Appendix: Common Currencies vs. Monetary Independence

A The infinite horizon model

This section defines the equilibrium of the infinity horizon model described in Section III of the paper and characterizes its main properties. We start with the case of policy competition and no commitment.

A.1 Policy equilibrium with discretion: Time-consistent policies

Because we limit the set of policy strategies to be Markov, the policy equilibrium is defined by a function $\Psi(\mathbf{s})$ that returns the current interest rates (R_1, R_2) as a function of the aggregate states $\mathbf{s}$. After normalizing all the nominal variables by the domestic stock of money, the aggregate states are given by the levels of technology, the normalized stock of deposits and the input ratios chosen by firms in the previous period. Formally, $\mathbf{s} = (A_1, A_2, \kappa_1, \kappa_2, D_1, D_2)$.

The equilibrium policy rule $\Psi(\mathbf{s})$ is the solution of the repeated game played by the two countries. The strategy of country i determines the current interest rate as a function of: (i) the aggregate states $\mathbf{s}$; (ii) the other country's interest rate R_{i^*}; and (iii) a policy rule $\Psi(\mathbf{s})$ determining future interest rates. We denote the policy strategy of country i by $R_i(\mathbf{s}, R_{i^*}; \Psi)$. An equilibrium for a given policy rule $\Psi(\mathbf{s})$ is defined as:

Definition 1 (Policy equilibrium for given Ψ) *A policy equilibrium for given Ψ is defined by "current" policy rules $\psi_i(\mathbf{s}; \Psi)$ and countries' policy strategies $R_i(\mathbf{s}, R_{i^*}; \Psi)$, such that:*

(a) The policy strategies $R_i(\mathbf{s}, R_{i^*}; \Psi)$ maximize the welfare of country i;

(b) The current policy rule ψ_i and the strategies R_i satisfy:

$$\begin{aligned} \psi_1(\mathbf{s}, \Psi) &= R_1(\mathbf{s}, \psi_2(\mathbf{s}; \Psi); \Psi) \\ \psi_2(\mathbf{s}, \Psi) &= R_2(\mathbf{s}, \psi_1(\mathbf{s}; \Psi); \Psi) \end{aligned}$$

The current policy rule $\psi_i(\mathbf{s}, \Psi)$ is the solution of the policy game in the current period when future interest rates are determined by the policy rule Ψ. Consistent with the definition of a Nash equilibrium, the strategies played by each country must be the optimal responses to the strategy played by the other country (condition (a)), and the equilibrium interest rates are such that countries do not have an incentive to deviate from these strategies (condition (b)).

The above definition imposes optimality in the choices of current policies but it takes as given the function Ψ that determines future policies. We have to make sure that future policies are also the results of optimal strategies played by the two countries in future periods. We then have the following definition of a discretionary policy equilibrium:

Definition 2 (Discretionary equilibrium) *A discretionary Markov perfect equilibrium is defined as a fixed point $\Psi^*(\mathbf{s})$ of the following mapping:*

$$\Psi(\mathbf{s}) = \psi(\mathbf{s}; \Psi)$$

In other words, the policy rule Ψ determining future policies must be equal to the current policy rule ψ. When this condition is satisfied, the policy rule that agents assume to determine future policies is the solution of the policy game played by the two countries in future periods.

Proposition 1 *The Markov perfect equilibrium of the repeated policy game is the equilibrium of the two-period model defined in Section II.1.*

Proof: *Assume that future interest rates are determined by the function $\Psi(\mathbf{s}')$, where $\mathbf{s}' = (A_1', A_2', \kappa_1', \kappa_2', D_1', D_2')$. In this economy the current equilibrium does not affect the future states $\mathbf{s}' = (A_1', A_2', \kappa_1', \kappa_2', D_1', D_2')$. In fact, even though the variables κ_1', κ_2', D_1' and D_2' are chosen in the current period, they only depend on the expectation of future policies. This can be seen by inspecting the firms' problem (15) and the households' first order condition (15). Therefore, the current interest rates do not affect next period states and next period equilibrium. This implies that the policy makers solve a static problem and the optimal strategies are independent of the function Ψ determining future policies.*

The key feature of this economy is that whatever is done in the current period will not affect the future states. The new input ratios and the new stocks of bank deposits will be affected only by future policies but are not affected by the current states and current policies. Because the current policies cannot affect the next period states, they also do not influence on the next period policies. Therefore, from the perspective of the policy makers, the function that determines future policies is irrelevant for the choice of the current optimal strategies.

