

Endogenous Market Incompleteness with Investment Risks*

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Abstract

This paper studies a general equilibrium economy in which agents have the ability to invest in a risky technology. The investment risk cannot be fully insured with optimal contracts because shocks are private information. We show that the presence of investment risks lead to under-accumulation of capital relative to an economy where idiosyncratic shocks can be fully insured. We also show that the availability of state-contingent (optimal) contracts—compared to simple debt contracts—brings the aggregate stock of capital close to the complete markets level. Institutional reforms that make possible the use of these contracts have important welfare consequences.

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1 Introduction

A large body of literature that studies the saving behavior in the presence of uninsurable idiosyncratic risks assumes that these risks are not associated with investment. As in Bewley (1986), the most common assumption is that earnings or endowments are subject to shocks that cannot be insured away. This is the assumption in papers such as Aiyagari (1994, 1995), Hansen & İmrohorođlu (1992), Huggett (1993, 1996), İmrohorođlu (1989), Ríos-Rull (1994). In this class of models, the inability to fully insure the idiosyncratic risk implies that the equilibrium interest rate is lower than in the complete markets economy, whether market incompleteness is taken as given or modeled endogenously. Because the interest rate is equal to the marginal productivity of capital, this also implies that the stock of capital is higher than in the complete markets economy (over-accumulation).

Although earnings or labor income uncertainty is an important source of idiosyncratic risk, investment activities are also subject to uninsurable risks. For instance, entrepreneurs invest heavily in their own business¹ and managers of corporations hold a large number of the company’s shares.² Even the return from investing in education is highly uncertain and cannot be insured away. What differentiates investment risks from earnings or endowment risks is that the agent can avoid these risks by choosing safer allocations of savings. On the contrary, earnings or endowment risks in the class of Bewley’s economies are beyond the control of the agent. The agent can only use the available markets to (incompletely) insure them.

The goal of this paper is to model explicitly investment risks. We consider three environments. In the first environment—the “Optimal Contract Economy”—agents can sign optimal state-contingent contracts. These contracts, however, cannot provide full insurance because there are agency problems in the form of asymmetric information. Therefore, in this economy market incompleteness is endogenous. In the second environment—the “Bond Economy”—agents cannot sign state-contingent contracts. Only non-contingent contracts (borrowing and lending) are available. In the third environment—the “Complete Markets Economy”—there are no agency problems, and therefore, full insurance against investment risks is possible.

By comparing these three economies we show that:

¹See Cagetti & DeNardi (2002), Carroll (2002), Gentry & Hubbard (2000), Hurst & Lusardi (2004) and Quadrini (1999).

²See Mikkelson, Partch, & Shah (1997) and Himmelberg, Hubbard, & Love (2000).

1. In the two economies with incomplete markets (the Bond Economy and the Optimal Contract Economy) the equilibrium risk-free interest rate is smaller than in the Complete Markets Economy. However, the aggregate stock of capital is smaller than in the Complete Markets Economy, *i.e.*, there is under-accumulation.
2. Even with very large agency problems, the availability of optimal contracts brings the aggregate stock of capital and the equilibrium interest rate very close to the corresponding levels in the Complete Markets Economy. As a result, the feasibility of optimal contracts increases welfare significantly.

The model studied in this paper has some similarities with the model studied in Khan & Ravikumar (2001). There are two important differences. The first difference is that their model allows for endogenous growth. Consequently, agency problems affect the long-term growth of the economy. In our model, instead, agency problems have level effects. In this respect, our set up is closer to Marcet & Marimon (1992). The second difference is that the paper by Khan and Ravikumar only compare the Optimal Contracts Economy to the Complete Markets Economy. In our paper, instead, we are primarily interested in comparing the Optimal Contract Economy to the economy in which state-contingent contracts are not available (the Bond Economy).

The comparison between the Bond Economy and the Optimal Contract Economy allows us to evaluate the possible implications of institutional reforms that make possible the use of state-contingent contracts. One of the reasons state-contingent contracts may not be extensively used in practice is because the enforcement system is highly inefficient and costly. This can be changed with the introduction of proper institutional reforms. Thus, by comparing the Bond Economy to the Optimal Contract Economy, our study provides a welfare assessment of institutional reforms—for example legal systems—leading to greater contract enforceability.

Our paper is also related to Angeletos (2003) who shows that uninsurable investment risks induce under-accumulation of capital. In Angeletos' paper, however, market incompleteness is not endogenous. Therefore, it does not address the question of whether the availability of state-contingent contracts has large macroeconomic and welfare implications in the presence of information asymmetries.

The plan of the paper is as follows. In the next section we describe the theoretical framework, characterize the agent's problem and define the general

equilibrium. Section 3 conducts a quantitative analysis using parameterized versions of the model. Section 4 extends the model in some important dimensions. Section 5 discusses possible implications of our results and provides concluding remarks.

2 The basic model

We start describing the economy with optimal contracts. In this economy there is a continuum of agents that maximize the expected lifetime utility:

$$E \sum_{t=0}^{\infty} \beta^t U(c_t), \quad U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \quad (1)$$

where c_t is consumption at time t and β is the intertemporal discount factor. Agents are endowed with one unit of time per period supplied inelastically at the market wage rate w_t .

Each agent can run a risky technology that returns $F(k_t, l_{t+1}, z_{t+1})$ in the next period with the inputs of capital k_t and labor l_{t+1} . The variable z_{t+1} is an idiosyncratic i.i.d. shock that is unknown when k_t is chosen but it is known when l_{t+1} is chosen. For simplicity we assume that the shock can take only two values, z_L and z_H , with $z_L < z_H$. The probability, denoted by p_i , with $i = L, H$, is strictly positive for both realizations of the shock. The function F is strictly concave in the production inputs and satisfies $\lim_{k_t \rightarrow 0} EF_k(k_t, l_{t+1}, z_{t+1}) = \lim_{l_t \rightarrow 0} EF_l(k_t, l_{t+1}, z_{t+1}) = \infty$.

The agent has the ability to divert the retained capital to get a private benefit. Diversion of capital is not observable and generates efficiency losses in the form of a lower probability of the good shock. More specifically, the probability of the good shock becomes zero if there is diversion. The private and unobservable return from diversion is additive to consumption. Given c_t the agent's consumption, the current utility is $U(c_t + \alpha k_t)$, where α is a utility parameter which is constant in the model. When we later specify the functional form for $F(k_t, l_{t+1}, z_{t+1})$, we will also impose some restrictions on the parameter α that guarantee the inefficiency of diversion.

