A Neoclassical Model of The Phillips Curve Relation

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Abstract

This paper integrates the modern theory of unemployment with a limited participation model of money and asks whether such a framework can produce correlations like those associated with the Phillips curve as well as realistic labor market dynamics. The model incorporates both monetary and real shocks. The response of the economy to monetary policy shocks is consistent with recent evidence about the impact of these shocks on the economy.

Key words: Monetary policy; liquidity effects; job creation and job destruction

JEL Classification: E3; E5; J6

1 Introduction

The simple stylized fact that is most closely associated with the notion of a Phillips Curve is that there is a positive correlation between inflation and employment. This is one of the robust monetary features of post-war U.S. data and it holds for most economies. Many economists view this as part of the essential core of economics and an important tool for the conduct of monetary policy. We start from the premise that understanding the origins of the correlations associated with the Phillips Curve is a crucial first step to understanding the implications of that relationship for economic policy. The Phillips Curve is fundamentally an empirical relation that links labor markets and monetary policy. We explore the source of that linkage by combining the modern theory of unemployment with a modern model of monetary transmission.

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In this paper we describe a model economy with a monetary sector that is in the spirit of the limited participation model of Christiano, Eichenbaum, & Evans (1996b, 1997) where government open market operations lead to an increase in the liquidity of the economy and drive the nominal interest rate down.[1] This increase in liquidity, in turn, has significant effects on the real variables of the economy. The real sector of the economy has a very detailed labor market in the spirit of search theoretic models, as in Mortensen & Pissarides (1994), where endogenous creation and destruction of jobs can occur in response to both aggregate and firm level shocks.

Before one can talk about the Phillips Curve relationship, one must understand what drives the tremendously volatile flows of jobs and workers that characterize the labor market. Richard Rogerson (1997) argues that understanding the dynamics of the formation and dissolution of employment matches is central to understanding correlations between unemployment (or employment) and inflation. We incorporate the dynamic features of the labor market in a general equilibrium business cycle model. Business cycle fluctuations are driven by two shocks: a technology shock, affecting the productivity of a firm-job, and a monetary policy shock, which has liquidity effects. These shocks lead to employment variation on both the intensive and extensive margins. This structure also permits us to study the affect of aggregate shocks on the creation and destruction of jobs.

We evaluate this model economy quantitatively by comparing its implications for both macroeconomic and firm level observations to U.S. data. We find that it captures many of the important features of U.S. data. Real and monetary shocks induce a different correlation structure (that is correlations at different lags and leads) between employment, inflation and the price level. When the economy is simultaneously hit by both shocks, the correlations between employment and inflation and between employment and the price level generated by the model, resemble the empirical correlations for the U.S. economy.

The response of the economy to a monetary policy shock is consistent with recent views about the impact of monetary shocks on the economy. In particular, we show that an unexpected positive monetary shock identified by an increase in the growth rate of money, induces persistent higher profits, higher stock market values of the firms, higher employment, higher hours worked and higher levels of output. The model economy also replicates other important cyclical facts such as the negative correlation between inflation and stock market returns and the positive correlation between money growth and stock market returns.[2]

[1] Christiano, Eichenbaum, & Evans (1996a), Hamilton (1997), Leeper, Sims, & Zha (1996) provide empirical evidence of the liquidity effect of monetary policy.

[2] These two empirical facts are documented by Marshall (1992) who also develops a monetary model which replicates these facts. Another study which concentrates

The organization of the paper is as follows. In section 2 we describe the basic structure of the model. To reduce the complexity of the model and allow for easier analytical intuition, we assume that agents are risk neutral and labor is the only input of production. In section 8, we extend the model by assuming risk averse agents and by introducing physical capital as a second input of production, and we show that the simplifying assumptions (risk neutrality and absence of physical capital) are not crucial for the main properties of the model. In sections 3 and 4 we describe the firm and household problems and define the general equilibrium for the economy, and section 5 develops the analytical intuition about how monetary policy shocks affect the economy. The calibration of the model is discussed in section 6, section 7 presents the main findings and section 9 concludes.

2 The economy

2.1 The monetary authority and the intermediation sector

We begin by assuming that the monetary authority controls the supply of liquidity (money) available for transactions by conducting open market operations, that is, by purchasing and selling government bonds. We assume that the total nominal stock of public debt is constant. Part of this stock is owned by the monetary authority and part is owned by the financial intermediaries. The nominal value of public debt or government bonds owned by the financial intermediaries is denoted by B. Transactions in government bonds take place between the monetary authority and the financial intermediaries (banks). For simplicity we assume that the interest paid on bonds owned by the private sector (banks) is financed with non-distorting taxes.

The quantity of liquid funds M available in the economy is constant. Part of these funds are held by households for transactions and the remainder is deposited with financial intermediaries. Financial intermediaries collect these deposits from households and use the funds to buy government bonds and to make loans to firms. Consequently, in each period, an amount $M - D$ of money—where D denotes the aggregate stock of nominal deposits—is available to the households for transaction and an amount $D - B$ is available to the firms. The sum of these two stocks gives the total amount of money used for transactions, that is, $M - B$.

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on the negative correlation between inflation and stock market returns is Jovanovic & Ueda (1998). They develop a monetary principal-agent model with optimal labor contracts which explains this negative correlation.

