Monetary Policy and The Financial Decisions of Firms[*]

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Abstract

In this paper we develop a general equilibrium model with heterogeneous, long-lived firms where financial factors play an important role in their production and investment decisions. When the economy is hit by monetary shocks, the response of small and large firms differs substantially, with small firms responding more than big firms. As a result of the financial decisions of firms, monetary shocks have a persistent impact on output. Another finding of the paper is that monetary shocks lead to considerable volatility in stock market returns.

Introduction

Empirical studies of the financial decisions of firms have documented important differences in the behavior of large and small firms. It has been shown by Fazzari, Hubbard, & Petersen (1988) and others that small firms are more profitable, pay fewer dividends, take on more debt, and invest more. Recent studies by Gertler & Gilchrist (1994), Gilchrist & Himmelberg (1995, 1998) have also shown that the investment decisions of small firms are more sensitive to cash flows and that they respond to monetary policy shocks very differently than do large firms. Because many of these authors identify small firms as *a priori* more likely to face financial constraints, these empirical features are widely interpreted as indirect evidence of frictions in financial markets. These frictions are conjectured to be an important channel for the propagation of monetary policy shocks.[1]

In this paper we argue that the many differences in behavior of large and small firms have a common explanation. It is that the financial decisions of firms will differ systematically with firm size as measured by the amount of equity in the firm. The higher sensitivity of small firms to monetary policy shocks derives from the fact that small firms take on more debt. Small firms choose higher debt-equity ratios because they are more profitable.

We study the financial decisions of firms that are heterogeneous in the amount of equity capital in their business. The capital structure of firms changes endogenously over time as a result of their financial decisions and in response to idiosyncratic shocks. The model is in the

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[1]See Bernanke & Gertler (1995) for a review of the "credit channel" of monetary transmission.

spirit of models by Jovanovic (1982) and Hopenhayn (1992), in that shocks affect the dynamics of firms over time. The key difference is that, in this environment, heterogeneity is not generated by technological differences. Rather, firms are heterogeneous because they face different financial conditions.[2]

We consider an economy where all firms have access to the same decreasing return-to-scale technology for producing a single homogeneous good. The firm's production plan is financed with funds borrowed from a financial intermediary. In deciding the optimal amount of debt, the firm faces a trade-off: on the one hand, more debt allows them to expand the production scale and to increase the expected profits; on the other hand, the increase in the amount of debt implies a higher volatility of profits to which the firm is averse. The aversion to the volatility of profits derives from the fact that the value of the firm is a concave function of profits. This trade-off induces firms to choose a different amount of financial leverage (debt-equity ratio) depending on the amount of equity they have in their business. Firms with less equity choose a higher debt-equity ratio to take advantage of the higher return associated with smaller size. This feature of the financial decision of firms plays a crucial role in differentiating the responses of small and large firms to monetary shocks.

Firm financing and investment decisions determine how firms grow over time. Accordingly, any successful treatment of the financing decisions of heterogeneous firms should be consistent with an important set of observations about industry dynamics. Studies of the relationship between firm size and growth have overturned the conclusion of Gibrat's Law which holds that firm size and growth are independent. Studies by Hall (1987) and Evans (1987), for example, show that the growth rate of manufacturing firms and their volatility is negatively associated with their initial age and size. Also, Davis, Haltiwanger, & Schuh (1996) find that the rates of job reallocation are decreasing in the firm size and age. In Cooley & Quadrini (1997) we study a similar environment but in a partial equilibrium setting and without aggregate shocks, and we establish that this model reproduces many of the salient features of industry dynamics that are observed in U.S. data. In particular, smaller firms grow faster, experience greater variability in their growth rates and they have higher rates of job reallocation. Moreover, the model is consistent with the observed financial and investment behavior of firms: we find that small firms take on more debt, distribute fewer dividends and their investment depends on the realization of cash flows, even after controlling for their future profitability.

In addition to replicating the observed features of industry dynamics, the model economy developed in this paper suggests that firm heterogeneity is an important channel of transmission for monetary policy. We find that there are significant differences in the responses of small and large firms to monetary shocks. Small firms are much more responsive. One consequence of this heterogeneous behavior is that a large fraction of the changes in aggregate output induced by monetary shocks derives from the reaction of small firms. Moreover, due to the persistence of this reaction, the response of the aggregate economy to monetary shocks is highly persistent with aggregate output that displays a hump-shape response.

The heterogeneous responses of firms to monetary policy occur for reasons related to the internal finance channel that has been analyzed in different contexts by Bernanke & Gertler (1989), Bernanke, Gertler, & Gilchrist (1998), Carlstrom & Fuerst (1997), Kiyotaki & Moore

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[2]Gomes (1997) studies a similar environment in which firm heterogeneity and financial decisions are central to understanding the behavior of firms investment behavior.

(1997). The firm's ability to finance its production plan is related to the value of its assets. When the value of these assets increases—either because the price of the assets increases or because the firm reinvests more profits—the firm is able to expand its production plan. In our model, a monetary shock affects investment through this mechanism: A fall in the nominal interest rate on loans decreases the interest payments of the firms and increases their profits. Because of reinvested profits, the next period financial capacity of the firms increases, which in turn allows them to expand production. This mechanism, however, is more important for small firms because they are more highly levered.

The central role played by the financial structure in the transmission of monetary shocks highlights an important fact about the economic conditions under which the economy is more vulnerable to such shocks. Monetary shocks will have a larger impact during periods in which firms are more heavily indebted.

Although monetary shocks have real consequences in this economy, the effects are quantitatively small. This is consistent with the results of Sims (1992) and Leeper, Sims, & Zha (1996). In spite of their limited impact on the real sector of the economy, monetary shocks have a significant impact on stock market variables: monetary shocks generate fluctuations of stock returns that are much larger than the fluctuation of profits, dividends and aggregate output. This latter finding is consistent with the empirical evidence.[3] The main mechanism through which monetary shocks influence the volatility of stock returns is by altering the factor with which agents discount dividends. Because dividends are paid with cash at the end of the period, the shareholder has to wait until the next period before being able to use these dividends for consumption. Consequently, changes in the nominal prices affect the real values of the dividends paid with cash. Because monetary shocks affect the inflation rate, in addition to the nominal lending rate, they have a significant impact on the market value of the firms, and therefore, on the stock market return. Thus, monetary shocks have a far greater impact on financial markets than their impact on the real economy would seem to warrant. This may provide some insights on the excess volatility puzzle of stock market returns as emphasized in LeRoy (1989) and Shiller (1981).

In the next section we describe the model economy to be studied and the decision problems facing households, firms, and financial intermediaries. We then describe the problem of the firms and the households in some detail and define the competitive equilibrium for this economy. After describing the calibration of the model, we present the properties of the artificial economy. In that section we describe the channels through which financial factors induce heterogeneous behaviors of firms over the business cycle.

1   The Model Economy

There are three sectors in the economy: the production sector, the household sector and the financial intermediation sector. Financial intermediaries intermediate liquid assets (money) between households and firms. Shares of the financial intermediaries and firms are owned by the households.

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[3]Thorbecke (1997) and Hinkelman (1997), for example, find that monetary policy shocks have a significant effect on the stock market and Shanken (1990) shows that it affects the betas of portfolios of firms.

1.1   Firms

In this section we describe the characteristics of existing (old) firms, that is, firms that were created in previous periods and have survived. The description of new firms is deferred to section 2.2.

At each point in time there is a continuum of firms that have access to the technology:

$$y = F(k, l, x, \varphi) \tag{1}$$

where φ is an idiosyncratic shock to technology, k is the input of capital which depreciates at rate δ, l is the input of labor and x is an intermediate input purchased from other firms. The shock is observed after the inputs of capital, labor and intermediate goods are employed in production. To simplify the analysis we assume that labor and the intermediate input are perfect complements to the input of capital. This implies that the chosen quantity of l and x will be proportional to k. We assume that γ_l units of labor and γ_x units of intermediate goods are required for each unit of capital, that is, $l = \gamma_l k$ and $x = \gamma_x k$. Given this assumption, the production technology can be rewritten as $y = F(k, \varphi)$.

The function F is strictly increasing and continuously differentiable in k and φ, strictly concave in k, and satisfies $F(0, \varphi) = F(k, 0) = 0$. The assumption that the function F is concave in k implies that the production technology displays decreasing returns-to-scale. The shock φ is assumed to be independently and identically distributed according to a log-normal distribution. Therefore, the domain of φ is $[0, \infty)$.

At each point in time, firms are characterized by the amount of capital, e, that they own. Henceforth, this capital is referred to as the *equity* of the firm. The amount of equity changes over time as firms reinvest profits. The value of a firm will thus depend on the realization of the idiosyncratic shock and its dividend policy. To keep the problem tractable, we assume that retained earnings is the only source of increased capital for the firm. We motivate this assumption by the observation that firms mainly rely on internal sources of funds for finance.[4]

In addition to the capital that the firms own, they can increase (decrease) the input of capital by renting it from (to) other firms at the rental rate r_k. By allowing firms to rent extra capital, we make sure that financial differences are the only factors that motivate firms to implement different production plans. Their production possibility is not technically constrained by the amount of physical capital they have accumulated until that point.

The purchase of the intermediate input has to be paid in advance and the firm borrows these funds from a financial intermediary.[5] Accordingly, the firm faces the constraint $b \geq x = \gamma_x k$, where b denotes the real value of liquid funds borrowed from the financial intermediary. By assuming that the intermediate input has to be paid in advance, the model captures the cost channel of monetary transmission that Barth & Ramey (1999) have shown to be empirically relevant for the propagation of monetary shocks in the economy.

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[4]Ross, Westerfield, & Jordan (1993) and Smith (1977) document that firms raise more than 80% of equity from internal sources. Theoretically, this could be justified by assuming that there is a sufficiently high proportional cost to funds raised with external sources.

[5]The assumption that the firm needs to finance the intermediate input, rather than the payment of wages as assumed in other models, is helpful for the calibration of the model. If the firm needs to borrow only to finance the advance payment of wages, in equilibrium the aggregate firms' debt would be small compared to their assets. By introducing the intermediate input and by assuming that the firm finances the purchase of this input with debt, we can calibrate the economy to obtain any desired aggregate debt to capital ratio.