For the analysis that follows it would be convenient to define the following function:

$$R(w_{t+1}; k_t, l_{t+1}, z_{t+1}) = F(k_t, l_{t+1}, z_{t+1}) - w_{t+1} l_{t+1} \quad (2)$$

This is the gross revenue net of the labor cost. Given the specification of the return from diversion, the optimal input of labor is fully determined by the

input of capital, the shock and the wage rate, that is, $l_{t+1} = l(k_t, w_{t+1}, z_{t+1})$. Therefore, we can eliminate l_{t+1} as an explicit argument of the gross revenue and write it simply as $R(w_{t+1}; k_t, z_{t+1})$.

In addition to the risky investment, there are state-contingent assets that pay b_i units of output in the next period conditional on the realization of the idiosyncratic shock z_i . Agents buy these assets from a financial intermediary. Because there is no aggregate shocks, by pooling many agents or contracts the intermediary does not face any uncertainty. The assumption that financial markets are competitive then implies that the current value of these assets is $\delta_t \sum_i p_i b_i$, where $\delta_t = 1/(1 + r_t)$ is the market discount rate and r_t is the equilibrium riskless interest rate. In other words, the equilibrium market price of one unit of consumption goods paid in the state z_i is $\delta_t p_i$. Of course, the quantities that each agent can buy are subject to some constraints that we define in the next section. Contracts are exclusive: once the agent signs a contract with one intermediary, it will be unable to sign another contract with a different intermediary.

2.1 The agent's problem

Denote by a the agent's wealth or net worth before consumption. Given the sequence of prices $P^t \equiv \{r_j, w_{j+1}\}_{j=t}^{\infty}$, the optimization problem can be written as follows:

$$V_t(a) = \max_{c, k, b_i} \left\{ U(c) + \beta \sum_{i=L, H} V_{t+1}(a_i) p_i \right\} \quad (3)$$

subject to

$$a = c + k + \delta_t \sum_{i=L, H} p_i b_i \quad (4)$$

$$a_i = w_{t+1} + b_i + R(w_{t+1}; k, z_i), \quad \text{for } i = L, H \quad (5)$$

$$U(c) + \beta \sum_{i=L, H} V_{t+1}(a_i) p_i \geq U(c + \alpha k) + \beta V_{t+1}(a_L) \quad (6)$$

$$a_i \geq \underline{a}_{t+1} \quad (7)$$

This is the optimization problem for any deterministic sequence of prices, not only steady states. This motivates the time subscript t in the value

function. Notice that z_i , with $i \in \{L, H\}$, denotes the next period realization of the shock which is unknown when the agent chooses the consumption and investment plan. Equation (4) is the budget constraint and Equation (5) is the law of motion for next period net worth before consumption.

Equation (6) is the incentive-compatibility constraint imposed by the intermediary. The left-hand-side of this constraint is the agent's lifetime utility if he or she does not divert the capital. The right-hand-side is the lifetime utility if the agent diverts capital. In this case the agent gets higher utility in the current period but the next period wealth will be a_L with probability 1. The intermediary will earn the expected return r_t from the contract only if the agent does not divert the capital and this is guaranteed by this constraint.³

The last constraint (7) imposes limited liability. Limited liability is justified by the assumption that the agent can renegotiate any liability for which the net worth is smaller than a minimum value \underline{a}_{t+1} . This lower bound depends on the particular assumptions about the penalty that an intermediary can impose on a defaulting agent. We assume that the intermediary can confiscate only the current net worth. After the confiscation the agent can continue to operate the risky technology and sign state-contingent contracts with other intermediaries (no market exclusion). This implies that the lower bound is $\underline{a}_{t+1} = 0$.

The structure of problem (3) is not standard because the unknown value function V_t enters the constraints of the problem and there are no guarantees that this function is concave. We will describe in the next section how we deal with this problem. For the moment we simply assume that a solution exists. This solution consists of the sequence of policy functions $\{c_j(a), k_j(a), b_j(a)(z_i)\}_{j=t}^{\infty}$. Denote by $M_t(a)$ the initial distribution of agents' assets. The general equilibrium can be defined as follows:

Definition 1 *Given the initial distribution $M_t(a)$, a general equilibrium is defined by (i) a sequence of agents' policy functions $\{c_j(a), k_j(a), b_j(a)(z_i)\}_{j=t}^{\infty}$*

³In deciding whether to divert the capital, the agent faces a trade-off. On the one hand diversion harms the agent because the expected revenues will be smaller. Although the agent also gets a temporary increase in utility, this is not sufficient to compensate the reduction in expected revenues. On the other, diversion increases the probability of the low shock which in general implies higher payments from the intermediary, *i.e.*, $b_L > b_H$. The incentive-compatibility constraint makes sure that the gains from diversion are smaller than the losses.

and labor demand $l(k, w, z_i)$; (ii) a sequence of value functions $\{V_j(a)\}_{j=t}^\infty$; (iii) a sequence of prices $P^t \equiv \{r_j, w_{j+1}\}_{j=t}^\infty$; (iv) a sequence of aggregate demands for labor $L(P^t) \equiv \{L_{j+1}(P^t)\}_{j=t}^\infty$; (v) a sequence of aggregate capital $K(P^t) \equiv \{K_j(P^t)\}_{j=t}^\infty$; and (vi) a sequence of aggregate consumption $C(P^t) \equiv \{C_j(P^t)\}_{j=t}^\infty$. These sequences must satisfy the following conditions: (i) the policy functions solve problem (3) at each point in time and $\{V_j(a)\}_{j=t}^\infty$ are the associated value functions; (ii) the aggregate demands of labor, capital and consumption are the aggregation of individual demands and they satisfy $L_{j+1}(P^t) = 1$ and $C_j(P^t) + K_j(P^t) = \int a M_j(da)$; (iii) the distributions $M_j(a)$, for $j > t$, evolve according to the individual policies and the stochastic properties of the idiosyncratic shock.

2.2 Complete Markets and Bond Economies

One of the goal of this paper is to compare the allocation obtained when state-contingent contracts are feasible with the allocations achieved in two alternative environments: when state-contingent contracts are not available (Bond Economy) and when shocks are public information (Complete Markets Economy).

Complete markets are achieved if $\alpha = 0$, that is, if there is no private benefit from diversion. In this case the incentive-compatibility constraint (6) will not be binding. Therefore, the agent's problem in the Complete Market Economy is still problem (3) but we can ignore the incentive-compatibility constraint (6).⁴ This allows the agent to fully insure against the investment risk and the first order conditions imply that $ER_k(w_{t+1}; k_t, z_{t+1}) = 1 + r_t$, where R_k is the derivative of the gross revenue with respect to k . In the steady state it must be that $1 + r_t = 1/\beta$ for all t .