Because we assume that M is constant, the monetary authority is able to modify the stock of money used for transactions by changing the stock of public debt owned by the intermediaries with open market operations. When the monetary authority purchases public bonds from the financial intermediaries, the quantity of loanable funds $D - B$ available to the intermediation sector increases (for a given stock of deposits D), and this has the potential to drive the interest rate down. [3]

To insure that open market operations change the supply of loanable funds, we need to impose some rigidity in the ability of the agents to adjust their stock of deposits. We assume that agents are able to change their stock of deposits at any moment but there is a readjustment cost associated with doing so. We denote this cost by $\tau(d, d')$ where d is the previous holding and d' the new chosen stock. The adjustment cost is continuously differentiable in both arguments and convex in the absolute change of the initial stock. We also assume that $\tau(d, d) = \tau_1(d, d) = \tau_2(d, d) = 0$, that is, the cost and the partial derivatives are zero when the $d = d'$. [4]

Our framework differs from the standard limited participation model where cash holdings cannot be changed in the current period but are perfectly flexible in the following period. In this paper we assume that households can adjust their stock of deposits at any moment, including the current period, but there are adjustment costs associated with doing so. The advantage of this approach over the standard limited participation model is that liquidity effects of monetary shocks are more persistent even though they may be smaller in the current period. Under the assumptions of the standard limited participation model, even if the transfers from the monetary authority are persistent, the greater availability of funds in subsequent periods will be mostly compensated for by a reduction in the stock of deposits owned by households. With adjustment costs, households do not completely adjust their nominal stock of deposits in the following period either, and this induces a more persistent effect of monetary policy shocks. As we will see, this persistence plays an important role on how monetary policy shocks affect employment.

The monetary authority controls the growth rate of the aggregate stock of money $M - B$ with open market operations. Monetary policy shocks are in-

[3] The competitiveness of the intermediation sector insures that the interest rate on deposits is equal to the interest rate on loans which in turn is equal to the interest rate on bonds.

[4] This cost can be justified by penalties that the intermediary charges on earlier withdrawals and by a lower interest rate earned in the first period in which new deposits are made. In turn, the charged penalty and the lower interest rate paid, are justified by costs that the intermediary faces when it readjusts its portfolio of loans. In this model deposits should not be interpreted as checking deposits but rather as less liquid deposits that earn a higher interest rate.

novations to the targeted growth rate g. We formalize the monetary policy rule with the process $\log(1+g') = \rho_g \log(1+g) + \epsilon'_g$, where the prime denotes the next period values and ϵ'_g is the monetary policy shock. Implicit in this specification of the policy rule is that the nominal stocks—specifically M, B and D—do not display any long-run trends. Extending the model to allow for upward trends in the nominal stocks is trivial but it would increase the notational complexity of the model without changing the results of the paper.

2.2 The household sector

There is a continuum of agents that maximize the expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t, \chi_t), \qquad u(c_t, \ell_t, \chi_t) = c_t + (1-\chi_t)a - \chi_t \frac{\ell_t^{\gamma}}{\gamma} \tag{1}$$

where c is consumption of market produced goods, ℓ is the time spent working, χ is an indicator function taking the value of one if the agent is employed and zero if unemployed, and a is homework production and consumption of an unemployed agent. In order to work, the agent needs to be employed and, if unemployed, the agent needs to search for a job. There is no cost to searching for a job and the probability of finding a job depends on the matching technology that will be described below.

Agents own three types of assets: cash, nominal deposits and firms' shares. In each period, agents are subject to the following cash-in-advance constraint:

$$p(c + \tau + i) \leq m - d' + \chi pw \tag{2}$$

and budget constraint:

$$p(c + \tau + i) + m' = m + rd' + \chi pw + pn\bar{\pi} - rB, \tag{3}$$

where primes denote the next period value. The variable p is the nominal price, i is the household's investment in the purchase of the shares of new firms, and τ is the cost of readjusting the nominal portfolio of deposits. The variable n identifies the number of firms' shares that the household owns. The average per-share dividend paid by these firms is $\bar{\pi}$. The wage received by an employed worker, denoted by w, is paid at the beginning of the period with cash and it enters the cash-in-advance constraint. The determination of the wage will be specified below. Finally, rB are the taxes that the household pays.

2.3 The production sector

The production technology displays constant returns to scale with respect to the number of employees. Without loss of generality, it is convenient to assume that there is a single firm or plant for each worker. The search for a worker involves a fixed cost κ and the probability of finding a worker depends on the matching technology $\Psi(V,U)$ where V is the number of vacancies (number of firms searching for a worker) and U is the number of workers searching for a job. The matching technology assumes the form $\Psi(V,U) = \mu V^{\alpha}U^{\zeta}$, with $\alpha, \zeta > 0$ and $\alpha + \zeta \leq 1$. Therefore, the matching technology is strictly increasing and concave in V and U, and for the moment we do not restrict the function to be homogeneous of degree one. The probability that a searching firm finds a worker is denoted by q and it is equal to $\Psi(V,U)/V$, while the probability that an unemployed worker finds a job is denoted by h and is equal to $\Psi(V,U)/U$.