The total amount of funds the firm can borrow is subject to an upper bound $\bar{b}(e)$ (borrowing limit), which depends on the firm's equity e. For simplicity, we assume that this borrowing limit is such that the firm is always able to repay the debt at the end of the period, for any realization of the shock. Although the imposition of an exogenous borrowing limit may seem arbitrary, it can be justified by an enforceability argument similar to Albuquerque & Hopenhayn (1997).[6] The presence of this limit imposes a lower bound to the size of the firm in terms of equity: a firm needs a positive value of equity in order to borrow and produce. However, as we will see, in the calibrated economy the borrowing limit is directly binding only for extremely small firms that, as a group, are quantitatively unimportant in terms of their contribution to aggregate output.[7]

The last assumption we make is that at the beginning of each period, and before implementing any production plan, the firm faces a probability η of becoming unproductive. In that case the firm is liquidated with residual value e (exogenous exit).

1.2 Households

There is a continuum of homogeneous households of total measure 1, that maximize the expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t) \qquad (2)$$

where c_t and l_t are consumption and labor at time t, and β is the households's intertemporal discount factor.[8] The utility function satisfies the standard properties. The household is endowed with one unit of working time that can be supplied to the market in return for the real wage rate w. Households' assets are of three types: cash, bank deposits, and a diversified portfolio of firm shares.

At the end of the period, each household holds an amount m of liquid assets (money). An amount d is deposited with a financial intermediary (bank) and earns the nominal interest rate r_d. The amount $m - d$ is available for transactions in the next period: the purchase of consumption goods requires money and the household faces the following cash-in-advance constraint:

$$pc \leq m - d \qquad (3)$$

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[6] For example, we can assume the following enforceability problem: after receiving the loan, a firm can distribute all the borrowed cash to the shareholders and then declare bankruptcy. In that case, if the liquidation value of the firm's capital is smaller than the debt, the bank will realize a loss with probability one, independently of the interest rate charged in the contract. In order to prevent this moral hazard problem, the bank is willing to make loans to the firm only up to a limit. The incentive-compatibility limit is equal to the value of the firm conditional on the firm implementing a non-deviating (moral hazard) policy. Because the value of a firm is an increasing function of the capital it owns, to simplify the analysis the value of the firm's capital is taken as proxy for the current value of the firm.

[7] Rather than imposing a borrowing limit exogenously, we could have assumed that there is asymmetric information between the firm and the intermediary and monitoring is costly. This is the assumption made, for example, in Carlstrom & Fuerst (1997) and Bernanke et al. (1998). The main properties of the model would not change, because, with costly state verification, the optimal "one-period" contract is the debt contract. We have chosen not to do this because it would complicate the structure of the model without changing its main properties.

[8] Throughout we use small letters to denote individual state and choice variables and prices, and capital letters to denote aggregate variables.

where p is the nominal price.

We assume that the stock of deposits cannot be changed before the end of the next period. This is the assumption typically made in the class of monetary models known as "limited participation" models. At the end of each period, the household has available its bank deposits plus the interest earned, wages, and dividends from the firms. In addition, it invests in new shares of firms. We denote the real resources invested in the purchase of new firms shares by i.[9] Denote with μ the initial portfolio of shares in existing firms. At the beginning of the period firms differ only over the amount of equity they own, and therefore, μ represents the measure of firms shares over e. Denote with $\pi(\mu)$ the dividends paid by this portfolio. Then the household's end-of-period money holding (assuming that the cash-in-advance constraint is binding) is:

$$m' = (1 + r_d)d + p(\pi(\mu) + wl - i) \tag{4}$$

The end-of-period stock of money m' is allocated to cash holding and deposits before the beginning of the next period.

The evolution of the portfolio of firms' shares owned by the representative household depends on the initial portfolio μ and the investment in new firms. Denoting with $\tilde{\mu}$ the shares of the new firms added to the initial portfolio μ, the household's portfolio evolves according to $\mu' = \psi(\mathbf{s}, \mu) + \tilde{\mu}$, where the function ψ defines the evolution of the old portfolio of shares. This function also depends on the set of aggregate states $\mathbf{s}$ as described below.

1.3   Financial intermediaries and the monetary authority

In this economy there is a continuum of competitive financial intermediaries. At the end of each period, the financial intermediaries collect deposits from households, and use these funds to make loans to firms at the beginning of the next period. They also receive injections of liquid funds from the monetary authority. The sum of deposits and monetary injection determines the total quantity of loanable funds. These funds are lent to firms through a standard one-period contract based on a non-contingent interest rate. The lending rate is denoted by r_l.

The intermediation sector is competitive, so, in equilibrium, intermediaries do not make profits. This implies that the interest on loans r_d paid to the households is implicitly determined by the zero-profit condition:

$$(1 + r_d)D = (D + \Delta M)(1 + r_l) \tag{5}$$

where D is the aggregate stock of deposits and ΔM is the aggregate transfer of money.

By controlling the monetary transfers to the intermediaries, the monetary authority controls the lending rate r_l. The action of the monetary authority is specified as an exogenous process for the lending rate r_l.

Our financial intermediation sector is somewhat different than in other limited participation models as, for example, in Christiano & Eichenbaum (1995) and Fuerst (1992). We assume that the monetary authority makes monetary transfers directly to the intermediaries and the

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[9]At the end of the period, households also trade in the shares of existing firms (old firms). However, because we assume that households own the market portfolio, in equilibrium no trade in existing firms takes place, and we neglect these transactions in the household's budget constraint. We will re-examine these transactions in later sections, when we derive the market value of the firms' shares.

intermediaries can use these transfers to remunerate the depositors. The more common approach is to assume the transfers are made directly to households. Our approach makes the control of the nominal interest rate easier through changes in the transfers ΔM.

2 The agents' problems

In this section we describe the optimization problems solved by the two main actors of this economy: firms and households. As is standard in monetary models, we normalize nominal variables (deposits, loans, cash holdings and the nominal price) by the pre-shock stock of money. The aggregate states of this economy are then given by the lending rate r_l, the beginning-of-period distribution of firms over equity represented by the measure μ, and the nominal stock of households deposits D. The set of aggregate state variables is denoted by $\mathbf{s} = (r_l, \mu, D)$.

2.1 The firm's Problem

The problem facing managers of firms is to choose capital, labor, the intermediate input, and the amount of borrowing from financial intermediaries to maximize the value of the firm for the shareholders. The value of the firm derives from the flows of dividends that are paid at the end of each period. Because dividends are paid at the end of the period with cash, the shareholders have to wait until next period to buy consumption goods. This implies that one unit of real dividends paid at time t allows the shareholder to buy $p_t/p_{t+1}(1 + g_t)$ units of consumption goods at time $t+1$. The term $(1 + g_t)$ derives from normalizing all nominal variables (including prices) by the pre-shock stock of money. Therefore, the expected utility at time t of one unit of real dividend paid at time t is equal to $\beta E_t(p_t u_{1,t+1}/p_{t+1}(1 + g_t))$, where $u_{1,t+1}$ is the marginal utility of consumption at time $t + 1$. The term $\beta E_t(p_t u_{1,t+1}/p_{t+1}(1 + g_t))$ will be denoted by $\omega(\mathbf{s})$ and it depends on the states of the economy $\mathbf{s}$. To determine the value in terms of utility of the firm's payments to the shareholders, we have to multiply these payments by $\omega(\mathbf{s})$.

Denote by $\Omega(\mathbf{s}, e)$ the value of a firm with equity e.[10] The optimization problem of a surviving firm can then be written as:

$$\Omega(\mathbf{s}, e) = \max_{k, b \leq \bar{b}(e)} E\left\{ \max_{e'} \left\{ \pi \cdot \omega(\mathbf{s}) + \beta\Omega(\mathbf{s}', e') \right\} \right\}(1 - \eta) + e \cdot \omega(\mathbf{s}) \cdot \eta \qquad (6)$$

subject to

$$b \geq \gamma_x k \qquad (7)$$

$$\pi(\mathbf{s}, e, k, b, \varphi) = (1 - \delta)e + F(k, \varphi) - w\gamma_l k - r_k(k - e) - \gamma_x k - r_l b - e' \geq 0 \qquad (8)$$

A productive firm solves two sequential problems at two different stages where each stage is characterized by a different information set. At the beginning of the period, and before observing

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[10]Measured in terms of utility for the representative household.

the shock φ, the firm decides the production scale by choosing k and b. By choosing k the firm also chooses l and x as they are perfect complements of k. At the end of the period, and after observing the shock φ, the firm decides the next period equity e'. Choosing e' is equivalent to choosing the dividend policy of the firm as specified in equation (8). The dividend cannot be negative which is equivalent to saying that the firm cannot raise equity with external sources. Equation (7) is the cash-in-advance constraint for working capital: in order to purchase the intermediate input, the firm needs to borrow liquid funds from the financial intermediary.

To solve this problem, the firm needs two other objects: the law of motion for aggregate states $H(\mathbf{s})$ and the function $Q(\mathbf{s})$ which gives the endogenous variables (w, r_k, g, p) as function of the states. The variable g is the growth rate of money which is determined endogenously as the monetary authority controls the lending rate r_l. These functions are taken as given by the firm.

In Cooley & Quadrini (1997) we show that the optimal dividend policy of the firm assumes a simple form. Firms will retain profits and build the equity of the firm until it reaches an optimal size $\bar{e}(\mathbf{s})$. The existence of an upper bound in the size of the firm derives from the fact that the value of the firm is increasing and concave. Consequently, the marginal increase in the value of the firm is decreasing in e and there is a point, $\bar{e}(\mathbf{s})$, for which the firm is indifferent between increasing its size and distributing dividends. This upper limit in the equity size of the firm is not exogenous but it depends on the aggregate states of the economy. The presence of the exogenous probability η of becoming unproductive guarantees the existence of this upper bound.[11]

By imposing sufficient conditions for which the cash-in-advance constraint (7) is binding and using the properties of the optimal dividend policy described above, the firm's choice reduces to the choice of b and the upper bound $\bar{e}$. The firm problem can then be reformulated as:

$$\Omega(\mathbf{s}, k) = \max_{\bar{e}, b \leq \bar{b}(e)} E\left\{\pi \cdot \omega(\mathbf{s}) + \beta \Omega(\mathbf{s}', k')\right\}(1 - \eta) + e \cdot \omega(\mathbf{s}) \cdot \eta \qquad (9)$$

subject to

$$\pi(\mathbf{s}, e, b, \varphi) = \max\left\{0 \, , \, (1 + r_k - \delta)e + F\left(\frac{b}{\gamma_x}, \varphi\right) - \left(\frac{(1 + r_l)\gamma_x + w\gamma_l + r_k}{\gamma_x}\right)b - \bar{e}\right\} \quad (10)$$

As in the previous program, the firm takes as given the law of motion for the aggregate states $H(\mathbf{s})$ and the price function $Q(\mathbf{s})$.