A.2 Equilibrium with commitment: Ramsey policies

The usual approach to the study of optimal policies with commitment in an infinite horizon setting is the Ramsey allocation. The monetary authorities choose the whole sequence of state contingent interest rates in the initial period, taking as given the sequence of state contingent interest rates chosen by the other country. Interest rates are state contingent because there are shocks.

Let $\mathbf{h}^t = \{z_{1,0}, z_{2,0}, ..., z_{1,t}, z_{2,t}\}$ be the history of shock realizations from time zero to t and let $\mathbf{H}^t$ be the collection of all possible histories. Denote by $R_i(\mathbf{h}^t)$, the sequence of state contingent interest rates in country i for all $\mathbf{h}^t \in \mathbf{H}^t$ and $t \geq 0$. A policy strategy with commitment for country i can be expressed as $r_i(\mathbf{h}^t; R_{i^*}(\mathbf{h}^t))$. Using this notation, an equilibrium with commitment is defined as:

Definition 3 (Commitment equilibrium) *An equilibrium with commitment is a sequence of state contingent interest rates $R_i(\mathbf{h}^t)$ and countries' strategies $r_i(\mathbf{s}; R_{i^*}(\mathbf{h}^t))$ such that:*

(a) The policy strategies $r_i(\mathbf{s}; R_{i^*}(\mathbf{h}^t))$ maximize welfare in country i;

(b) The state contingent interest rates $R_i(\mathbf{h}^t)$ satisfy:

$$\begin{aligned} R_1(\mathbf{h}^t) &= r_1(\mathbf{s}; R_2(\mathbf{h}^t)) \\ R_2(\mathbf{h}^t) &= r_2(\mathbf{s}; R_1(\mathbf{h}^t)) \end{aligned}$$

for all $\mathbf{h}^t \in \mathbf{H}^t$ and $t \geq 0$.

This is the standard definition of a Nash equilibrium. The only complication derives from the fact that the players' action is a sequence of functions.

Proposition 2 *The Ramsey policy equilibrium is the equilibrium under policy commitment of the two-period model defined in Section II.2.*

Proof: *Because all the interest rates are chosen in advance, firms are not committed yet to the long-term plans. Therefore, the short-term technology becomes irrelevant. The same argument used in the proof of Proposition 1 then shows that the interest rates chosen in a particular contingency do not affect future equilibria. Therefore, the equilibrium in the two-period model is also the equilibrium in the infinite horizon model.*

A.3 Equilibrium with policy coordination: Common currencies

We assume that the adoption of the common currency is accompanied by a process of financial integration. In the model this is captured by allowing firms to borrow indifferently from domestic and foreign banks. With multiple currencies, banks have relationships only with domestic firms because they lend only in the currency in which their deposits are denominated and firms borrow only in domestic currency. With a common currency, however, the nationality of the currency disappears and the interest rate in the two countries will be equalized.

The common currency also implies the unification of the monetary authorities. The new authority will maximize the weighted welfare of the households of the two countries. The relative weight used for country 1 is α.

In this new environment all households receive the same transfers from the unified monetary authority. This is unlikely to be the case, especially when one country unilaterally adopts the currency of the other country. However, the assumption that monetary policy interventions take the form of monetary transfers is made only for analytical convenience. We should think of these interventions as open market operations conducted by the monetary authority(s) with banks. In this paper we neglect possible seigniorage gains of one country over the other.

In the environment with a common currency, the equilibrium conditions in the goods markets do not change and they are still given by equations (5) and (6). The loans markets instead, become unified and conditions (23) and (24) are replaced by the new equilibrium condition:

$$P_1X_{11} + P_2X_{12} + P_2X_{22} + P_1X_{21} = D_1 + T_1 + D_2 + T_2 \tag{A.1}$$

where now P_1 and P_2 are denominated in the same currency. Although prices have the same denomination, they are not necessarily the same because the two goods are not perfect substitutes. We have the following proposition:

Proposition 3 (Optimal policy with common currencies) *With common currencies the optimal policy is the Friedman rule of a zero nominal interest rate independent of the weight assigned by the monetary authority to the two countries and independent of whether the monetary authority can commit to future policies.*