If the search process is successful, the firm operates the technology $y = A\ell^{\nu} - \varphi$, where A is the aggregate level of technology, ℓ is the working time of the worker, and φ is the cost of an intermediate input or the non-labor cost of production. The aggregate technology level A is equal to $\bar{A}e^{z}$ where z is an aggregate technology shock which follows the first order autoregressive process $z' = \rho_z z + \epsilon_z'$. The cost φ is idiosyncratic to the firm and is assumed to be independently and identically distributed across firms and times with distribution function $F : [0,\infty] \rightarrow [0,1]$.[5] The cost of the intermediate input is observed before the firm rents capital and starts production. If the realization of φ is sufficiently high, the firm (and the worker) may prefer to discontinue the match and shut down rather than pay the cost. The value of φ above which the firm decides to shut down is denoted by $\bar{\varphi}(s)$ and it is a function of the state of the economy s. Therefore, the probability of job separation is equal to $1 - F(\bar{\varphi}(s))$, and it depends on the aggregate state of the economy s.

The contract signed between the firm and the worker specifies the working time ℓ and the wage w so that the worker gets a share η of the surplus generated by the match. The assumption of a constant sharing fraction of the surplus is standard in these types of models and it is motivated theoretically by assuming a Nash bargaining process between the firm and the worker where η denotes the bargaining power of the worker relative to the firm. At each point in time the matching surplus depends on the state of the economy s and on the firm specific shock φ. Therefore, the wage is also a function of these variables and we will denote it by $w(s,\varphi)$.

[5] By assuming that the idiosyncratic shock is in the form of an intermediate cost, rather than multiplicative to the production function, we avoid the problem of having excessive differences in the cross sectional distribution of hours worked.

Wages have to be paid in advance with money. Firms finance these advance payments by borrowing from a financial intermediary at the nominal interest rate r.

3 The firm's problem

In this economy firms post vacancies and implement optimal production plans so as to maximize the welfare of their shareholders. Denote with $J(s, \varphi)$ the value of a match for the firm measured in terms of current consumption. This is given by:

$$J(s, \varphi) = \tilde{\pi}(s, \varphi) + \beta E \int^{\bar{\varphi}(s')} J(s', \varphi') dF(\varphi') \tag{4}$$

The function $\tilde{\pi}(s, \varphi)$ is defined as $E[\beta p(s)\pi(s, \varphi)/p(s')]$, where $\pi(s, \varphi)$ are the dividends paid by the firm to the shareholders at the end of the period. The function expresses the current value for the shareholder of the dividend paid by the firm. Because dividends are paid in cash at the end of the period, the agent needs to wait until the next period to transform this cash into consumption. This implies that the current value in terms of consumption of one unit of cash received at the end of the period is equal to $\beta p(s)/p(s')$.

The dividends paid to the shareholders are equal to the output produced by the firm minus the cost for the intermediate input φ and the labor cost $w(1+r)$:

$$\pi(s, \varphi) = A\ell^{\nu} - \varphi - w(1 + r) \tag{5}$$

Notice that the labor cost is given by the wage plus the interest paid on the loan used to finance the advance payment of the wage. The determination of the wage w and hours worked ℓ will be specified below.

Given $J(s)$, the value of a vacancy $Q(s)$ is defined as:

$$Q(s) = -\kappa + q(s)\beta E \int^{\bar{\varphi}(s')} J(s', \varphi') dF(\varphi') + (1 - q(s))\beta E Q(s') \tag{6}$$

Because the value of a vacancy must be zero in equilibrium, that is, $Q(s) = Q(s') = 0$, equation (6) becomes:

$$\kappa = q(s)\beta E \int^{\bar{\varphi}(s')} J(s', \varphi') dF(\varphi') \tag{7}$$

Equation (7) is the arbitrage condition for the posting of new vacancies, and accordingly, for the creation of new jobs. It simply says that, in equilibrium, the cost of posting a vacancy, κ, is equal to the discounted expected return from posting the vacancy.

Consider now the value of a match for a worker. Define $W(s,\varphi)$ and $U(s,\varphi)$ to be, respectively, the value of a match and the value of being unemployed in terms of current consumption. They are defined as:

$$W(s,\varphi) = w(s,\varphi) - \frac{\ell^{\gamma}}{\gamma} + \beta E \int^{\bar{\varphi}(s')} [W(s',\varphi') - U(s')]dF(\varphi') + \beta EU(s') \tag{8}$$

$$U(s) = a + h(s)\beta E \int^{\bar{\varphi}(s')} [W(s',\varphi') - U(s')]dF(\varphi') + \beta EU(s') \tag{9}$$

where a is home production and consumption of an unemployed worker. Notice that we have defined the value in terms of consumption of being employed, net of the disutility from working ℓ^{γ}/γ. Adding equations (4) and (8), and subtracting equation (9), gives the total surplus generated by the match, that we denote by $S(s,\varphi)$. The surplus is shared between the worker and the firm according to the fixed proportion η, that is, $W(s,\varphi) - U(s) = \eta S(s,\varphi)$ and $J(s,\varphi) = (1-\eta)S(s,\varphi)$. Using this sharing rule and equation (7), the surplus of the match can be written as:

$$S(s,\varphi) = \tilde{\pi}(s,\varphi) + w(s,\varphi) - a - \frac{\ell^{\gamma}}{\gamma} + \frac{(1-\eta h(s))\kappa}{(1-\eta)q(s)} \tag{10}$$