The optimization problem solved in the bond economy is obtained by restricting $b_L = b_H = b$. In this case the incentive-compatibility constraint (6) never binds and the optimization problem simplifies to:

$$V_t(a) = \max_{c, k, b} \left\{ U(c) + \beta \sum_{i=L, H} V_{t+1}(a_i) p_i \right\} \quad (8)$$

subject to

⁴Constraint (7) is still relevant because eliminates Ponzi games, although it will not be binding in equilibrium.

$$a = c + k + \delta_t b \quad (9)$$

$$a_i = w_{t+1} + b + R(w_{t+1}; k, z_i) \quad (10)$$

$$a_L \geq 0 \quad (11)$$

Notice that the limited liability constraint is imposed only in the case in which $z = z_L$. In fact, if this constraint is satisfied for $z = z_L$, it is also satisfied for $z = z_H$.

The above optimization problem is a standard concave problem. We can then establish the following properties:

Proposition 1 *For any sequence of prices, there is a unique solution to problem (8) and the function $V_t(a)$ is strictly increasing, concave and differentiable at all t .*

Proof 1 *It can be verified that the feasible set in problem (8) is convex and the objective function is strictly concave. Therefore, if V_{t+1} is concave, V_t is strictly concave. Moving backward we can establish that $\lim_{t \rightarrow -\infty} V_t$ is concave. Because the objective in problem (8) is strictly concave, the solution is unique. Standard arguments can be used to prove that the value function is differentiable. Q.E.D.*

Given Proposition 1, the solution to problem (8) is characterized by the following first order conditions:

$$U'(c_t) = \beta (1 + r_t) E\{U'(c_{t+1})\} + (1 + r_t)\lambda_t \quad (12)$$

$$U'(c_t) = \beta E\{U'(c_{t+1}) \cdot R_k(w_{t+1}; k, z)\} + \lambda_t \cdot R_k(w_{t+1}; k, z_L) \quad (13)$$

where λ_t is the Lagrange multiplier associated with the limited liability constraint (11). The multiplier is positive if the solution is binding.

Conditions (12) and (13) make clear that the expected return from the risky investment is always greater than the return from the risk-free asset, that is, $1 + r_t < ER_k(w_{t+1}; k, z)$. To see this, consider the case in which the limited liability constraint is not binding. Conditions (12) and (13) imply that:

$$(1 + r_t) \cdot EU'(c_{t+1}) = ER_k(w_{t+1}; k, z) \cdot EU'(c_{t+1}) + \text{Cov}\left(R_k(w_{t+1}; k, z), U'(c_{t+1})\right) \quad (14)$$

Because $U'(c_{t+1})$ is negatively correlated with $R_k(w_{t+1}; k, z)$, the last term on the right-hand-side is negative, and therefore, $1 + r_t < ER_k(w_{t+1}; k, z)$.

Let's compare this to the case in which $z_L = z_H = z$ (no shocks). In this case the covariance term in equation (14) is zero and the marginal returns from the two investments are equal, that is, $1 + r_t = ER_k(w_{t+1}; k, z)$. This environment is similar to the one studied in Aiyagari (1995). The only difference is that w_{t+1} is not deterministic in Aiyagari. However, even if w_{t+1} is stochastic at the individual level, the condition $1 + r_t = ER_k(w_{t+1}; k, z)$ still holds. Because in the steady state equilibrium the interest rate is smaller than the intertemporal discount rate, that is, $r < 1/\beta - 1$, the model with only earnings risks generates an over-accumulation of capital.⁵

With investment risks, the result that the interest rate is lower than the intertemporal discount rate still holds. However, the marginal return on capital is not necessarily smaller than the intertemporal discount rate for all agents. This implies that in the aggregate economy there is under-accumulation of capital relative to the complete markets level. We will show this result numerically in Section 3.

2.3 Optimal contract economy

One of the complication in solving problem (3) is that the unknown function V_i enters the constraints of the problem. It is then convenient to study the dual problem which minimizes the cost of providing a certain level of utility to the agent.

Denote by v_t the lifetime utility of the agent and by $A_t(v_t)$ the cost for the intermediary. This is defined as:

$$A_t(v) = \min_{c, k, v_i} \left\{ c + k + \delta_t \sum_{i=L, H} \left[-w_{t+1} - R(w_{t+1}; k, z_i) + A_{t+1}(v_i) \right] p_i \right\} \quad (15)$$

subject to

$$v = U(c) + \beta \sum_{i=L, H} v_i p_i \quad (16)$$

⁵This result, however, may not apply when the supply of labor is elastic. Pijoan-Mas (2003) shows that precautionary savings could be negative in this case.

$$U(c) + \beta \sum_{i=L,H} v_i p_i \geq U(c + \alpha k) + \beta v_L \quad (17)$$

$$v_i \geq \underline{v}_{t+1}, \quad \text{for } i = L, H \quad (18)$$

Equation (16) is the promise-keeping constraint, equation (17) is the incentive-compatibility constraint and equation (18) imposes limited liability. The lower bound \underline{v}_{t+1} is the equivalent of \underline{a}_{t+1} imposed in the original problem. However, this lower bound is no longer exogenous but it is determined by the condition $A(\underline{v}_{t+1}) = 0$. This guarantees that the limited liability constraint $a_i \geq 0$ is satisfied in the original problem.

This is the problem solved by a financial intermediary that enters into a long-term contractual relation with the agent. In the original problem (15), instead, the intermediary was signing only a one-period or short-term contract with the agent. Therefore, the solutions are not necessarily equivalent. However, if we can show that the long-term contract is equivalent to a sequence of short-term contracts, we can claim that the solution of the dual problem is equivalent to the solution of the original problem.

As shown in Fudenberg, Holmstrom, & Milgrom (1990), if the utility frontier is downward sloping, the long-term contract is free from renegotiation and can be implemented with a sequence of short-term contracts. In our model, the utility frontier is represented by the negative of the function $A_t(v)$. Therefore, it is enough to show that the negative of $A_t(v)$ is not increasing for all $v > \underline{v}_t$. In Section 3 we will show this result numerically for the particular parameterizations of the model considered in this paper.

Once we have (numerically) established that the solution of the dual problem (15) is equivalent to the solution of the original problem (3), we can easily see the correspondence between the two problems. More specifically, the cost value $A_t(v)$ is equal to the net worth a in the original problem. Likewise, the agent's value $V_t(a)$ in the original problem corresponds to the agent's promised utility v in the dual problem. Therefore, $a = A_t(v)$ and $v = V_t(a)$.

In solving the optimization problem (15), however, we face an important difficulty: the constraint set of this problem is not convex. Consequently, we cannot prove that the problem is concave and we cannot use first order conditions to characterize the solution. Therefore, in solving the problem we use a direct optimization technique that we describe in the Appendix.