2.2 Entry of new firms

The creation of a new firm requires an initial investment κ, which is sunk. We also assume that, once a new firm is created, it becomes productive only with probability λ. If the firm becomes

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[11]Notice that in equilibrium firms always prefer to accumulate physical capital, rather than liquid funds. If firms prefer to accumulate liquid funds, then the demand for rental capital would be higher than its supply and this would drive the rental rate up. As the rental rate of capital increases, firms will find more convenient to keep their equity in the form of physical capital rather than liquid funds.

productive, it has access to the same production technology and faces the same decisions as existing firms.

The presence of the fixed cost κ is motivated by technical considerations. Without this cost, the optimal size of a new firm in terms of capital would be zero in the limit and the return from creating new firms would go to infinity. The size of all firms would collapse to the size of new entrants and there would be no firm heterogeneity. The exogenous probability of failure is introduced to control for the optimal size of new entrants. As we will see below, when λ is one the optimal size of new entrants would be the maximum size $\bar{e}$, while in the data new firms are generally small. With a sufficiently small λ, however, the size of new firms will be small. We justify this probability with the empirical observation that young firms face a lower survival rate than old firms.

In each period there is an optimal size (in terms of equity) of new firms denoted by e_0. This is the equity size that maximizes the present return from creating new firms.[12] Because the households purchase a diversified portfolio, the cost (in terms of utility) of a new firm of size e is $(\kappa + e)\omega(\mathbf{s})$. The value of owning it is the next period value of an existing firm of that size, discounted to the current period, that is, $\lambda\beta E\Omega(\mathbf{s}', e)$. Therefore, the present return from creating a new firm is $\lambda\beta E\Omega(\mathbf{s}', e)/(\kappa + e)\omega(\mathbf{s})$ and the optimal size of a new firm is determined by solving the following optimization problem:

$$e_0 \;=\; \arg\max_{e \succ 0}\left\{\frac{\lambda\beta E\Omega(\mathbf{s}', e)}{(\kappa + e)\omega(\mathbf{s})}\right\} \tag{11}$$

Figure 1 illustrates graphically how the optimal size of new firms is determined. The figure plots the term $\lambda\beta E\Omega(\mathbf{s}', e)$, which is the current value (in terms of utility) of a newly created firm. The concavity of Ω implies that this term is also concave in e. The optimal size of new firms is given by the value of capital for which the line departing from $-\kappa\omega(\mathbf{s})$ is tangent to the curve $\lambda\beta E\Omega(\mathbf{s}', e)$. This is because the current return from creating a new firm of size e is equal to the slope of the line departing from $-\kappa\omega(\mathbf{s})$ and crossing the curve at the point e. As can be verified from the picture, the line with the highest slope is the one tangent to the value curve $\lambda\beta E\Omega(\mathbf{s}', e)$.

New firms are created until the value of a new firm of size e_0 is equal to its cost, and in equilibrium the following arbitrage condition has to be satisfied:

$$(\kappa + e_0)\omega(\mathbf{s}) \;=\; \lambda\beta E\Omega(\mathbf{s}', e_0) \tag{12}$$

Therefore, in equilibrium, the slope of the tangent line to the curve $\lambda\beta E\Omega(\mathbf{s}', e)$ departing from $-\kappa\omega(\mathbf{s})$ is equal to $\omega(\mathbf{s})$. Simple comparative static using figure 1 shows that an increase in λ raises the value curve of new firms and increases the optimal size of new entrants. When $\lambda = 1$, the optimal size of new firms is equal to the upper bound $\bar{e}$. This is because the upper bound is at the point in which the slope of $\beta E\Omega(\mathbf{s}', e)$ is equal to $\omega(\mathbf{s})$. Both variables e_0 and $\bar{e}$ depend on the aggregate states of the economy and they fluctuate over the business cycle.

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[12]The optimal size of new firms is determined by the value of e that maximizes the value of the portfolio of new firms obtained with the investment of a fixed amount of funds. Because there is no limit in the number of new firms that can be created, the problem consists of maximizing the surplus of the portfolio of all new firms that can be obtained with a fixed amount of resources.

Figure 1: Optimal size of new entrant firms.
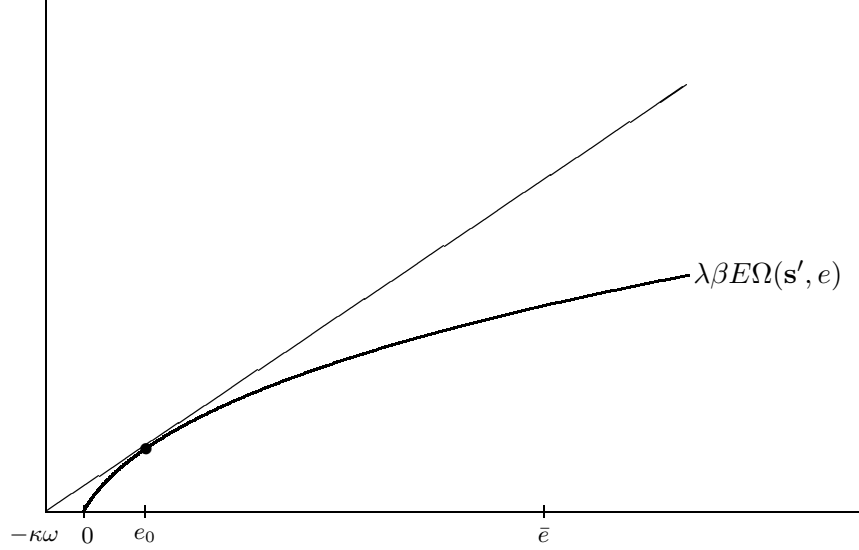
2.3 The household's Problem and general equilibrium

The households in this economy choose labor supply, consumption, investment in new firms and their initial size, deposits and cash holding.[13] Denote by $\hat{\mathbf{s}}$ the set of individual states for the households. They are given by the initial portfolio of shares, μ, the nominal stock of deposits, d, and the stock of nominal assets m (cash and deposits), that is, $\hat{\mathbf{s}} = (\mu, d, m)$. The recursive formulation of the household's problem is:

$$V(\mathbf{s}, \hat{\mathbf{s}}) = \max_{c,l,d',e_0,i} \left\{ u(c, 1 - l) + \beta EV(\mathbf{s}', \hat{\mathbf{s}}') \right\} \tag{13}$$

subject to

$$pc \leq m - d \tag{14}$$

$$p(c + i) + (1 + g)m' = m + pwl + r_d d + p \int_e \pi(\mathbf{s}, e)\mu(de) \tag{15}$$

$$\mu' = \psi(\mathbf{s}, \mu) + \tilde{\mu}, \qquad \tilde{\mu} = \frac{\lambda i}{\kappa + e_0} \tag{16}$$

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[13] As pointed out previously, households also trade in shares of existing firms. However, because in equilibrium households hold the same portfolio of shares, in this section we ignore these potential transactions.

Equation (14) is the cash-in-advance constraint for the household, and equation (15) is the budget constraint. The end-of-period stock of nominal assets m' is multiplied by the gross growth rate of money $1 + g$ as a result of normalizing all nominal variables by the aggregate pre-shock stock of money M. Equation (16) defines the evolution of the household's portfolio of the firms' shares. The portfolio of new firms' shares is a mass point at e_0, with mass equal to $\lambda i/(\kappa + e_0)$. The households, as the firms, also use the law of motion for aggregate states $H(\mathbf{s})$ and the function $Q(\mathbf{s})$ to solve this problem.

Under the condition for which the cash-in-advance constraint is binding, the first order conditions of the household's problem with respect to l, d', e_0 and i are:

$$u_{2,t} - w \cdot \omega(\mathbf{s}_t) = 0 \tag{17}$$

$$E_t \left\{ \frac{u_{1,t+1}}{p_{t+1}} - \beta \frac{(1 + r_{d,t+1})u_{1,t+2}}{p_{t+2}(1 + g_{t+1})} \right\} = 0 \tag{18}$$

$$\lambda \beta E_t \Omega_2(\mathbf{s}_{t+1}, e_0) - \omega(\mathbf{s}_t, e_0) = 0 \tag{19}$$

$$\lambda \beta E_t \Omega(\mathbf{s}_{t+1}, e_0) - (\kappa + e_0)\omega(\mathbf{s}_t) = 0 \tag{20}$$

The function Ω_2 is the derivative of Ω with respect to its second argument, that is, e_0. The function $\omega(\mathbf{s})$ has been defined previously and it is the value in terms of utility of one unit of real resources paid at the end of the period with cash. Notice that equation 19 was implicitly derived in section 2.2 in which we discussed the entry of new firms. Equation (20) is the arbitrage condition derived in (12).

The definition of equilibria follows.

Definition 2.1 (Recursive equilibrium) *A recursive competitive equilibrium for this economy consists of: (a) Households' decision rules $l(\mathbf{s}, \hat{\mathbf{s}})$, $d'(\mathbf{s}, \hat{\mathbf{s}})$, $e_0(\mathbf{s}, \hat{\mathbf{s}})$, $i(\mathbf{s}, \hat{\mathbf{s}})$, and households' value function $V(\mathbf{s}, \hat{\mathbf{s}})$; (b) Firms' decision rule $b(\mathbf{s}, e)$ and firms' value function $\Omega(\mathbf{s}, e)$; (c) Aggregate demand of loans $B(\mathbf{s})$, rental capital $K(\mathbf{s})$ and labor $L(\mathbf{s})$ from firms; (d) Aggregate supplies of labor $L_h(\mathbf{s})$ and deposits $D_h(\mathbf{s})$, and aggregate investment in new firms' shares $I(\mathbf{s})$ each of size e_0 from households; (e) Function $Q(\mathbf{s})$ for (w, r_k, g, p); (f) Law of motion $H(\mathbf{s})$ for aggregate states $\mathbf{s} = (r_l, \mu, D)$. Such that: (a) The decision rules $l(\mathbf{s}, \hat{\mathbf{s}})$, $d'(\mathbf{s}, \hat{\mathbf{s}})$, $e_0(\mathbf{s}, \hat{\mathbf{s}})$, $i(\mathbf{s}, \hat{\mathbf{s}})$ solve the household's problem (13) and $V(\mathbf{s}, \hat{\mathbf{s}})$ is the associated household's value function; (b) The decision rule $b(\mathbf{s}, e)$ solves the firm's optimization problem (9) and $\Omega(\mathbf{s}, e)$ is the associated firm's value function; (c) Prices are competitive and the markets for loans, rental capital, labor, and final goods clear; (d) The laws of motion for aggregate states are consistent with individual decision rules of households, firms and financial intermediaries.*

3   Calibration

Given the complexity of the model, an analytical solution is not feasible. Accordingly, we solve the model using numerical methods. These methods are described in the appendix.