Moreover, by equating $W(s,\varphi) - U(s)$ to $\eta S(s,\varphi)$, and using (5), we derive the wage $w(s,\varphi)$ which is equal to:

$$w(s,\varphi) = \frac{\eta(A\ell^{\nu} - \varphi)E\left(\frac{\beta p(s)}{p(s')}\right) + (1-\eta)\left(a + \frac{l^{\gamma}}{\gamma}\right) + \frac{\eta h(s)\kappa}{q(s)}}{\left[1 - \eta + \eta(1+r)E\left(\frac{\beta p(s)}{p(s')}\right)\right]} \tag{11}$$

The wage $w(s,\varphi)$, as well as the surplus generated by the match, depend on the labor input ℓ. Because the firm and the worker are splitting the surplus, the optimal input of labor is determined by maximizing this surplus. Based on this principle of optimality, we have:

Proposition 1 *The optimal input of labor is given by:*

$$\ell(s) = \left(\frac{\nu A}{1+r}\right)^{\frac{1}{\gamma-\nu}} \tag{12}$$

PROOF. By differentiating the surplus in equation (10) after eliminating w using equation (11), we get (12).

According to equation (12), the labor input, and therefore, firm's output, is decreasing in the nominal interest rate r. This is because the interest rate increases the marginal cost of labor. This has important implications for the impact of monetary policy shocks on real activities.

A successful match is endogenously discontinued when the realization of the shock makes the value of the surplus zero or negative, and the condition $S(s, \bar{\varphi}) = 0$ implicitly defines the upper bound shock $\bar{\varphi}(s)$.

Using equations (7) and (4) we derive:

$$\frac{\kappa}{q(s)} = \beta E \int^{\bar{\varphi}(s')} \tilde{\pi}(s', \varphi') dF(\varphi') + \beta E \left(\frac{\kappa F(\bar{\varphi}(s'))}{q(s')} \right) \tag{13}$$

where as before, $\tilde{\pi}(s, \varphi)$ is the value in terms of current consumption of dividends distributed by the firm at the end of the period. Using forward substitution and the law of iterated expectations, we obtain:

$$\frac{\kappa}{q(s_t)} = \beta E_t \sum_{j=1}^{\infty} \left(\prod_{i=1}^{j-1} \beta F(\bar{\varphi}(s_{t+i})) \right) \int^{\bar{\varphi}(s_{t+j})} \tilde{\pi}(s_{t+j}, \varphi) dF(\varphi) \tag{14}$$

From this equation we see that an increase in the expected sum of future dividends (properly discounted) induces a reduction in the current value of q. If separation was exogenous, this would imply an increase in the number of vacancies which will increase the next period employment. With endogenous separation, however, the impact on the employment rate is more complex given that the decrease in q could be driven by an increase in the number of searchers if the rate of job separation increases. To prevent this it is sufficient that the current rate of separation does not increase following the increase in the right-hand-side of equation (14). For example, if the increase in the discounted sum of future dividends is driven by the persistence of a good shock in the current period, then it is likely that the fall in q is associated with a fall in the separation rate.

4 The household's problem and general equilibrium

In this section we describe the household problem written in recursive form after normalizing all nominal variables by M. The aggregate states of the economy are the technology shock z, the growth rate of money g, the stock of government bonds B owned by the intermediaries, the stock of nominal deposits D, and the number of workers N that at the beginning of the period are matched with a firm. The individual states are the occupational status χ, the stock of liquid assets m, the stock of nominal deposits d, and the number of shares n owned by the household. We will denote the set of individual states with $\hat{s} = (\chi, m, d, n)$. Denoting with $\Omega(s, \hat{s})$ the household's value function, the household's problem is:

$$\Omega(s,\hat{s}) = \max_{m',d',v,c} \left\{ c + (1-\chi)a - \chi \frac{\ell^{\gamma}}{\gamma} + \beta \, E \, \Omega(s', \hat{s}') \right\} \tag{15}$$

subject to

$$c = \frac{m - d'}{p} + \chi w - \tau(d, d') - v\kappa \tag{16}$$

$$m' = (1+r)d' + p\, n \, \bar{\pi} - rB \tag{17}$$

$$n' = nF(\bar{\varphi}) + vq \tag{18}$$

$$\bar{\pi} = \int^{\bar{\varphi}(s)} \pi(s, \varphi) dF(\varphi) \tag{19}$$

$$s' = H(s) \tag{20}$$

The variable n denotes the number of shares of firms matched to a worker that are owned by the household and v the purchase of new firm shares or vacancies. Only a fraction $F(\bar{\varphi})$ of old firms survive to the next period and only a fraction q of new firms (vacancies) will find a worker. We assume that households own a portfolio representative of the market, and therefore, the dividend payment $\bar{\pi}$, as well as the the next period portfolio of firm shares, do not depend on the idiosyncratic risk of each firm. The function H in equation (20) defines the law of motion for the aggregate states s.