3 Numerical analysis

The goal of this section is to show numerically the macroeconomic and welfare implications of market incompleteness. Although the analysis is not aimed at matching specific observations, it provides important information about the potential magnitude of these implications.

Parameterization: We assign the following parameter values. The period in the model is one year and the intertemporal discount rate is $\beta = 0.95$. The risk aversion parameter is $\sigma = 1.5$.

We assume that the shock affects the efficiency units of capital. More specifically, if the investment at time t is k_t , the efficiency units of capital at the beginning of the next period (before choosing labor) is $\tilde{k}_{t+1} = z_{t+1}k_t$. The total resources returned by the risky technology is:

$$F(k_t, l_{t+1}, z_{t+1}) = \tilde{k}_{t+1} + (\tilde{k}_{t+1}^\epsilon l_{t+1}^{1-\epsilon})^\theta$$

The first component is capital net of depreciation and the second component is production. After setting $z_L = 0.5$ and $z_H = 1.0$, the probability of the low shock is chosen to have an expected depreciation rate of 8 percent, that is, $p_L \cdot z_L + (1 - p_L) \cdot z_H = 0.92$. This implies that, with 16 percent probability, capital depreciates by 50 percent and with 84 percent probability there is no depreciation. A sensitivity analysis will be conducted by changing the value of z_L (keeping the average depreciation rate constant). The return-to-scale parameter is set to $\theta = 0.95$ and the parameter $\epsilon = 0.35$. This implies a labor income share of 60 percent. Finally we set $\alpha = 0.2$. This value guarantees that diversion is always inefficient. We will also conduct a sensitivity analysis with respect to this parameter. Table 1 reports the full set of parameter values for the baseline economy.

Steady state properties: Figure 1 plots several variables for an agent in the steady state equilibrium. The left panels for the Bond Economy and the right panels for the Optimal Contract Economy. The first two panels plot the agent's value as a function of assets, that is, the function $V(a)$. In the case of optimal contracts, this is the inverse of the function $A(v)$ derived from solving the dual problem. Because $V(a)$ is monotonically increasing, the function $A(v)$ is also increasing. This implies that the utility frontier, $-A(v)$, is downward sloping. As Fudenberg et al. (1990) shows, this guarantees that

Table 1: Parameter values for the baseline economy.

Discount rate	β	0.95
Risk-aversion	σ	1.50
Diversion parameter	α	0.20
	θ	0.95
	ϵ	0.35
Technology $zk + [(zk)^\epsilon l^{1-\epsilon}]^\theta$	z_L	0.50
	z_H	1.00
	p_L	0.16

the long-term contract is free from renegotiation and can be implemented as a sequence of short-term contracts. Therefore, the solution of problem (15) is equivalent to the solution of the original problem (3).

The other panels plot the investment in the risky technology, k , the investment in the state-contingent asset, b_i , and the next period wealth a_i . In the Bond Economy there are no state contingent assets and b represents the investment in the riskless asset or bond. In both economies the next period wealth depends on the realization of the shock. It is interesting to observe that state-contingent contracts reduce significantly the volatility of assets, and therefore, the risk of investing in the risky technology (see the last two panels of Figure 1). In other words, the availability of state contingent contracts allows for a better insurance of the investment risk. This better insurance encourages more investment in the risky technology and explains why the availability of these contracts can have substantial macroeconomic and welfare consequences.

Table 2 reports the steady state interest rate, aggregate capital and the concentration of wealth as measured by the Gini index. In the Complete Markets Economy the interest rate is equal to the intertemporal discount rate and the stock of capital (normalized to 1) satisfies $ER_k(w; k, z) = 1/\beta$. In the two versions of market incompleteness, instead, the interest rate is smaller than the intertemporal discount rate. This is not surprising given the results of Huggett (1993) and Aiyagari (1994). What differs here is that the aggregate stock of capital is smaller than in the Complete Markets Economy. In other words, market incompleteness leads to under-accumulation of capital. This is the direct consequence of the fact that the accumulation of

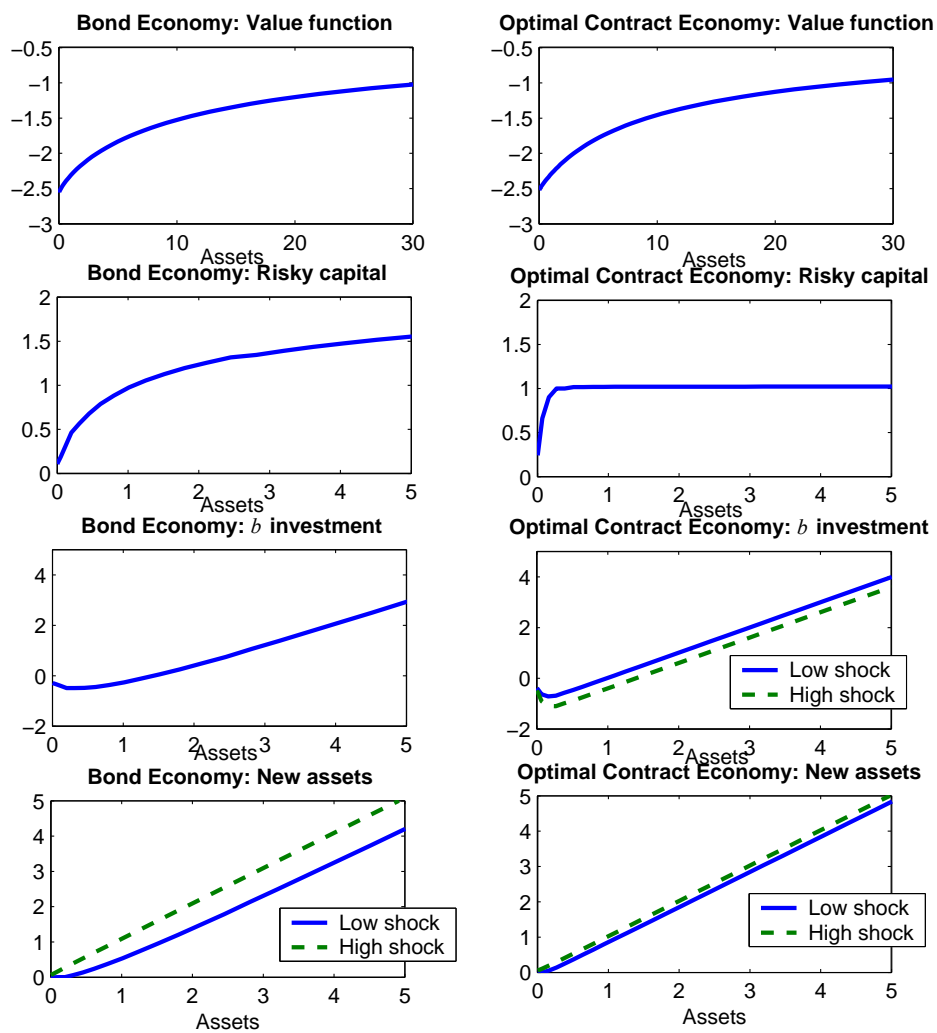


Figure 1: Value function and policy rules in the Bond Economy and in the Optimal Contract Economy.

capital is risky and agents require a risk premium.