We calibrate the model assuming that a period is a quarter and the discount factor β is equal to 0.985. The household's per-period utility is specified as $u(c, 1-l) = \alpha \log(c) + (1-\alpha) \log(1-l)$. The parameter α is calibrated so that in the steady state the representative household spends 33% of the available time in market activities.

The production technology is specified as $F(k, \varphi) = \varphi h(k)$ where the shock φ is log-normally distributed with parameters $\bar{\varphi}$ and σ_φ. The function h is assumed to be quadratic, that is, $h(k) = k - Ak^2$. The parameter A affects the degree of concavity of the production technology and we set it to 0.006. This parameter is not important for the main properties of the model. The other two parameters that characterize the production technology are γ_l and γ_x, that is the factors of proportionality between the inputs of labor and intermediate goods, and the capital input. After imposing an upper bound for equity $\bar{e} = 1,000$ (this acts like a normalization factor), these two parameters are calibrated jointly with the parameters $\bar{\varphi}$ and σ_φ by imposing the following conditions: (a) the steady state capital-output ratio is equal to 12; (b) the largest firm employs 10,000 workers; (c) the average debt-equity ratio in the economy is 0.35; (d) small firms choose a leverage that is about twice the leverage chosen by the largest firm.

By imposing that the largest firm employs 10,000 workers, the model captures the range of firms that are thought to face stricter financial conditions. (See Gertler & Gilchrist (1994) for example.) The parameter that is specially important in differentiating the leverage of small and large firms is σ_φ. A larger value of σ_φ implies smaller differences between small and large firms.

The depreciation rate of capital is set to 0.025 which is consistent with the values used in the business cycle literature. The exit probability η is set equal to 0.012 which is the average exit rate in the sample of manufacturing firms analyzed by Evans (1987), on a quarterly basis.

The survival probability of a newly created firm, λ, is important in determining the size of new entrants: smaller values of λ imply a smaller size of new entrants. By fixing the value of this parameter at 0.6, we keep the size of new entrants relatively small as in the data. Although the survival probability of new firms in the first quarter of life is larger than 0.6, we should interpret this value as the survival probability of new firms in the first years of life, given that in the model we do not keep track of the age of the firm. The set-up cost κ is determined residually so that the arbitrage condition for the entrance of new firms in the steady state equilibrium is satisfied, given the imposed distributional range of firms over equity e.

Finally, the lending rate is assumed to follow a first order autoregressive process with correlation parameter $\rho_m = 0.9$ and standard deviation $\sigma_m = 0.0025$. The full set of parameter values are reported in table 1.

4   Steady state properties

We begin by describing the main features of the steady state equilibrium. Some of the properties of the steady state equilibrium will help to clarify why firms of different size respond differently to monetary shocks.

Figure 1a reports the value of debt and the borrowing limit as functions of equity. As can be seen from this figure, the value of debt is increasing in the size of the firm. However, the fact that debt is increasing in equity is not a "direct" consequence of the borrowing limit. In fact, the borrowing limit is binding only for extremely small firms, and most of the firms choose to borrow less than their limit.

Table 1: Calibration values for the model parameters.

Intertemporal discount rate	β	0.985
Consumption/leisure share	α	0.393
Technology parameter	A	0.006
Technology parameter	γ_l	5.000
Technology parameter	γ_x	0.340
Mean idiosyncratic shock	$\bar{\varphi}$	0.490
Standard deviation shock	σ_φ	0.400
Depreciation rate	δ	0.025
Probability of exit	η	0.012
Survival probability of new firms	λ	0.600
Firm initial set-up cost	κ	13.079

To understand why firms do not borrow up to the borrowing limit, consider the trade-off that they face in deciding their production plans. By borrowing more the firm can expand the scale of production and increase the expected profits. On the other hand, a larger production scale implies an increase in the volatility of profits. Because the next period equity depends on the current realization of profits and the value of the firm is a concave function of equity (see figure 1e), the firm is averse to fluctuations of profits. Therefore, in deciding whether to expand the scale of production by borrowing more, the firm compares the marginal increase in the expected profits with the marginal increase in its volatility (and therefore, in the volatility of next period equity). Because of the decreasing return-to-scale property of the production function, as the firm increases its equity and implements larger production plans, the marginal expected profits from increasing the production scale further decreases. This implies that the firm becomes more concerned about the volatility of profits and borrows less in proportion to its equity. Thus, as the firm grows, the composition of the sources of finance changes in favor of internal sources.[14]

Figure 1b plots the firm's leverage, the ratio of debt to equity. Smaller firms choose a higher ratio. Although the increased leverage allows small firms to be more profitable (see figure 1c), their profits are more vulnerable to changes in the lending rate. This implies that monetary shocks will affect small firms more heavily than large firms. This observation will be important in understanding the impulse responses of small and large firms to monetary shocks that are described in the next section.

It is also interesting to note that small firms rent capital from large firms, that is, the capital input of small firms is larger than the capital they have previously accumulated. If we interpret rental capital as commercial credit, then in the model larger firms provide commercial credit to smaller ones, which is consistent with the allocation of commercial credit across firms of different size observed in the real economy. For this reason, large firms can be considered to be more

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[14]The fact that the borrowing limit is not directly binding does not mean that this limit is irrelevant in the model. Even though the firm is not currently constrained, there is always a non-zero probability that the firm could experience a sequence of bad shocks and its equity approaches zero. At this point the borrowing limit becomes binding. The possibility of being constrained prevents the firm from borrowing too much, as more debt makes this possibility more likely.

liquid than smaller firms.

Figure 1d plots the expected dividend rate, that is, the amount of dividends as a fraction of equity that the firm expects to pay before the realization of the idiosyncratic shock. The dividend rate is increasing in the size of the firm. This is because, as observed in section 2.1, the optimal policy consists in retaining the earnings until the firm has reached the maximum size $\bar{e}$. This policy derives from the concavity of the firm's value, as shown in figure 1e, which in turn derives from the concavity of the production function. Note also that, in this economy, firms with higher profits invest more, independently of their future profitability. In this respect the model is consistent with the empirical findings of Fazzari et al. (1988) and Gilchrist & Himmelberg (1995, 1998) that cash flow has a significant impact on firms' investment, even after controlling for their future profitability.

Finally figure 1f plots the steady state size distribution of firms. If we exclude the largest class, the steady state distribution is skewed toward small firms which is also an empirical regularity of the data. The concentration of firms in the largest class occurs because in the model there is an upper bound to the size of the firm. In the data, of course, there exist firms that employ many more workers than the largest firm in the model (10,000 workers). Although the number of these firms is relatively small, they account for a large fraction of aggregate production. We interpret the largest firms in the model as representing the production of very big firms: the large share in production of these big firms is accounted for in the model by an increase in the number of firms rather than their size.

5   The response to monetary shocks

Figures 2a-2b report the impulse response of small and large firms after a shock that initially increases the lending rate r_l by 25 basis points. Small firms are those employing fewer than 5,000 employees. The figures show the responses of real debt and output as deviations from the steady state. As can be seen from the figure, the reaction of small firms is significantly greater than large firms. This is consistent with the empirical facts outlined in Gertler & Gilchrist (1994).

This asymmetric response occurs for the reasons articulated earlier: a monetary shock leads to proportionately greater growth in the profits of small firms. The increase in the interest rate decreases the firms profits, which in turn reduces their next period equity. Given the reduced value of equity, firms borrow less in the next period. This is the internal finance mechanism that has been emphasized in Bernanke & Gertler (1989), Bernanke et al. (1998), Carlstrom & Fuerst (1997), Kiyotaki & Moore (1997). As observed in the previous section, small firms are more heavily indebted than large firms (higher leverage). Consequently, changes in the interest rate imply a larger impact on the interest burden of small firms (again, in proportion to the equity of the firm), which in turn implies a larger impact on their next period equity.

Figures 3a-3h show the aggregate response of several variables to a monetary shock. The pattern of the lending rate is plotted in figure 3a. Because the lending rate follows an exogenous autoregressive process, after the unexpected increase, it returns asymptotically to the steady state. In keeping with the limited participation structure of the monetary sector, the increase in the nominal interest rate requires a fall in the growth rate of money as shown in figure 2b. Following a monetary contraction, output, hours, wages, profits and prices fall. These responses are consistent with the empirical findings of Christiano, Eichenbaum, & Evans (1996). It is

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Table 2: Business cycle properties of the artificial economy.

| | Standard deviation | | | |
	Money shock	Real shock	Both shocks	U.S. Economy
Output	0.11	1.61	1.62	1.67
Consumption	0.62	1.26	1.40	0.84
Investment	2.38	3.77	4.49	8.24
Wage rate	0.51	1.22	1.33	0.86
Profits	3.08	5.34	6.20	9.15
Capital share	0.84	1.12	1.46	0.70
Price index	0.74	0.75	1.06	1.39
Inflation	0.64	0.72	0.97	0.57

| | Correlation with output | | | |
	Both shocks	Money shock	Real shock	U.S. Economy
Consumption	0.28	0.94	0.84	0.84
Investment	-0.06	0.89	0.74	0.91
Wage rate	0.84	0.93	0.88	0.27
Profits	-0.06	0.90	0.77	0.74
Capital share	-0.85	0.54	0.40	0.15
Price index	0.21	-0.85	-0.59	-0.51
Inflation	0.44	-0.55	-0.39	0.34

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 200 periods and repeating the simulation 200 times. The statistics are averages over these 200 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4. Consumption includes consumer expenditures in non-durable and services. Consumer durables are classified as investment. Wages is the index of real compensation per hour in the non-farm business sector. Profits are corporate profits and the price index is the CPI index.

important to observe the persistent response of aggregate output which follows a hump-shaped pattern. This is a consequence of the internal finance mechanism, as outlined above, through which shocks get propagated in the economy. Also interesting is that job destruction reacts more than job creation and in the opposite directions.[15] This implies that the variability of job creation is smaller than the variability of job destruction and that they are negatively correlated. This is consistent with the findings of Davis et al. (1996) who document the dynamics of job reallocation in the manufacturing sector.