Proof: *From the cash-in-advance constraints we have* $P_1C_1 = M - D_1 - D_2 - N_2$ *and* $P_2C_2 = N_2$. *Because all the right-hand-side variables are determined at the beginning of the period, the nominal value of consumption is also determined, that is:*

$$\frac{P_2C_2}{P_1C_1} = \bar{e}\frac{C_2}{C_1} = \frac{N_2}{M - D_1 - D_2 - N_2} \tag{A.2}$$

Consumption in the two countries is still given by (18) and (19). Because there is only one interest rate, the ratio C_2/C_1 is independent of R. Moreover, given the technology levels, this ratio only depends on $\bar{e}$. Given this result, equation (A.2) uniquely determines the real exchange rate (or price ratio) $\bar{e}$ and this rate does not depend on R. Therefore, the common interest rate does not affect consumption through the real exchange rate. This implies that we can determine the effect of R on C_1 and C_2 using equations (18) and (19), keeping $\bar{e}$ constant. It can be easily verified that R has a negative effect on the consumption of both countries. Therefore, the optimal policy is to choose the minimum value of the interest rate, that is $R = 0$, independently of the welfare weights.

The intuition for this result derives from the fact that the interest rate acts as a production tax, implying that a positive value of R is inefficient. For given values of the shocks, the net production of both countries is affected negatively by a higher interest rate. Therefore, independently of the weights given to the welfare of the two countries, the policy maker will set the interest rate to zero.

B International mobility of capital

In this section we extend the infinite horizon model by allowing for international mobility of capital. In this environment households can hold financial assets (deposits) in foreign banks. To simplify the analysis, we assume that foreign deposits are always denominated in the currency of country 1. By making this assumption, we need to keep track only of the net foreign position of country 2. The net stock of foreign deposits of households in country 2 (foreign deposits if positive and foreign debt if negative) is denoted by $\tilde{d}_2$.

For households in country 1 the cash-in-advance constraint is still $P_1c_1 = n_1$. Households in country 2, however, also hold foreign currency that they retained at the end of the previous period. Denoting by $\tilde{n}_2$ the liquid funds retained in foreign currency, the cash-in-advance constraint is $P_2c_2 \leq n_2 + \tilde{n}_2/e$. Notice that $\tilde{n}_2$ derives from the previous liquidation or acquisition of foreign deposits and can be negative.[1] The beginning-of-period financial assets in country 1 are equal to the retained liquidity plus the nominal value of domestic deposits, that is, n_1+d_1. In country 2, the beginning-of-period financial assets, denominated in the currency of country 2, are given by $n_2 + d_2 + (\tilde{n}_2 + \tilde{d}_2)/e$.

The choice variables for households in country 2 now also include the deposits in foreign currency. The optimality condition for the choice of $\tilde{d}_2$ is:

$$E\left(\frac{u_c(c_2')}{P_2'}\right) = \beta E\left(\frac{(1+R_1')u_c(c_2'')P_1'\bar{e}'}{P_2'P_1''(1+g_1')\bar{e}''}\right) \qquad \text{(B.1)}$$

[1] Households in country 2 use all the foreign currency owned at the beginning of the period to buy consumption goods. Therefore, their end-of-period currency position is equal to the payments received from foreign deposits, that is, $\tilde{d}_2(1+R_1)$. Given this position, households decide the new foreign deposits $\tilde{d}_2'$. The difference is the new foreign currency position, that is, $\tilde{n}_2' = \tilde{d}_2(1+R_1) - \tilde{d}_2'$. In the next period this currency will be sold (or purchased if $\tilde{n}_2' < 0$) in the exchange rate market.

The optimality condition for domestic deposits is still (22).