Table 2: Steady state interest rate, capital stock, and wealth inequality for different degrees of market completeness.

	Interest rate (%)	Aggregate capital	Gini index (%)
Bond Economy	4.22	0.911	43.8
Optimal Contract Economy	5.21	0.995	42.4
Complete Markets Economy	5.26	1.000	

Table 2 also shows that the availability of state-contingent contracts brings the steady state capital very close to the complete markets level. While in the Bond Economy the stock of capital is about 9 percent smaller than in the Complete Markets Economy, with optimal contracts aggregate capital is only 0.5 percent smaller. This finding parallels the result of Khan & Ravikumar (2001) for the effects of market incompleteness on the long-run growth of the economy.

The availability of state-contingent contracts also reduces the inequality in the distribution of wealth but only slightly. Notice that in the Complete Markets Economy the distribution of wealth is not determined: any distribution of wealth is a steady state equilibrium as long as aggregate wealth does not change. See Chatterjee (1994) for a proof of this result.

The Gini index for wealth is relatively small relative to the data. This is because shocks are i.i.d. and there is no other sources of heterogeneity. If we assume that only a sub-group of agents have access to the risky technology—as we will do in the next section—the model will generate a much higher concentration of wealth.

Institutional reforms and welfare: The steady state comparisons conducted above show that market incompleteness may have substantial macroeconomic consequences in the absence of state-contingent contracts. We now study the welfare implications. We will ask the following question: Assuming the existence of institutions that make the use of state-contingent contracts feasible, what are the welfare consequences of introducing these institutions?

Figure 2 plots the transition dynamics induced by the unanticipated introduction of state-contingent contracts. After this change, the interest rate increases sharply and then it converges gradually to the new steady state level. The introduction of state-contingent contracts increases the demand of capital immediately but the supply responds only gradually through capital accumulation. This explains the overshooting. As panel (c) shows, the aggregate stock of capital converges to a higher level only gradually. As capital increases, the demand of labor also increases and to clear the labor market the wage rate must rise (see panel (b)). The increase in the wage rate reduces profits, and therefore, the propensity to invest in the risky technology. This effect, however, does not totally offset the higher incentive induced by better insurance possibilities provided by state-contingent contracts. Panel (d) shows that the concentration of wealth, measured by the Gini index, declines slightly.

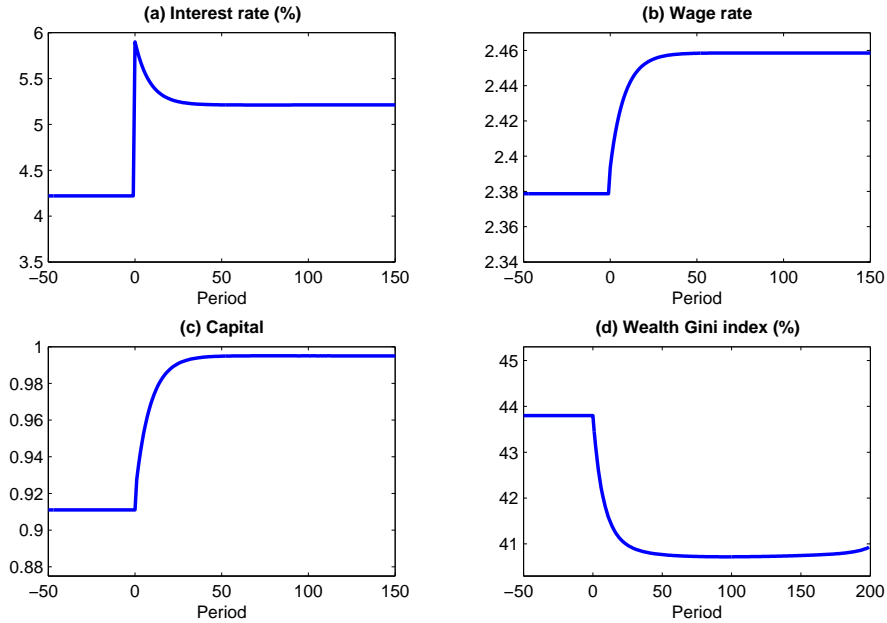


Figure 2: Transition to the steady state with state-contingent contracts.

The welfare consequences are calculated as the aggregate additional consumption (appropriately distributed among agents) required to make all agents indifferent between remaining with the existing institutions (and being unable to use state-contingent contracts) and undertaking a transition

to the new steady state equilibrium after the reform (giving access to state contingent contracts).

Let $V^{Bond}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{Bond})$ be the expected lifetime utility of an agent with net worth a that lives in the steady state of the Bond Economy. The distribution of agents over a is denoted by $M(a)$. Moreover, define $V^{OptCon}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{OptCon})$ to be the expected lifetime utility of an agent with net worth a after the introduction of state-contingent contracts (therefore, after undertaking the transition to the new steady state). The consumption gain from transition, denoted by $g(a)$, is determined by the following condition:

$$V^{OptCon}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{Bond} \cdot (1 + g(a))) = (1 + g(a))^{1-\sigma} \cdot V^{Bond}(a)$$

In other words, the consumption gain is determined by equalizing the lifetime utility achieved in the transition with the lifetime utility obtained by increasing the consumption in the Bond Economy by $c_t^{Bond} g(a)$ for all t .

The aggregate consumption gains are given by:

$$\text{Gains} = \frac{\int_a c^{Bond}(1 + g(a))M(da)}{\int_a c^{Bond}M(da)} - 1$$

The average gains in the baseline economy are 2.32 percent of aggregate consumption. We have also computed the welfare gains from the Optimal Contract Economy to the economy with complete markets. In this case the welfare gains are only 0.15 percent.

Although the average gains are positive, these gains are not uniformly distributed across agents. The top panel of Figure 3 plots the welfare gains as a function of the initial wealth. It is interesting to note that the gains are larger for (initially) wealthier agents. For example, an agent with the average wealth would gain less than 2 percent. For an agent with 10 times the average wealth, the welfare gains are 12 percent. The bottom panel plots the initial and final distribution of agents over assets. This informs us about the relative importance of poorer agents (who do not gain much from the transition) and wealthier agents (who are the largest beneficiaries).