We summarize the business cycle properties of this economy in the first column of Table 2, which reports the standard deviations and correlations with output of several aggregate variables. As can be seen from this table, the model generates significant volatility in prices and inflation, but does not generate the volatility of output we observe in the data (see last column of the table). The finding that monetary policy shocks contribute only marginally to output fluctuations, is consistent with the empirical studies of Sims (1992) and Leeper et al. (1996).

To investigate the impact of other shocks on the business cycle properties of the artificial

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[15]The definition of job reallocation follows the definition adopted by Davis et al. (1996): job creation is computed by summing the employment gains of expanding firms, and job destruction is computed by summing the employment losses of contracting firms.

economy, we extend the model by introducing an aggregate shock z to the production technology. The technology becomes $y = \varphi h(zk)$. The shock follows a first order autoregressive process as in the standard real business cycle model. The autoregressive parameter is set to 0.95 and the standard deviation to 0.036. With this variability, the model generates output volatility that is similar to the data. Of course, we are not evaluating the performance of the model according to its ability to generate the empirical volatility of output.

The business cycle statistics of the model with only technology shocks and with both monetary and technology shocks are reported in the second and third columns of Table 2. Once we consider both shocks, this model generates business cycle properties that are close to those observed in the data. It is interesting to note that the model also generates a positive correlation between output and capital incomes share, which is also a feature of the data.

6   The response of financial markets

We now consider the response of financial markets to aggregate shocks. Stock market transactions take place at the end of the period, after the payment of dividends. Define $q(\mathbf{s}, e)$ to be the real market price of a share of a firm with total equity e at the end of the period—i.e. after the payment of dividends. This price can be derived using the function $\Omega(\mathbf{s}, e)$, defined in (6). The function $\Omega(\mathbf{s}, e)$ is the utility value of a firm with total equity e at the beginning of the period, before uncertainty is resolved. These two functions are related by the following arbitrage condition:

$$q(\mathbf{s}, e)\omega(\mathbf{s}) = \beta E\Omega(\mathbf{s}', e) \tag{21}$$

The left hand side of the equation is the cost in terms of utility of paying the price $q(\mathbf{s}, e)$ at the end of the period. The function $\omega(\mathbf{s})$ multiplying the price has been defined previously and it is equal to $E(\beta p u_1'/p'(1+g))$. The term on the right hand side is the expected present value (also in terms of utility) of owning a share of the firm. In equilibrium these must be the same. Consequently, the equilibrium market price of the firm is:

$$q(\mathbf{s}, e) = \frac{\beta E\Omega(\mathbf{s}', e)}{\omega(\mathbf{s})} \tag{22}$$

Using $q(\mathbf{s}, e)$, we can price any portfolio of firms' shares. We concentrate the analysis on a particular portfolio, the one composed of the steady state distribution of firms. The composition and size of this portfolio is kept constant. The value of this portfolio, denoted by SM, is the sum of the real market prices of the firms' shares traded at the end of the period, weighted by the steady state distribution of firms. This measure of the stock market value is the closest to standard stock market indices like the S&P 500 index.[16] Formally, we compute it as:

$$SM_t = \int_0^{\bar{e}} q(\mathbf{s}_t, e)\, \mu^*(de) \tag{23}$$

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[16]We have also considered an alternative portfolio consisting of the market portfolio. The difference between the market portfolio and the one consider in the paper is that in the former the composition and dimension change as the distribution and mass of firms in the economy change, while in the latter the composition and dimension are kept constant. Because the properties of these two portfolios are very similar, we report the results only for the steady state portfolio.

where $\mu^*(e)$ is the steady state measure of firms. We will refer to SM as the stock market index.

To compute the stock market return we need to compute the market price of this portfolio at the end of the next period, inclusive of the dividends that the portfolio will pay in that period. The value of this portfolio, denoted by $\overline{SM}_{t+1}$, is simply given by:

$$\overline{SM}_{t+1} = \int_0^{\bar{e}} \frac{\Omega(\mathbf{s}_{t+1}, e)}{\omega(\mathbf{s}_{t+1})}\, \mu^*(de) \tag{24}$$

The ex-post stock market return index is:

$$r_{t+1} = \frac{\overline{SM}_{t+1} - SM_t}{SM_t} \tag{25}$$

Figures 4a-4f report the impulse responses of stock market prices, returns and dividend yields after a monetary shock that increases the lending rate (left graphs) and after a negative technology shock (right graphs). Each graph on the left-hand side uses the same scale of the graph on right-hand side. Consequently, the responses of the same variable to different shocks are comparable in magnitude.

As can be seen, both a contractionary monetary policy and a negative technology shock, induce a fall in the stock market value, return and dividend yield. Moreover we observe that monetary and real shocks produce fluctuations in stock market returns of similar magnitude. This is in contrast to the contribution of monetary shocks to the fluctuation of output in which real shocks are far more important than monetary shocks.

The first section of Table 3 reports standard deviations of the stock market index, return, profits and dividend yields for three versions of the economy: the economy with only monetary shocks, the economy with only real shocks, and the economy with both monetary and real shocks. The comparisons of these three versions of the model allow us to evaluate the importance of monetary and real shocks for stock market fluctuations. As was expected from the analysis of the impulse responses, monetary shocks generate significant stock market fluctuations even though they are relatively unimportant for generating fluctuations in aggregate output. We also observe that, as in the data, the model generates much more volatility in stock market returns than in dividend yields.

The sensitivity of the stock returns to monetary shocks derives from the impact that these shocks have on the factor $\omega(\mathbf{s})$, which is the factor with which current dividends are evaluated. Recall that, with some simplification, the value of a firm can be written as $\Omega = \pi \cdot \omega + \beta \Omega'$. Therefore, even though the dividend π does not change, the value of the firm changes if ω changes. Because ω is affected by the inflation rate, and monetary shocks have a direct impact on this rate, the value of the firm is very sensitive to these shocks. Another way to see this is by defining the value of the firm in terms of the discounted value of dividends. This value, denoted by $\tilde{\Omega}$, is given by:

$$\tilde{\Omega} = \pi + \beta E\left\{\left(\frac{\omega'}{\omega}\right)\tilde{\Omega}'\right\} \tag{26}$$

This is the pre-dividend market price of the firm. This price is given by the current dividend plus the discounted price of future dividends. The key point is that future dividends are discounted with a variable factor which depends on ω. Therefore, changes in ω induce large changes in the

Table 3: Cyclical properties of stock market prices, returns and dividend yield.

	Money shock	Real shock	Both shocks
	Standard deviation		
Stock market prices	0.07	0.11	0.13
Stock market returns	4.43	7.27	8.55
Dividend yields	0.31	0.90	0.95
	Correlation with money growth		
Stock market prices	0.76	-0.61	0.01
Stock market returns	0.71	-0.60	-0.01
Dividend yields	0.79	-0.60	-0.14
	Correlation with inflation		
Stock market prices	0.85	-0.69	-0.14
Stock market returns	0.80	-0.70	-0.16
Dividend yields	0.87	-0.68	-0.29

NOTES: In the data standard deviation of real stock market return 8.2%. Standard deviation of real dividend yield 0.7%. Correlation money growth-stock market return 0.16; correlation inflation-stock market return -0.28. Monthly observations from 1957.1 through 1994.4 compounded to quarterly frequencies.

market price of a firm. Because ω is sensitive to the changes in nominal prices, then monetary shocks have a large impact on stock market prices by impacting on the inflation rate.

Finally the second and third section of Table 3 reports the correlations of stock market values, returns, and dividend yields with money growth and inflation. In the data stock market returns are positively correlated with the growth rate of money and negatively correlated with the inflation rate. These features of the stock markets returns have been documented and analyzed in Jovanovic & Ueda (1998) and Marshall (1992).

The economy with only monetary shocks is able to generate the positive correlation of money growth with the stock market returns but does not generate the negative correlation with inflation. This is because a negative monetary shock is associated with a reduction in the growth rate of money and inflation rate, and it has a negative impact on the stock market returns. On the other hand, the economy with only technology shocks generates the opposite results with the stock market returns that are negatively correlated with both money growth and inflation. This is because a negative technology shock has a negative impact on the stock market returns and generates a persistent increase in the nominal prices and inflation. To prevent the nominal interest rate from raising (through the inflation expectations), the growth rate of money has to be increased. Therefore, the growth rate of money and the inflation rate are both negatively correlated with the stock market returns.

When we consider both shocks, the signs of these correlations depend on the relative importance of the two shocks. For the particular calibration, we have that inflation is negatively correlated with stock market returns and the correlation with the growth rate of money is basically zero. Finally, we should emphasize that if the monetary shocks take the form of innovations to the growth rate of money, rather than to the nominal interest rate, then with sufficient vari-

ability in the growth rate of money the model with both types of shocks will be able to generate the right correlations of the stock market returns with the growth rate of money and the inflation rate.

7 Conclusion

We have developed a general equilibrium model with heterogeneous, long-lived firms where financial factors play an important role in differentiating the production and investment decisions of the firms, and their response to monetary shocks. We find that the response of small firms to monetary shocks is larger than big firms. The mechanism that generates this heterogeneous response is the internal finance channel. Through this propagation mechanism, the response of the economy to monetary shocks follows a hump-shape pattern.

The aggregate impact of monetary shocks on the real sector of the economy is not large. Nevertheless, monetary shocks cause considerable volatility in financial markets, particularly in stock market returns.

A   Appendix: Computational procedure

We describe first the computational procedure to solve for the steady state equilibrium and then the algorithm used to compute the equilibrium with aggregate uncertainty.