In equilibrium we have that households are indifferent in the allocation of cash between consumption and the purchase of firms' shares. This result derives from the assumption that the utility function is linear in consumption. Consequently we have an infinite number of equilibria corresponding to different distributions of firms' shares among households. Because the aggregate behavior of the economy is independent of this distribution, we concentrate on a particular equilibrium. This is the equilibrium in which all agents make the same portfolio choices of deposits and shares of firms. This implies that differences in earned wages between agents give rise to different consumption levels rather than differences in asset holdings. We refer to this equilibrium as the *symmetric equilibrium*. We then have the following definition:

Definition 2 (Symmetric recursive equilibrium) *A symmetric recursive competitive equilibrium is defined as a set of functions for (i) household decisions $m'(s,\hat{s})$, $d'(s,\hat{s})$, $v(s,\hat{s})$; (ii) labor input $l(s)$, wage $w(s,\varphi)$ and exit decision $\bar{\varphi}(s)$; (iii) aggregate deposits $D(s)$, banks' holding of government bonds $B(s)$, loans $L(s)$ and employment $N(s)$; (iv) interest rate $r(s)$ and nominal price $p(s)$; (v) law of motion $H(s)$. Such that: (i) the household's decisions are the optimal solutions to the household's problem (15); (ii) the labor input and exit condition maximize the surplus of the match and the wage is such that the worker obtains a fraction η of that surplus; (iii) the market for loans clears, that is $D(s) - B(s) = L(s)$, and $r(s)$ is the equilibrium interest rate; (iv) the law of motion of aggregate states $H(s)$ is consistent with the individual decisions of households and firms; (v) all agents choose the same holdings of deposits and firm shares (symmetry).*

After substituting the cash-in-advance constraint and the budget constraint in the household's utility, the household's problem reduces to the choice of the variables d' and v. Differentiating with respect to d' we get:

$$1 = (1+r)E\left(\frac{\beta p}{p'}\right) - p\left[\tau_2(d,d') + \beta E\tau_1(d',d'')\right] \tag{21}$$

This equation is complicated by the presence of the adjustment cost. Without this cost the equation would reduce to $1 = \beta(1+r)E(p/p')$ which is the usual Euler equation. This equation will also hold with adjustment costs in a steady state equilibrium because we are assuming that $\tau_1 = \tau_2 = 0$ when $d' = d$. The first order conditions with respect to the number of new firm shares v is:

$$\frac{\kappa}{q} = \beta\, E\left(\frac{\beta p'\bar{\pi}'}{p''}\right) + \beta\, E\left(\frac{\kappa F(\bar{\varphi}')}{q'}\right) \tag{22}$$

which is the same equation we found before in (13).

4.1 Steady state equilibrium

In a steady state equilibrium all variables are constant. The steady state interest rate can be derived from equation (21) and it is given by $r = 1/\beta - 1$. Once we know the interest rate r, we are able to determine the steady state labor input ℓ from equation (12). Then the steady state equilibrium can easily be characterized using the following system of six equations in the six unknowns

$V, U, N, D, p, \bar{\varphi}$. All nominal variables are normalized by M.

$$\frac{\kappa}{q(V,U)} = \beta^2 \int^{\bar{\varphi}} \pi(\varphi) dF(\varphi) + \frac{\beta \kappa F(\bar{\varphi})}{q(V,U)} \tag{23}$$

$$\beta(\bar{A}\ell^{\nu} - \bar{\varphi}) - a - \frac{l^{\gamma}}{\gamma} + \frac{\kappa(1 - \eta h(V,U))}{(1-\eta) q(V,U)} = 0 \tag{24}$$

$$pN \int^{\bar{\varphi}} (\bar{A}\ell^{\nu} - \varphi) dF(\varphi) = (1 - D) + pN \int^{\bar{\varphi}} w(\varphi) dF(\varphi) \tag{25}$$

$$pN \int^{\bar{\varphi}} w(\varphi) dF(\varphi) = D - B \tag{26}$$

$$h(V,U)U = (1 - F(\bar{\varphi}))N \tag{27}$$

$$U = 1 - NF(\bar{\varphi}) \tag{28}$$

Equation (23) is derived from the first order condition of the household (22). Equation (24) is the exit condition $S(\bar{\varphi}) = 0$. Equation (25) is the aggregate cash-in-advance constraint for the households and (26) is the equilibrium condition in the market for loans. Equation (27) is the flow of workers in and out of employment in the steady state equilibrium. The wage $w(\varphi)$ is derived from equation (11) which in the steady state becomes $w(\varphi) = \beta\eta(\bar{A}\ell^{\nu} - \varphi) + (1-\eta)(a + \ell^{\gamma}/\gamma) + \eta h \kappa / q$.

5 The impact of monetary and real shocks

To help develop some analytical intuition about how a monetary policy shock affects the economy, we consider here a simplified version of the model in which the idiosyncratic shock always assumes the mean value. This eliminates the possibility of endogenous exit, that we replace with the exogenous separation rate λ. In addition, we also assume that firms get all the surplus of the match, that is, $\eta = 0$. Under those conditions equation (14) becomes:

$$\frac{\kappa}{q(s_t)} = \beta E_t \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} \left(\frac{\beta p(s_{t+j})}{p(s_{t+j+1})} \right) \pi(s_{t+j}) \tag{29}$$

With $\eta = 0$, the dividend distributed by the firm is equal to:

$$\pi(s) = \left(1 - \frac{\nu}{\gamma}\right) A \left(\frac{\nu A}{1+r}\right)^{\frac{\nu}{\nu-\gamma}} - a(1+r) - \varphi \tag{30}$$

where now φ is a constant.