The distribution of the welfare gains can be explained as follows. After the introduction of state-contingent contracts, the aggregate demand of capital increases. Because the supply responds slowly, the interest rate increases

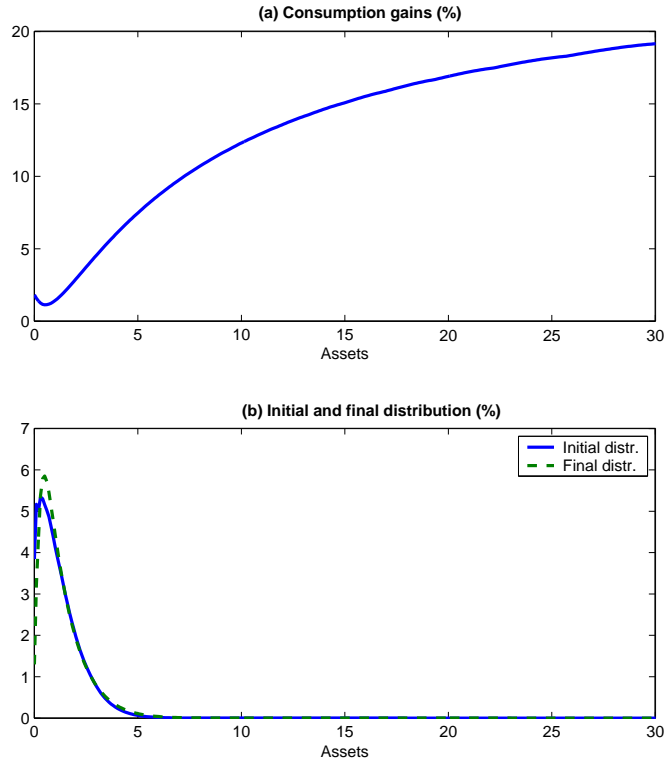


Figure 3: Distribution of the welfare gains following the introduction of state-contingent contracts.

(see the first panel of Figure 2). The increase in the interest rate is beneficial for the holders of wealth, that is, the richest agents. For the poorer agents, instead, the increase in the interest rate represents an increase in the cost of financing because they are net borrowers. This finding may appear surprising: we could have thought that the financial markets improvement was more beneficial for poorer agents given that they face tighter constraints. This would have been the case if the interest rate had remained constant. However, due to general equilibrium effects, the interest rate increases and this benefits those who receive interest payments, that is, the wealthy.

Sensitivity analysis: We close this section by conducting a sensitivity analysis with respect to some key parameters. We start with the utility parameter for diversion, α , the concavity of the production function, θ , and the

volatility of the shock, $z_H - z_L$. Key statistics for the steady state equilibrium and the welfare gains from the transition are reported in Table 3.

Table 3: Sensitivity analysis: Steady state values and welfare gains from transition.

	Interest rate (%)	Aggregate capital	Gini index (%)	Welfare gains (%)
Baseline, $\alpha = 0.2, \theta = 0.95, z_L = 0.5$				
Bond Economy	4.22	0.911	43.8	
Optimal Contract Economy	5.21	0.995	42.4	2.32
Complete Markets Economy	5.26	1.000		
Higher utility from diversion, $\alpha = 0.3$				
Bond Economy	4.22	0.911	43.8	
Optimal Contract Economy	5.18	0.992	41.1	2.19
Complete Markets Economy	5.26	1.000		
Higher curvature of production, $\theta = 0.915$				
Bond Economy	4.19	0.911	38.8	
Optimal Contract Economy	5.21	0.994	41.7	2.25
Complete Markets Economy	5.26	1.000		
Higher volatility of shocks, $z_L = 0.25$				
Bond Economy	2.96	0.832	48.3	
Optimal Contract Economy	5.23	0.997	42.1	4.73
Complete Markets Economy	5.26	1.000		

First, we observe that the higher utility from diversion does not affect significantly our results. A similar conclusion seems to hold per the curvature of the production function. Now the Gini index for the Bond economy is smaller but the difference is not large. The volatility of the shock, instead, seems to play an important role. The increase in the volatility has significant macroeconomic consequences when state-contingent contracts are not available. For example, the aggregate stock of capital drops by 8 percent when the low realization of the shock changes from 0.5 to 0.25. The drop in the risk-free interest rate is also large. However, the availability of state-contingent contracts still brings the aggregate stock of capital very close to the complete markets level. As a result, the introduction of state-contingent

contracts leads to much larger welfare gains, almost 5 percent.

Table 4 reports key statistics for different values of the risk aversion parameter σ . First we observe that the under-accumulation result is robust to different curvatures of the utility function. With larger values of σ the equilibrium steady state of capital declines in the Bond Economy but it does not change significantly in the optimal contract economy. As a result, the welfare gains from contingent contracts are larger when agents are very averse to risk.

Table 4: Sensitivity analysis with respect to risk aversion.

	Interest rate (%)	Aggregate capital	Gini index (%)	Welfare gains (%)
Lower relative risk aversion, $\sigma = 1$				
Bond Economy	4.62	0.934	48.3	
Optimal Contract Economy	5.22	0.994	41.2	1.63
Complete Markets Economy	5.26	1.000		
Higher relative risk aversion, $\sigma = 1.5$				
Bond Economy	4.22	0.911	43.8	
Optimal Contract Economy	5.21	0.995	42.4	2.32
Complete Markets Economy	5.26	1.000		
Higher relative risk aversion, $\sigma = 3$				
Bond Economy	3.62	0.828	37.1	
Optimal Contract Economy	5.22	0.996	42.2	4.29
Complete Markets Economy	5.26	1.000		

Table 5 reports key statistics for different values of the intertemporal discount rate β . In this case we observe that the under-accumulation result is still maintained for alternative values of β . Compared to the level of capital in the Complete Markets Economy, capital accumulation declines with higher values of β . The decline, however, is not large. As a result, the welfare gains from the introduction of state-contingent contracts are not very sensitive to β .

We can conclude this section by observing that, although we do not have a theorem about the effects of investment risks on the accumulation of capital, the sensitivity analysis shows that the under-accumulation result is quite

Table 5: Sensitivity analysis with respect to the discount factor.

	Interest rate (%)	Aggregate capital	Gini index (%)	Welfare gains (%)
Lower discount factor, $\beta = 0.93$				
Bond Economy	6.47	0.922	41.2	
Optimal Contract Economy	7.47	0.993	42.3	2.07
Complete Markets Economy	7.53	1.000		
Higher discount factor, $\beta = 0.95$				
Bond Economy	4.22	0.911	43.8	
Optimal Contract Economy	5.21	0.995	42.4	2.32
Complete Markets Economy	5.26	1.000		
Higher discount factor, $\beta = 0.98$				
Bond Economy	1.22	0.907	43.0	
Optimal Contract Economy	2.01	0.997	42.2	2.68
Complete Markets Economy	2.04	1.000		

robust. The same conclusion applies to the finding that the availability of state-contingent contracts brings the steady state level of capital very close to the complete markets level.