A.1   Solving for a steady state equilibrium

The computational procedure to solve for the steady state equilibrium consists of the following steps.

1. Guess initial values for w, r_k, and D. Then using equations (5), (14), (17) and (18) compute the steady state values of r_d, r_l, p and C.

2. Guess the steady state upper bound for equity $\bar{e}$ and choose a discrete grid in the space of firms' equity, *i.e.*, $e \in \mathcal{E} \equiv \{e_1, ..., e_n\}$. In this grid, $e_1 = 0$ and $e_n = \bar{e}$.

3. Guess initial steady state values of debt b_i^*, for $i \in \{1, .., n\}$.

4. Guess initial values of Ω_i, for $i \in \{1, .., n\}$.

5. Approximate with a second order Taylor expansion the function $\tilde{\Omega}_i$ around the guessed values b_i^*. This function is defined as:

$$\tilde{\Omega}_i(b) = \beta \sum_{j=1}^{n-1} \int_{e_j}^{e_{j+1}} \left[\Omega_j + \left(\frac{\Omega_{j+1} - \Omega_j}{e_{j+1} - e_j}\right)(x - e_j)\right]\Phi_i(b, dx) \qquad (27)$$
$$+ \int_{e_n}^{\infty} [(x - e_n) \cdot \omega + \beta \Omega_n]\Phi_i(b, dx)$$

where $\Phi_i(b, x)$ is the density function for the end-of-period firm's resources, that is, $x = (1 - \delta)e_i + F(k, \varphi) - r_k(k - e_i) - wl - (1 + r_l)b$. The function $\tilde{\Omega}_i$ is the approximated utility value of the firm with equity e_i and debt b, conditional on being productive (which happens with probability $1 - \eta$). The factor ω is constant in the steady state and it is equal to $\beta u_1/(1 + g)$. The value function Ω is approximated with piece-wise linear functions joining the grid points in which the value function is computed. The definition of $\tilde{\Omega}_i$ given in (27), takes as given the dividend policy of the firm consisting in retaining all profits until the firm reaches the size e_n.

6. Solve for the firm's policy b_i by differentiating the function $\tilde{\Omega}_i(b)$ with respect to b and eliminate b using this policy rule. Take the resulting values to guess new values of Ω_i and restart the procedure from step 5 until all firms' value functions have converged.

7. After value function convergence, check whether the firm policies found above reproduce the guesses for the steady state values of debt b_i^*. If not, update this guesses and restart the procedure from step 4 until convergence.

8. Check the optimality of the upper bound $\bar{e}$ by verifying the following condition:

$$\beta \frac{\partial \Omega(e)}{\partial e}\bigg|_{e = \bar{e}} = \omega \qquad (28)$$

To check this condition, we compute the numerical derivative of $\Omega(e)$ at e_n, taking as given the value of b_n^* found above. If this condition is not satisfied, change the guess for $\bar{e}$ and restart the procedure from step 2 until convergence.

9. Find the optimal size of new firms e_0 using the condition:

$$\lambda \beta \frac{\partial \Omega(e)}{\partial e}\bigg|_{e = e_0} = \omega \qquad (29)$$

10. Given the size of new entrants, iterate on the measure of firms until convergence. In each iteration, it is assumed that the measure of new entrants, denoted by $\tilde{\mu}$, is equal to the measure of exiting firms. Using $\tilde{\mu}$, we determine the investment in the creation of new firms, that is, $I = \tilde{\mu}(\kappa + e_0)$.

11. Using the steady state distribution of firms, compute the demand for labor L^d, rental capital K^d and the dividends Π. The measure of firms is rescaled so that the demand for labor is equal to the guessed value L.

12. At this point we verify three equilibrium conditions: 1) the aggregate budget constraint $(1 + g) = (1 + rd)D + p(\Pi + wL) - pI$; 2) the equilibrium condition in the market for rental capital $K^d = 0$; 3) the arbitrage condition for the creation of new firms $\beta\lambda\Omega(e_0) = (\kappa + e_0)\omega$. If these conditions are not satisfied, we update the initial guesses for w, r_k, D, and restart the procedure from step 1 until convergence.

A.2   Solving for the equilibrium with aggregate uncertainty

Given the steady state equilibrium variables, we seek for Markovian decision rules of the firms and of the representative household. At each point in time the states of the economy are: (i) the lending rate r_l; (ii) the distribution of firms over real equities represented by the measure μ; (iii) the nominal stock of deposits D. The set of states are denoted by $\mathbf{s} = (r_l, \mu, D)$.

Given the computational difficulties in deriving functions of measures, the distribution of firms μ has to be approximated with some of its moments. We approximate the distribution with two variables: the aggregate value of equity owned by new firms (new entrants), denoted by E^0, and the aggregate value of equity owned by old firms (firms with more than one period of age), denoted by E^1. Therefore, as in Krusell & Smith (1998), we approximate the distribution of agents (in our case young and old firms) with the first moments E^0 and E^1. However, the procedure followed to derive the laws of motion of these moments is slightly different, and it consists in the aggregation of the individual decision rules. Following is the description of the whole procedure.

1. Discretize the state space of firms' equity as in the computation of the steady state equilibrium described in the previous section.

2. Guess a linear function $H(\mathbf{s})$ for the law of motion of the aggregate states and a linear function $Q(\mathbf{s})$ for the variables (w, r_k, g, p, ω). The function H maps the current states $\mathbf{s} = (r_l, E^0, E^1, D)$ into next period states and Q maps the current states into the vector of variables $\mathbf{h} = (w, r_k, g, p, \omega)$. Given these functions, the firm's problem is well-defined.

3. Guess initial quadratic (firms' value) functions $\Omega_i(\mathbf{s})$, $i = \{1, ..., n\}$.

4. Given the guesses for $\Omega_i(\mathbf{s})$, approximate with a second order Taylor expansion the function $\tilde{\Omega}_i$ defined as:

$$\tilde{\Omega}_i(\mathbf{s}, \mathbf{h}, b, \mathbf{s}') = \tag{30}$$

$$\beta \sum_{j=1}^{n-1} \int_{e_j}^{e_{j+1}} \left[\Omega_j(\mathbf{s}') + \left(\frac{\Omega_{j+1}(\mathbf{s}') - \Omega_j(\mathbf{s}')}{e_{j+1} - e_j} \right)(x - e_j) \right] \Phi_i(\mathbf{s}, b, dx)$$

$$+ \int_{e_n}^{\infty} \left[(x - e_n) \cdot \omega + \beta\Omega_n(\mathbf{s}) \right] \Phi_i(\mathbf{s}, b, dx)$$

for $i \in \{1, .., n\}$, where b is the real firm's debt and $\Phi_i(\mathbf{s}, b, x)$ is the density function for the end-of-period resources. A simplifying assumption, here, consists of taking the size of the largest firm fixed at the steady state value $\bar{e}$.

5. Using the law of motion H, eliminate the next period states $\mathbf{s}'$. Then taking derivatives with respect to b of the functions $\tilde{\Omega}_i$, derive the firm's decision rules $b_i(\mathbf{s}, \mathbf{h})$, for $i = \{1, .., n\}$, which are linear functions of $\mathbf{s}$ and $\mathbf{h}$. If the firm is financially constrained, then the policy rule is simply given by a linear approximation of the borrowing constraint.

6. Using the decision rules $b_i(\mathbf{s}, \mathbf{h})$, eliminate the variable b from the firms' objective function and, using the function Q, eliminate the vector of variables $\mathbf{h}$. After reducing, the functions $\tilde{\Omega}_i$ depend only on the states $\mathbf{s}$. These are used to form the new guesses for the firms' value functions. The procedure is then restarted from step 4 until all value functions have converged.

7. Using the decision rules found in step 5, derive the aggregate demand for loans by summing the individual decision rules of new and old firms weighted by the steady state distribution. In that way we get a linear demand for loans from new firms $B_0^d(\mathbf{s}, \mathbf{h})$ and a linear demand for loans from old firms $B_1^d(\mathbf{s}, \mathbf{h})$. These two demands are then multiplied by the factors E^0/E^{0*} and E^1/E^{1*} respectively, and they are linearized around the steady state. By summing these two linear functions we get the aggregate demand for loans $B^d(\mathbf{s}, h)$. The computational approximation, here, consists in assuming that when the mean values of equity for new and old firms increase, then the mass of all new and old firms increases proportionally. Using the demand for loans and the firms' demands for labor and rental capital, as well as the function linking the supply of loans to the supply of deposits, approximate with linear functions the demands for labor $L^d(\mathbf{s}, \mathbf{h})$, for rental capital $K_d^d(\mathbf{s}, \mathbf{h})$ and deposits $D^d(\mathbf{s}, \mathbf{h})$. Furthermore, given the optimal decision rules of the firms, linearize the function for the next period value of equity of existing firms $E^1(\mathbf{s}, \mathbf{h})$, and the function of dividends distributed by existing firms $\Pi(\mathbf{s}, \mathbf{h})$. The derivation of these functions is similar to the derivation of the aggregate demands.

8. Using the derived demands for labor, rental capital, loans, the linearized household's budget constraint and $\omega = \beta u_1 p/p'$, impose market clearing conditions and compute the vector of variables $\mathbf{h} = (w, r_k, g, p, \omega)$ as a function of the states $\mathbf{s}$, the labor supply L and the household dividends Π. Denote this function by $\mathbf{h} = \tilde{Q}(\mathbf{s}, L, \Pi)$.

9. Given the function $\tilde{Q}(\mathbf{s}, L, \Pi)$, the dividend function $\Pi(\mathbf{s}, \mathbf{h})$ and the next period equity of all firms $E^1(\mathbf{s}, h)$, the household's decision rules for the variables L, $E^{0'}$ and D' are derived using the household's first order conditions, that is, equations (17), (18), and (19). The decision rules are computed with the method of undetermined coefficient after linearizing these conditions around the steady state. In equation (19) it is assumed that the size of new entrants is fixed at the steady state value.

10. The solution of the household problem allows the derivation of L, E_0', D' as linear functions of the states $\mathbf{s}$. These functions, together with the functions $\tilde{Q}(\mathbf{s}, L, \Pi)$, $\Pi(\mathbf{s}, \mathbf{h})$ and $E^1(\mathbf{s}, \mathbf{h})$, allows us to derive the law of motion $H(\mathbf{s})$ and the function $Q(\mathbf{s})$. These functions are then used as new guesses and the procedure is restarted from step 3 until full convergence.