First, we observe that according to equation (29), an increase in the expected sum of future dividends, weighted by the factor $\beta p_t/p_{t+1}$, is associated with an increase in the factor κ/q_t, that is a decrease in the probability that a vacancy is successfully matched. Given the specification of the matching function, this requires an increase in the number of vacancies, and therefore, an increase in the number of employed workers next period.

Monetary policy shocks affect the employment rate through this mechanism. From equation (30) we observe that the firm dividend is decreasing in r, and a positive monetary shock which reduces the nominal interest rate has the effect of increasing the dividends distributed by the firm. Moreover, if the fall in the interest rate is persistent, it also increases future dividends. It is in this respect that the presence of rigidness in the households' ability to adjust their portfolio plays an important role, because it generates a persistent fall in the nominal interest rate. For that reason our approach carries some advantages over the standard limited participation model in which the interest fall is only temporary. Consequently, if we neglect the impact of monetary policy shocks on future rates of inflation, then a persistent shock increases future expected dividends and increases employment. However, this impact is not unambiguous given that monetary policy shocks, that is changes in r, also affect the factor $\beta p(s_{t+j})/p(s_{t+j+1})$ as can be seen from equation (21). However, assuming that changes in this factor are not too important, then a reduction in the nominal interest rate induces an increase in the number of vacancies and a reduction in the next period's unemployment rate.

A monetary shock also affects the value of the stock market. The equilibrium condition for the posting of new jobs is $\kappa = q(s)\beta EJ(s')$. (See equation (7) under the assumption of exogenous exit). Because a positive monetary shock causes a reduction in q, the expected next period value of a firm, given by the term $EJ(s')$, must increase. This in turn implies that the current value of an existing firm must also increase, leading to a positive correlation between money growth and stock market returns.

A similar mechanism works with aggregate technology shocks: as can be seen from equation (30), a persistent shock to the technology increases current and future dividends. Assuming that the induced changes in the nominal interest rate and in the price level are not too important, this will induce an increase in the number of employed workers and in the stock market value.

When we consider the general model with endogenous exit and a positive share of the surplus for the worker, the impact of monetary policy shocks and real shocks is more complex. Nevertheless, the numerical solution of the model shows that the mechanism works as described here.

6 Calibration

In this section we discuss the calibration of the model's parameters. We fix the discount factor at $\beta = 0.98$. This implies an interest rate of approximately 2 percent per quarter. Given that in our economy there is no growth in nominal variables, the steady state nominal interest rate is equal to the steady state real interest rate. The other parameter of the utility function is the working disutility parameter γ. We assume that the disutility function is quadratic, and therefore, we set $\gamma = 2$. The home production a is assumed to be zero.

The matching technology is characterized by three parameters: μ, α and ζ. In the baseline model we take $\alpha = 0.6$ and $\zeta = 0.4$, which are consistent with the estimates of Blanchard & Diamond (1989). Then, after imposing steady state values of $q = 0.7$ and $h = 0.6$, we are able to determine μ as well as the implied steady state vacancy-unemployment ratio. The value of q is similar to the value used by Den-Haan, Ramey, & Watson (1997) and the value of h implies an average duration of unemployment of 1.67 as reported by Cole & Rogerson (1996). Although in the baseline model we assume that the matching function is linearly homogeneous in V and U, we will also consider alternative calibrations.

Regarding the sharing parameter η, we try different values and we report the sensitivity of the results to this parameter. As we will see, this parameter is important for the volatility of employment but not for the shape of the response of employment to shocks.

The production function is characterized by two parameters: the scale parameter ν and the technology level $\bar{A}$ (in addition to the stochastic properties of the aggregate and idiosyncratic shocks). It is reasonable to assume that if a worker works longer at the same intensity, his or her production increases proportionally. Therefore, we set $\nu = 1$. Then, given the steady state interest rate r, the parameter $\bar{A}$ is determined by imposing the condition that each employed worker spends, on average, one third of the available time working (see equation (12)).

For analytical simplicity, we assume that the intermediate cost φ has a distribution function which is exponential, that is, $\varphi \sim e^{-\varphi/\theta}/\theta$. The parameter θ is determined jointly with the parameter κ by imposing that the steady state unemployment rate equals 6 percent and the arbitrage condition for the creation of new vacancies is satisfied. Notice that we define the unemployment rate as the number of workers that at the beginning of the period are not matched with a firm, that is, $1 - N$. This is different from the number of searching workers, which is equal to $1 - NF(\bar{\varphi})$, because some of the matched workers discontinue the match and search for a new job in the same period.

The ratio of the stock of public debt to aggregate final output is assumed to be 0.5. This value, however, does not affect the properties of the economy.

The growth rate of money follows the process $\log(1+g') = \rho_g \log(1+g) + \epsilon'_g$, with $\epsilon_g \sim N(0, \sigma_g^2)$. The parameter values are $\rho_g = 0.49$ and $\sigma_g = 0.00623$, which are the values used by Cooley & Hansen (1989).