4 Extensions of the model

The model studied in the previous sections is very stylized. We have assumed that all agents have access to the risky investment. It may be more natural to assume that only a sub-group of agents have access to this type of investment. We have also assumed that agents do not face any earnings risks. Another assumption is that labor supply is fixed while in an actual economy it may respond to wages. This section extends the model along these dimensions and shows that the main results are robust to these extensions.

4.1 Only a fraction of the population has access to the risky technology

One possible interpretation of the risky investment is that it captures the risk associated with entrepreneurial activities. We can then assume that the agents investing in the risky technology are the ones engaged in entrepreneurial activities. In line with this interpretation we assume that 10 percent of agents are in the position to invest in the risky technology.⁶ We will refer to these agents as “entrepreneurs” and to the others as “workers”.

Entrepreneurs solve the same problem we have studied earlier. Workers, instead, solve a simpler problem. Because they face no risk, the consumption path can be easily determined using the Euler equation, the budget constraint, and the law of motion for wealth, that is:

$$\begin{aligned} U'(c_t) &\leq \beta(1+r_t)U'(c_{t+1}) \\ a_t &= c_t + \delta_t b_t \\ a_{t+1} &= w_{t+1} + b_t \end{aligned}$$

The Euler equation is satisfied with the inequality sign if $a_{t+1} = 0$, that is, if the borrowing limit is binding. In the steady state the interest rate is lower than the intertemporal discount rate and the liability constraint binds, that is, $a_t = 0$ for all t . The level of consumption is then equal to $c_t = \delta w$, where δ and w are constant in a steady state. Table 6 reports some steady state statistics.

Table 6: Steady state values and welfare gains from transition when 10 percent of the population has access to the risky investment.

	Interest rate	Aggregate capital	Gini index	Welfare gains
Bond Economy	1.84	0.873	95.1	
Optimal Contract Economy	5.24	0.993	94.9	5.81
Complete Markets Economy	5.26	1.000		

The basic results do not change by assuming that only a fraction of the population has access to the risky investment. In particular, the aggregate

⁶See Quadrini (1999) for a documentation of the share of entrepreneurs in the population.

stock of capital is still smaller than in the Complete Markets Economy and the availability of optimal contracts brings the aggregate stock of capital close to the complete markets level. The most notable change is the increase in the Gini index. This is because only a small fraction of agents (the entrepreneurs) save. Although the model is stylized, this shows how entrepreneurial activities can generate a much higher concentration of wealth. This point is also made in Quadrini (2000) and Cagetti & DeNardi (2002). Also significant is the increase in the welfare gains from the introduction of state-contingent contracts. These larger gains come from the increase in the wage rate. Because 90 percent of the population are workers with low levels of consumption, the increase in the wage rate, and therefore, consumption, has an important impact on their utilities.

4.2 Agents also face earnings risks

Would the result change if agents also face idiosyncratic risks to earnings as in the Bewley economy? To investigate this question we assume that agents have different earnings abilities ε . Individual labor income is then the product of the earnings ability with the wage rate, that is, εw . Earnings abilities follow a two-state Markov process with symmetric transition probability $\Gamma(\varepsilon'/\varepsilon)$.

To keep the problem simple, we assume that earnings abilities are observable. This implies that with state-contingent contracts the earnings risk is insurable. Therefore, the problem solved in the Optimal Contract Economy is the same problem solved with no earning risks. In the Bond Economy the optimization problem is also similar. The only difference is that now we take expectations also with respect to ε .

In Table 7 we report the results for the economy with earnings risks. The process for the earnings ability has been calibrated by assuming an autocorrelation of 0.5 and a standard deviation of 0.33. These are the baseline numbers used in Aiyagari (1994). We report the results for two different cases: when all agents have access to the risky investment and when the risky investment is available only to 10 percent of the population.

Even if agents face earnings risks, the aggregate stock of capital is smaller than in the Complete Markets Economy. However, we observe that the under-accumulation of capital is somewhat reduced. This is because the presence of uninsurable earnings risks brings an extra incentive to save. This reduces the equilibrium interest rate. The lower interest rate then facilitates more

Table 7: Steady state values and welfare gains from transition when agents face earnings risks.

	Interest rate	Aggregate capital	Gini index	Welfare gains
All agents have access to risky investment				
Bond Economy	3.09	0.972	44.5	
Optimal Contract Economy	5.21	0.995	42.4	5.97
Complete Markets Economy	5.26	1.000		
Only 10% have access to risky investment				
Bond Economy	0.01	0.931	88.6	
Optimal Contract Economy	5.24	0.993	94.9	9.33
Complete Markets Economy	5.26	1.000		

investment in the risky technology.

4.3 Elastic labor supply

In this section we consider the case in which the labor supply is elastic. We limit the analysis to the extreme case in which labor is perfectly elastic. A simple way to incorporate this in the model is by assuming that the utility function takes the following form: $U(c - \varphi \cdot l)$. This implies that the equilibrium wage rate is fixed and equal to the parameter φ .

As shown in Table 8, market incompleteness has a much bigger impact on the macroeconomy when labor is elastic. In particular, compared to the case with inelastic labor supply, the aggregate stock of capital is substantially smaller relative to the complete markets level: 35 percent smaller in the Bond Economy and 3 percent smaller in the Optimal Contracts Economy. With inelastic labor supply they were 9 percent and 0.5 percent respectively. This is because, with inelastic supply, the fall in the demand of labor induces a fall in the equilibrium wage rate which in turn increases the return from the risky investment (that is, the expected profit rate increases). This reduces the fall in the demand of risky capital and in equilibrium the capital stock is higher. When the supply is perfectly elastic, instead, the lower demand of labor does not lead to lower wages. Consequently, the fall in investment is bigger.

Table 8: Steady state values and welfare gains from transition when labor is elastic and all agents have access to the risky technology.

	Interest rate	Aggregate capital	Gini index	Welfare gains
Bond Economy	4.57	0.650	42.5	
Optimal Contract Economy	5.21	0.970	42.4	1.37
Complete Markets Economy	5.26	1.000		

Figure 4 plots the transition path of several variables for the cases of elastic and inelastic labor supply. The plots are constructed using the baseline economy in which all agents have access to the risky investment. The plots for the case in which only a fraction of agents invest in this technology are qualitatively similar (which we omit for economy of space).