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Figure 1a – Short-term debt

Borrowing limit
Short-term debt

Firm's size (Equity)
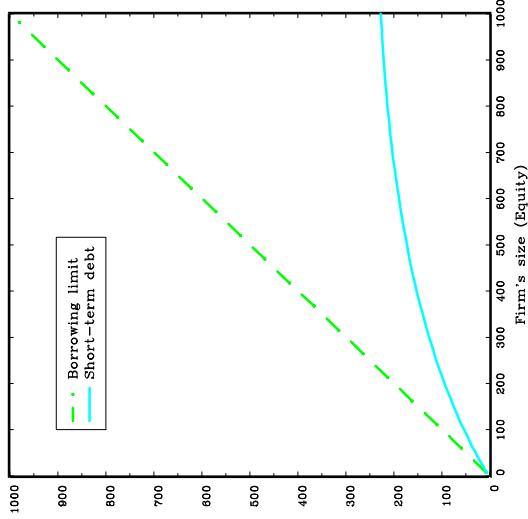
Figure 1b – Debt–Equity ratio

Firm's size (Equity)
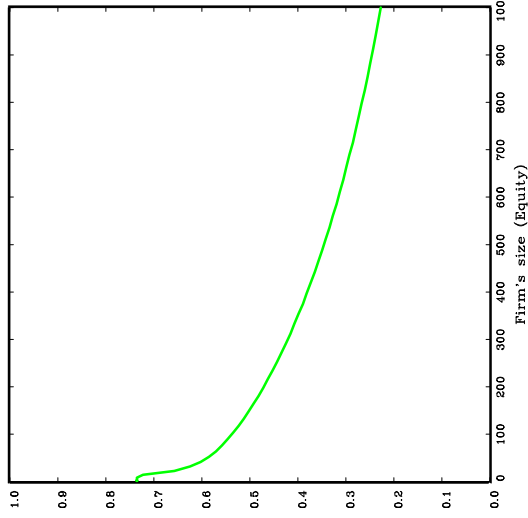
Figure 1c – Expected profit rate

Firm's size (Equity)
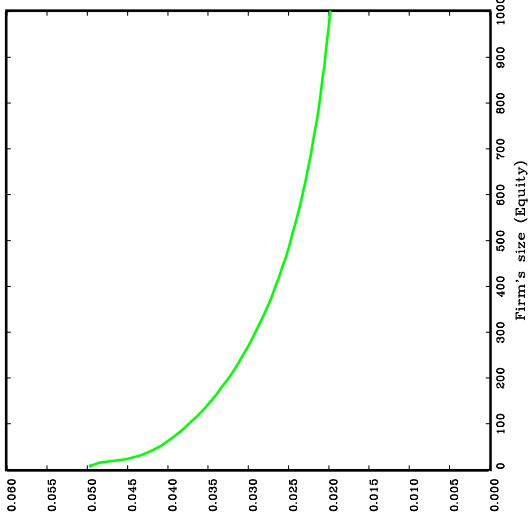
Figure 1d – Expected rate of dividend

Firm's size (Equity)
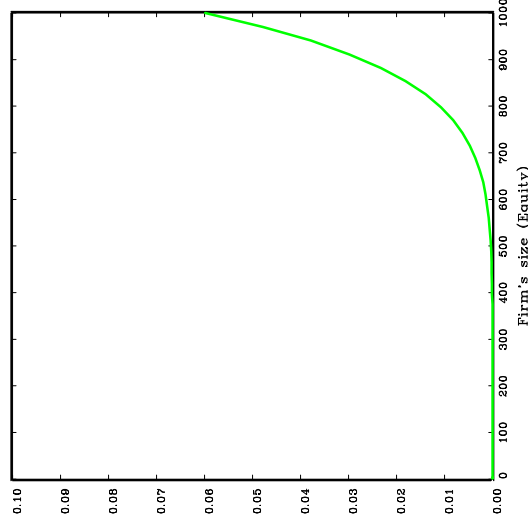
Figure 1e – Firm's value

Firm's size (Equity)
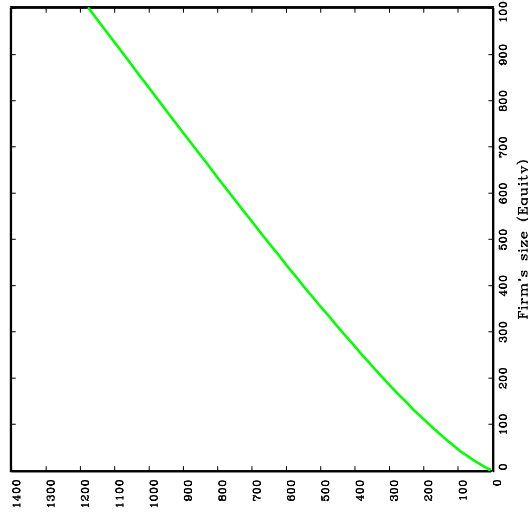
Figure 1f – Stationary distribution of firms

Measure of firms

Firm's size (Equity)
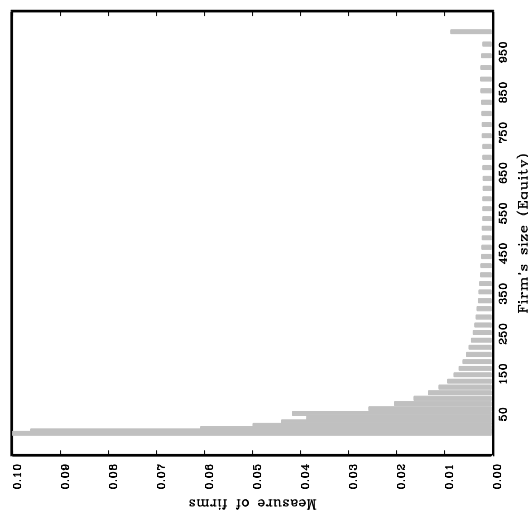
Figure 2a − Real debt
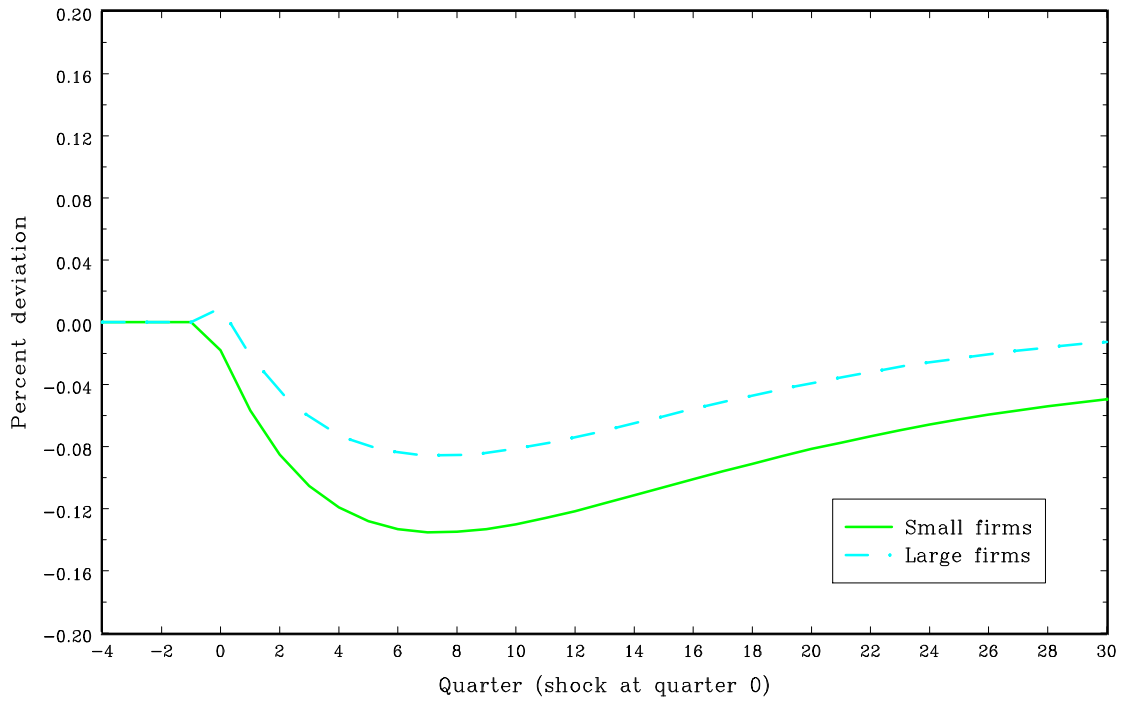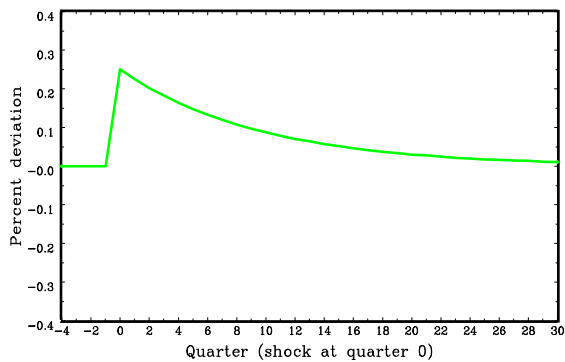