As before, banks make loans only in the currency in which they receive deposits. Because firms contract loans that are denominated in domestic currency, this implies that domestic firms borrow only from domestic banks.[2]

The equilibrium conditions in the goods markets do not change. In the loan and exchange markets, instead, they are slightly different. More specifically, the equilibrium condition in the loan market of country 1 is:

$$P_1(X_{11} + \bar{e} \cdot X_{12}) = \tilde{D}_2 + D_1 + T_1 \tag{B.2}$$

where now the supply of loans from domestic banks also includes the deposits of foreign residents $\tilde{D}_2$. In country 2, instead, we still have:

$$P_2(X_{22} + X_{21}/\bar{e}) = D_2 + T_2 \tag{B.3}$$

Finally, the equilibrium condition in the exchange rate market is:

$$P_1 \cdot \bar{e} \cdot X_{12} + \tilde{N}_2 = P_1 \cdot X_{21} \tag{B.4}$$

The exchange rate market takes place at the beginning of the period after the government transfers. The supply of country 1's currency derives from the imports of country 1 (exports of country 2) and the foreign currency retained by households in country 2. The demand derives from the imports of country 2 (exports of country 1).

It can be verified that lemma 1 still holds with international mobility of capital and the interest rate is given by $R_i = \frac{1+g_i}{D_i/M_i+g_i} - 1$. Therefore, there is a unique correspondence between the domestic interest rate and the domestic growth rate of money. This implies that the specification of the monetary policy instrument in terms of money growth rate or interest rate is still equivalent.

B.1 Policy equilibrium with international mobility of capital

With international mobility of capital the set of state variables also includes $\tilde{N}_2$ and $\tilde{D}_2$. The whole states for the normalized model are $\mathbf{s} = (A_1, A_2, \kappa_1, \kappa_2, D_1, D_2, \tilde{N}_2, \tilde{D}_2)$. One important difference respect to the case of financial autarky is that now monetary policy affects the real value of the financial wealth that households own in foreign currency. In our model, households in country 2 owns deposits in country 1 (or borrows if those are negative) and they hold foreign currency from the previous liquidation of foreign deposits. The foreign currency will be converted in domestic currency and will affect

[2] We allow households to hold foreign deposits but banks can not make loans to foreign firms. Allowing banks to make loans to foreign firms would not change the results as long as the balance sheet position of the banks is covered.

the nominal exchange rate. In fact, from the equilibrium condition in the exchange rate market we have:

$$\bar{e} \cdot X_{12} + \frac{\tilde{N}_2}{P_1} = X_{21} \tag{B.5}$$

From this condition we see that the determination of the real exchange rate depends on the volume of currency owned by foreigners, $\tilde{N}_2$, and on the nominal price P_1. Therefore, the policy equilibrium will depend on the state variable $\tilde{N}_2$. However, an important property of this model is that the next period value of $\tilde{N}_2$ does not depend on the current policies R_1 and R_2. To see this notice that $\tilde{N}'_2$ is determined by the equation:

$$\tilde{N}'_2 = \tilde{D}_2(1 + R_1) - \tilde{D}'_2 \tag{B.6}$$

Although R_1 enters this equation, whatever the value of the term $\tilde{D}_2(1+R_1)$, the value of $\tilde{D}'_2$ will be chosen to satisfy the first order condition (B.1). This condition depends only on the next period states A'_1, A'_2, $\tilde{N}'_2$. Therefore, $\tilde{D}'_2$ will be chosen such that $\tilde{N}'_2$ satisfy the first order conditions independently of the value of $\tilde{D}_2(1 + R_1)$. In other words, the equilibrium policies do not depend on $\tilde{D}_2$.

If the next period states do not depend on the current policies, then the current policies can not affect the next period policies. These implies that, when we limit our analysis to Markov policy strategies, the equilibrium interest rates do not depend on the policy rule $\Psi(\mathbf{s})$ assumed to determine future interest rates. The determination of the policy fixed point $\Psi^*(\mathbf{s})$ is then trivial. This fix point is a function of $(A_1, A_2, \kappa_1, \kappa_2, \tilde{N}_2)$. Therefore, the only difference respect to the economy without mobility of capital is that the policy function also depends on $\tilde{N}_2$.

Although the policy rule Ψ takes a simple form, the international mobility of capital makes the dynamic system derived from equations (22) and (B.1), unstable. There is a stationary steady state, but a small perturbation from the steady state will put the variable $\tilde{D}_2$ in a diverging path.

One way to eliminate the non-stationarity in the financial positions of the two countries, is by assuming that there are some frictions in international financial markets. For example, we could assume that the expected return from foreign investments (foreign deposits) decreases when the international position of the country increases. This assumption can be justified with the possibility of default. We have solved the model numerically after making this assumption and found that the international mobility of capital does affect in a significant way the results of the economy with financial autarky.