The adjustment cost function is specified as $\tau(d, d') = \phi \cdot ((d' - d)/d)^2$ and the value of the parameter ϕ is determined to obtain the desired volatility of the nominal interest rate: The higher ϕ is, the higher the volatility of the interest rate. The value chosen for the baseline model is $\phi = 3$.

Finally, the technology shock, z, follows the first order autoregressive process $z' = \rho_z z + \epsilon'_z$, with $\epsilon_z \sim N(0, \sigma_z^2)$. The parameter ρ_z is assigned the value 0.95, which is in the order of values commonly used in business cycle studies. The parameter σ_z, instead, is set so that the standard deviation of output is similar to that observed in U.S. data. Of course, the volatility of output is not a dimension along which we evaluate the performance of the model. In the baseline model $\sigma_z = 0.0033$.

7 Findings

Figures 1a-1h show the response of several variables to a monetary shock (increase in the growth rate of money), and Figures 2a-2h show the responses to a real shock (increase in the aggregate technology level). A monetary shock increases the liquidity in the economy and causes a persistent decrease in the nominal interest rate. As a consequence of the fall in the interest rate, the number of hours worked and employment increase. Of particular interest is the response of job flows: after a fall in the interest rate, there is an increase in job creation and a decrease in job destruction. The fall in job destruction is greater than the increase in job creation. Moreover, while the decrease in job destruction persists for several periods, job creation falls below the steady state level after the first period. Therefore, most of the increase in employment is due to the response of job destruction, rather than job creation. Figure 1e reports the responses of advertised vacancies, the job finding rate h, and the probability q that a vacancy is successfully filled. The ratio between h and q is equal to the ration between vacancies and the number of searching workers. As can be seen from this figure, a positive shock to the growth rate of money induces an increase in the number of posted vacancies and in the finding rate h, and a decrease in probability q. Finally, as shown by figures 1g and 1h, the monetary shock also induces an increase in the price level and generates inflation.

Fig. 1. Impulse response of the economy to a monetary shock.

Fig. 2. Impulse response of the economy to a technology (real) shock.

The response of the economy to a real shock is qualitatively similar for several variables. As shown in Figures 2a-2h, output, hours, employment, and the ratio between the number of vacancies and the number of searching workers increase. Also, the increase in job creation is dominated by a decrease in job destruction. However, the responses of prices and inflation are different: while a monetary shock drives prices up, a real shock is deflationary. From a quantitative point of view, the impact of a technology shock on the real variables is more important than the impact of a monetary shock. From this observation we can anticipate that the employment rate is negatively correlated with the price level. In the absence of the technology shock, however, the price level would be positively correlated with employment.

Table 1 reports standard correlations at different lags and leads of employment, job creation and destruction with inflation and nominal prices, as generated by the calibrated economy and for the U.S. economy. These statistics are for the economy with a low value of the surplus share $\eta = 0.01$. The sharing parameter is not important for the statistics of Table 1, so we don't report these statistics for alternative values of η. The sharing parameter, however, is important for some of the statistics of Table 3 where we report these statistics for different values of η.

Table 1 shows that the model successfully replicates the positive correlation of employment with the current inflation rate, and the negative correlation with the price level. In addition the model also captures the positive correlation of employment with future inflation. Therefore, the model replicates a version of the Phillips curve relation in the sense of generating a positive contemporaneous correlation between inflation and employment.

Regarding the correlations of inflation and job flows, we observe that the model predicts that inflation has a contemporaneous negative impact on both

Table 1
Correlation with inflation and prices at different lags and leads.

	Correlation of current employment with						
	t-3	t-2	t-1	t	t+1	t+2	t+3
Model Economy							
Inflation	-0.24	-0.25	-0.15	0.22	0.29	0.28	0.23
Price index	-0.42	-0.57	-0.66	-0.53	-0.36	-0.19	-0.06
U.S. Economy							
Inflation	0.08	0.21	0.38	0.51	0.51	0.48	0.44
Price index	-0.72	-0.65	-0.50	-0.30	-0.10	0.10	0.27

	Correlation of current job creation with						
	t-3	t-2	t-1	t	t+1	t+2	t+3
Model Economy							
Inflation	0.20	0.08	-0.34	-0.27	-0.22	-0.17	-0.13
Price index	0.53	0.58	0.38	0.22	0.09	-0.01	-0.09
U.S. Economy							
Inflation	-0.71	-0.28	-0.34	-0.14	-0.12	-0.12	0.06
Price index	0.13	0.02	-0.12	-0.17	-0.22	-0.27	-0.25

	Correlation of current job destruction with						
	t-3	t-2	t-1	t	t+1	t+2	t+3
Model Economy							
Inflation	0.18	0.23	0.29	-0.14	-0.24	-0.24	-0.21
Price index	0.27	0.41	0.59	0.50	0.36	0.21	0.09
U.S. Economy							
Inflation	0.39	0.30	-0.10	-0.12	-0.09	-0.11	-0.45
Price index	0.46	0.57	0.56	0.53	0.49	0.45	0.27

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 200 periods and repeating the simulation 100 times. The statistics are averages over these 100 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4.

job creation and destruction as in the data. However, the model does not generate the negative correlation between the price level and job creation.