Of course, the assumption that labor is perfectly elastic is an abstraction. When the elasticity of labor is positive but not infinitely elastic, the effects of market incompleteness on the accumulation of capital are smaller. However, the point we would like to make here is that the elasticity of labor tends to increase the under-accumulation of capital when markets are incomplete.

5 Conclusion

In this paper we have studied an economy in which agents have investment opportunities in a risky technology but the risk cannot be fully insured. The consideration of uninsurable investment risks overturns the previous conclusion that uninsurable risks induce agents to over-accumulate capital. As in Angeletos (2003), we have shown that with investment risks the equilibrium stock of capital is smaller than in the complete markets economy.

This result may have important policy implications. Aiyagari (1995) shows that in a model with uninsurable earning risks, a positive capital income tax is desirable in the long-run because it reduces the over-accumulation of capital. Golosov, Kocherlakota, & Tsyvinski (2003) also show that a positive capital income tax may improve the allocation when market incompleteness is endogenous.

The result that investment risks lead to an under-accumulation of capital may bring into question the conclusion about the desirability of long-term

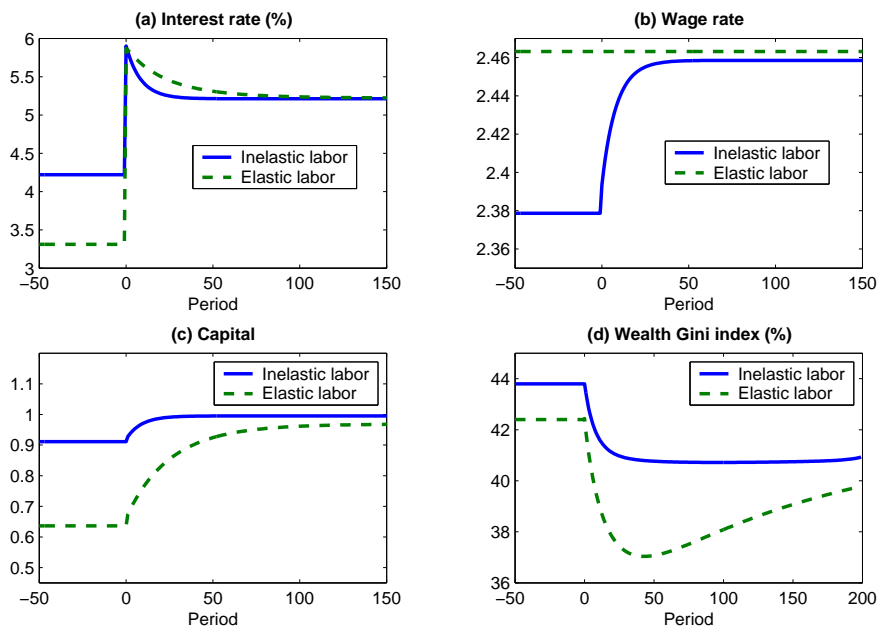


Figure 4: Transition to the Optimal Contract Economy for different degrees of labor elasticity.

capital taxes. Because in Aiyagari (1995) the optimality of capital taxes derives from the over-accumulation of capital, the rationale for this type of taxation may vanish if the model does not generate over-accumulation. We leave for future research the full investigation of this conjecture.

We have also compared economies with different degrees of market incompleteness. We have placed particular attention to economies in which state-contingent contracts are available but they cannot provide full insurance due to information asymmetries. Even if agency problems are quite severe in the sense that agents can obtain large gains from diverting resources, the use of state-contingent contracts can lead to an aggregate stock of capital that is very close to the complete markets level and substantially higher than the capital that would prevail when debt contracts are the only feasible contracts. We have also seen that the availability of optimal contracts can have important welfare consequences.

This result points out the importance of factors that make state-contingent contracts feasible. Among these factors, formal and informal institutions play a central role. The reason state-contingent contracts may not be extensively

used in practice is because the enforcement system is highly inefficient and costly. For instance, the resolution of contractual disputes may be extremely long and uncertain. There is now a substantial cross-country evidence that the degree of contract enforcement is correlated with the degree of financial development. See Levine (1997) and Dolar & Meh (2002) for reviews of the empirical literature. In our model, the economy with state-contingent contracts can be interpreted as an economy in which financial markets are more developed in part as a result of a more efficient institutional enforcement. Thus, our study provides a welfare assessment of institutional reforms—for example legal systems—leading to greater contract enforceability. The next step, then, is to understand which types of institutions facilitate or make possible the use of these contracts.

Another important question is whether the presence of agency problems in the management of business investments affects the cyclical properties of the economy in response to aggregate shocks. This has been the focus of recent papers such as Covas (2004) and Philippon (2003).

Appendix: Computation of the equilibrium

Steady state for the Bond Economy: We start the procedure by guessing the steady state interest and wage rates. Given the prices, we solve problem (8) on a grid of points for the asset holdings a using value function iteration. After guessing the next period values of $V(a)$ at each grid point, we approximate this function with a quadratic polynomial. Given the next period value function, problem (8) is solved at each grid point using a maximizing routine that do not requires smoothness of the value function. We use the Fortran routine BCPOL.

Once the iteration on the value function has converged, we use the agents' policy rules to find the invariant distribution of agents over a . Starting from an initial distribution we iterate until convergence. After aggregating using the invariant distribution, we verify the clearing conditions in the capital and labor markets. Based on these conditions we update the prices and restart the procedure until all markets (labor and capital) clear.

Steady state for the Optimal Contract Economy: The numerical procedure is similar to the procedure used to solve for the steady state of the Bond Economy based on value function iteration. Because we solve for the dual problem (15), the agent's problem is solved at each grid point of v . In forming the grid for v , however, we do not know the lower bound \underline{v} . Therefore, when we guess the prices r and w we also guess the value of \underline{v} , which is the first point of the grid. After solving for the individual problem on all grid points we verify whether $A(\underline{v}) = 0$. If this condition is not met, we update the guess for \underline{v} and repeat the whole procedure until convergence.

Transition equilibrium: To compute the transition from the steady state of the Bond Economy to the steady state of the Optimal Contracts Economy, we start the procedure by guessing sequences of prices, r and w , and lower bounds, \underline{v} , for a certain number of periods. The number of periods is sufficiently long for the economy to get close to the new steady state equilibrium. Given the guessed sequences, we solve the agents' problem backward at each grid point starting from the final transition period. In the final period the economy is supposed to have converged to the new steady state, and therefore, we already know the solution. Once we have solved for all transition periods, we start from the initial period and compute the market clearing conditions and check the condition $A_t(\underline{v}_t) = 0$. We then update the guessed sequences and continue until all markets clear and the condition $A_t(\underline{v}_t) = 0$ is satisfied in all transition periods.

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