy*, Vol. 87: 940-71.

Battaglini, Marco and Stephen Coate. 2008. "A Dynamic Theory of Public Spending, Taxation, and Debt." *American Economic Review*, 98(1): 201-236.

Caballero, Ricardo J., Emanuel Farhi, and Pierre-Olivier Gourinchas. 2008, "An Equilibrium Model of Global Imbalances and Low Interest Rates." *American Economic Review*, 98(1): 358-93.

Caballero, Ricardo and Pierre Yared. 2008. "Future Renting-Seeking and Current Public Savings." NBER Working Paper No. 14417

Chang, Roberto. 1990. "International Coordination of Fiscal Deficits." *Journal of Monetary Economics*, 25(3): 347-66.

Chari, V. V., Lawrence J. Christiano, and Patrick J. Kehoe. 1994. "Optimal Fiscal Policy in a Business Cycle Model." *Journal of Political Economy*, 102(4): 617-52.

Chinn, Menzio and Hiro Ito. 2008. "A New Measure of Financial Openness." *Journal of Comparative Policy Analysis*, 10(3): 309-22.

Cooper, Rusell, Hubert Kempf, and Dan Peled. 2008. "Is It or Is It Ain't My Obligation? Regional Debt in a Fiscal Federation." *International Economic Review*, 49(4): 1469504.

Corbae, Dean, Pablo D'Erasmo, and Burhanettin Kuruscu. 2009. "Politico-Economic Consequences of Rising Wage Inequality." *Journal of Monetary Economics*, 56(1): 43-61.

Farhi, Emmanuel and Ivan Werning. 2008. "The political economy of nonlinear capital taxation." Mimeo.

Fogli, Alessandra and Fabrizio Perri. 2006. "The Great Moderation and the U.S. External Imbalance." *Monetary and Economic Studies*, Institute for Monetary and Economic Studies, Bank of Japan, 24(S1): 209-25.

Golosov, Mikhail, Narayana Kocherlakota, and Aleh Tsyvinski. 2003. "Optimal Indirect and Capital Taxation." *Review of Economic Studies*, 70(3): 569-87.

Ilzetzki, Ethan. 2011. "Rent-Seeking Distortions and Fiscal Procyclicality." *Journal of Development Economics*, 96(1): 30-46.

Kaminsky, Graciela, Carmen Reinhart, and Carlos Végh. 2004 "When it rains, it pours: Procyclical capital flows and macroeconomic policies." *NBER Macroeconomics Annual 2004*, ed. Mark Gertler and Ken Rogoff, 11-82. Cambridge,

Kehoe, Patrick J. (1989). "Cooperation Among Benevolent Governments May Be Undesirable." *Review of Economic Studies*, 56(2): 28996.

Kocherlakota, Narayana 2007. "Money and Bonds: An Equivalence Theorem." Working Paper.

Krueger, Dirk, and Fabrizio Perri. 2006. "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory." *Review of Economic Studies*, 73(1): 163-93.

Krusell, Per, and Jose Victor Rios-Rull. 1999. "On the Size of U.S. Government: Political Economy in the Neoclassical Growth model.", *American Economic Review*, 89(5): 1156-81.

Krusell, Per, Fernando Martin, and Jose Victor Rios-Rull. 2006. "Time Consistent Debt.", Unpublished manuscript.

Lucas, Robert E. Jr., and Nancy L. Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary Economics*, 12(1): 55-93.

Marcet, Albert and Andrew Scott. 2009. "Debt, Deficits, and the Structure of Bond Markets." *Journal of Economic Theory*, 144(2): 473-501.

Mendoza, Enrique, Vincenzo Quadrini, and Jose-Victor Rios-Rull. 2009. "Financial Integration, Financial Development and Global Imbalances." *Journal of Political Economy*, 117(3): 371-416.

Mendoza, Enrique, and Linda Tesar. 2005. "Why Hasn't Tax Competition Triggered a Race to the Bottom? Some Quantitative Lessons from the EU." *Journal of Monetary Economics*, 52(1): 163-204.

Obstfeld, Maurice, and Alan M. Taylor. 2005. *Global Capital Markets Integration, Crisis, and Growth.* Cambridge University Press, New York.

Persson, Torsten and Lars Svensson. 1989. "Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences." *The Quarterly Journal of Economics*, 104(2): 325-345.

Persson, Torsten and Guido Tabellini. 2000. *Political Economics: Explaining Economic Policy.* Cambridge, Mass: MIT Press

Quadrini, Vincenzo. 2005. "Policy Commitment and the Welfare Gains from Capital Market Liberalization." *European Economic Review*, 49(8): 1927-51.

Reinhart, Carmen and Kenneth Rogoff. 2011. "From Financial Crash to Debt Crisis." *American Economic Review*, 101(5): 1676-706.

Shin, Yongseok. 2006. "Ramsey Meets Bewley: Optimal Government Financing with Incomplete Markets. " Unpublished manuscript, Washington University in St. Louis.

Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti. 2012. "Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt." *Econometrica*, forthcoming.

Yared, Pierre. 2011. "Public Debt under Limited Private Credit." *Journal of the European Economic Association*, forthcoming.