Figure 2b − Output
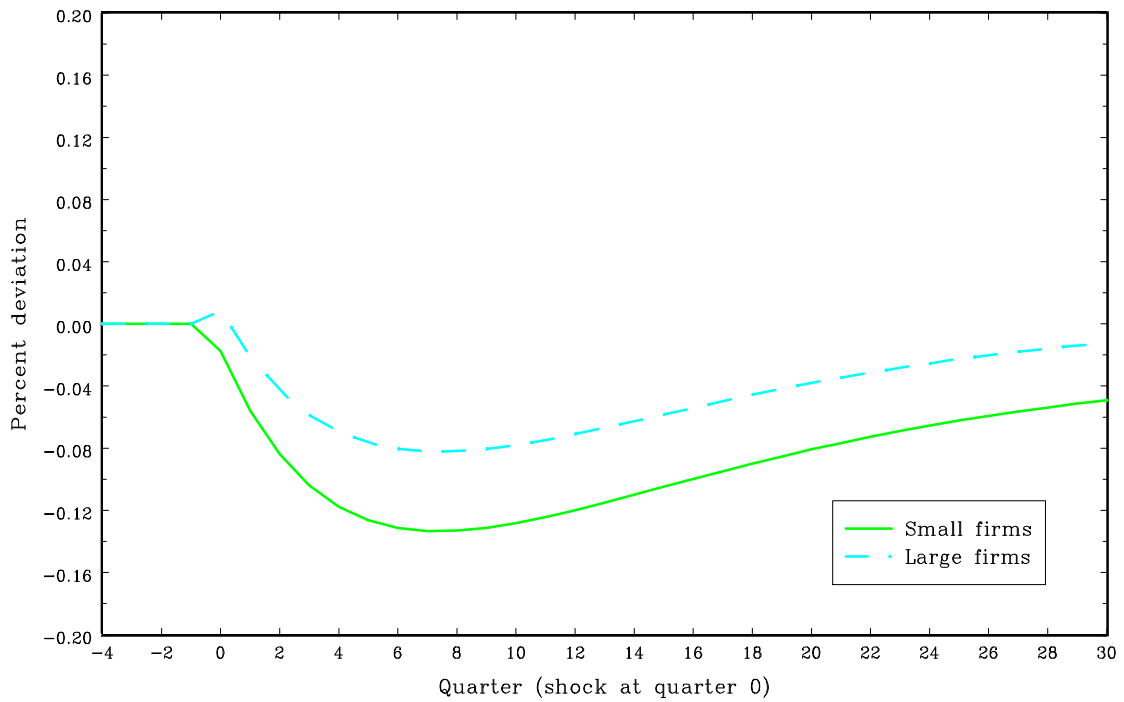
Figure 3a – Interest rate on loans

Figure 3b – Growth rate of money
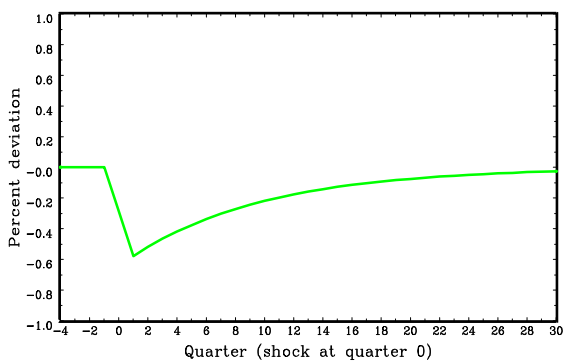
Figure 3c – Output
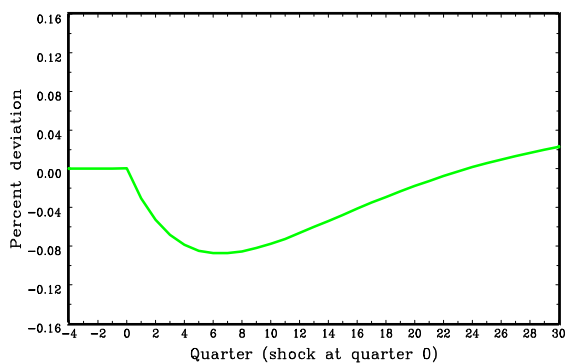
Figure 3d – Labor variables

Hours worked
Job creation
Job destruction
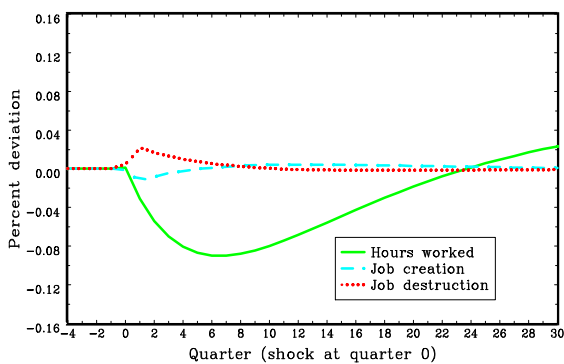
Figure 3e – Wages
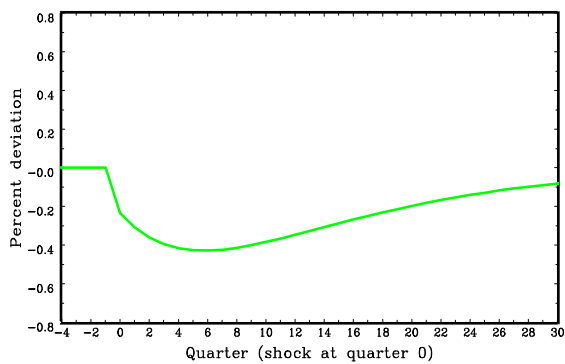
Figure 3f – Profits
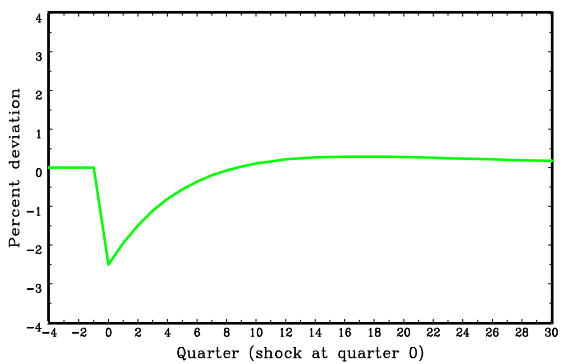
Figure 3g – Nominal price
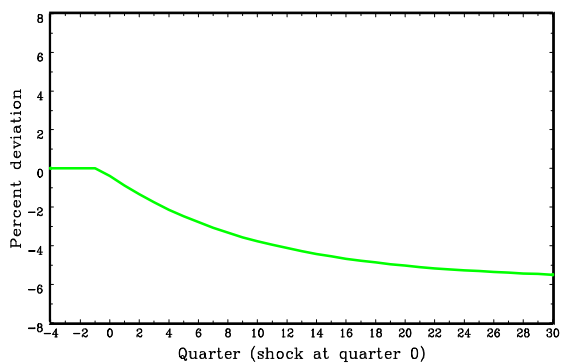
Figure 3h – Inflation rate
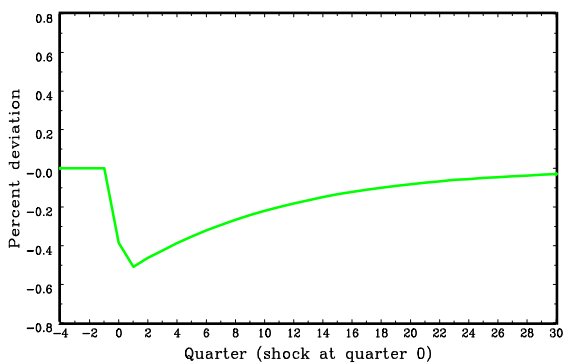
Figure 4a — Money shock: Stock index value
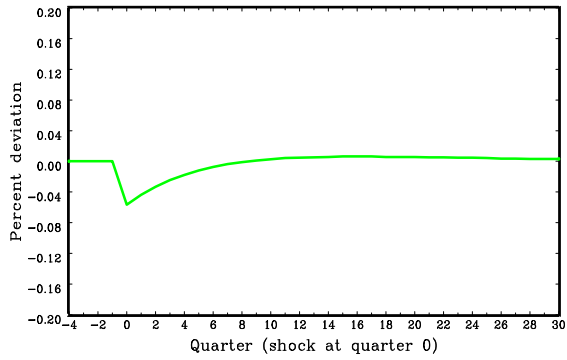
Figure 4b — Real shock: Stock index value
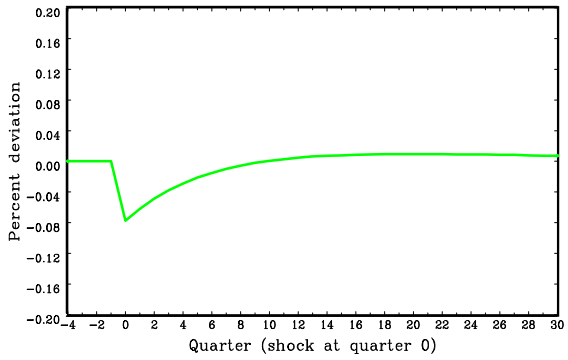
Figure 4c — Money shock: Stock index return
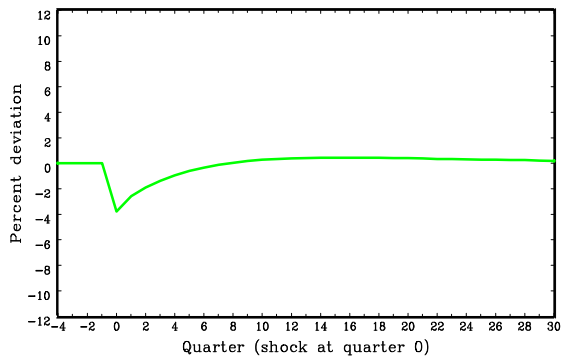
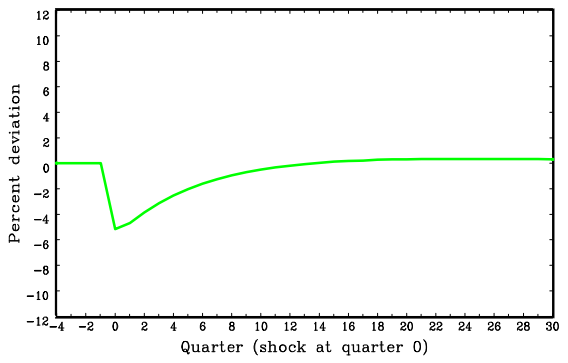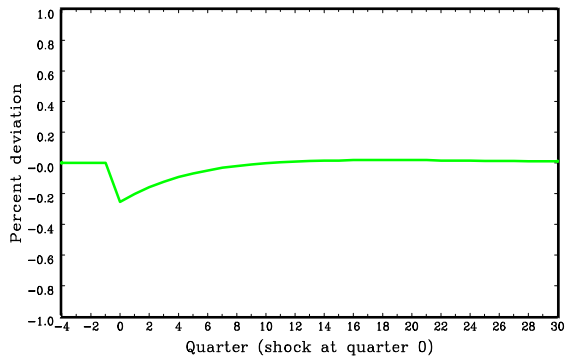
Figure 4d — Real shock: Stock index return

Figure 4e — Money shock: Dividends

Figure 4f — Real shock: Dividends