Table 2 reports the cross correlations between employment, job creation and job destruction. If we compare the statistics of the model as shown in the first section of the table (model economy A) with the statistics computed from the data collected by Davis, Haltiwanger, & Schuh (1996), we observe that the model does not generate a negative correlation between job creation and destruction. The model fails along this dimension because it does not generate a persistent increase in job creation after a positive shock. This is because the

Table 2
Cross-correlation of employment, job creation and job destruction.

	Correlation at lags and leads k						
	t-3	t-2	t-1	t	t+1	t+2	t+3
Model Economy A							
corr(Cre_{t+k}, Emp_t)	-0.06	-0.23	-0.44	-0.69	-0.96	-0.85	-0.62
corr(Des_{t+k}, Emp_t)	-0.40	-0.63	-0.85	-0.95	-0.65	-0.40	-0.20
corr(Cre_{t+k}, Des_t)	-0.03	0.11	0.28	0.50	0.92	0.87	0.66
Model Economy B							
corr(Cre_{t+k}, Emp_t)	0.18	0.05	-0.13	-0.36	-0.67	-0.79	-0.77
corr(Des_{t+k}, Emp_t)	-0.68	-0.77	-0.78	-0.66	-0.35	-0.11	0.07
corr(Cre_{t+k}, Des_t)	-0.30	-0.30	-0.30	-0.29	0.21	0.49	0.60
U.S. Economy							
corr(Cre_{t+k}, Emp_t)	0.27	0.15	0.04	-0.19	-0.58	-0.68	-0.60
corr(Des_{t+k}, Emp_t)	-0.63	-0.65	-0.59	-0.35	-0.01	0.29	0.45
corr(Cre_{t+k}, Des_t)	-0.39	-0.44	-0.47	-0.43	-0.14	0.18	0.34

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 200 periods and repeating the simulation 100 times. The statistics are averages over these 100 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4.

high probability of finding a job and the decrease in the number of jobs that are destroyed, reduces the pool of workers searching for a job, and therefore, reduces the probability that an advertised job is successfully filled. (See figures 1e and 2e) Given the fall in the probability of filling a vacancy, firms drastically decrease the number of posted jobs after the first period.[6]

To allow for a more persistent response of job creation, it is necessary to reduce the dependence of the probability of finding a worker on the number of searchers. This can be accomplished by assuming a matching function in which the coefficient ζ is relatively small. The second section of Table 2 (model economy B) is constructed with $\alpha = 0.6$ and $\zeta = 0.1$. For this model we also readjust the standard deviation of the technology shock so that the model generates a similar volatility of output. Because the volatility of output increases when we reduce ζ, in order to maintain the same output volatility we reduce σ_z from 0.0033 to 0.002. As can be seen from the second section of table 2, the correlation between job creation and job destruction is now negative and the correlation structure between job flows and employment is closer to the empirical correlations.[7] The smaller value of ζ induces a more persistent re-

[6] This feature of the Mortensen and Pissarides framework is discussed in considerable detail by Cole & Rogerson (1996).

[7] As with Table 1, the statistics reported in Table 2 do not depend crucially on the sharing parameter η.

Fig. 3. Impulse response of the economy to monetary and real shocks when $\zeta = 0.1$.

sponse of job creation as shown by figures 3d and 3h which report the impulse responses to monetary and real shocks when the matching function displays decreasing returns to scale. With this new parameterization, the response of job creation goes in the opposite direction of job destruction for more than one period. As shown by figures 3c and 3g this is because the lower sensitivity of q on the number of searching workers induces a more persistent response of the number of new vacancies.

How can we justify a low value for ζ when the existing empirical studies seem to have found values on the order of 0.4? The fact is that our model does not capture the effect of changes in the labor force participation on the probability that an advertised job is filled, by changing the pool of searchers in the market. In the model, when the number of employed workers increases, the number of workers searching for a job necessarily decreases. This reduces the probability that an advertised job is filled. In reality, it is possible that an increase in the number of employed workers does not give rise to a decrease in the number of searchers (or at least there is not a one-to-one decrease), if the labor force participation responds positively to an improvement in the labor market. This is particularly important when the labor force participation responds with some lag. Because in the model the labor force participation rate is constant and equal to 1, one way to take into account this effect is by assuming a small value of ζ. Whether or not the increase in labor force participation required to justify this amount of decreasing returns is plausible is an open question. But the essential point is that, if the reduction in the number of searchers has a large impact on the probability that a vacancy is successfully filled, it is impossible in this simple model to generate a negative correlation between job creation and destruction. In order to generate this negative correlation, either the decrease in the number of searchers has a small impact on q, or the increase in the number of employed workers is not compensated by a one-to-one reduction in the number of unemployed workers due to the expansion of the labor force.

Table 3 reports standard deviations of different variables and some selected correlations. In that table two different versions of the model economy are considered. We first consider the baseline model with a low value of η. For this model we observe that job creation is not significantly more volatile than job destruction and this is a weakness of the model. At the same time, how-

Table 3
Business cycle properties of **Model Economy A** ($\zeta = 0.4$) and U.S. economy.

	$\eta = 0.01$			$\eta = 0.10$			U.S.
	M&R	Money	Real	M&R	Money	Real	Economy
Standard deviations							
Output	1.60	0.40	1.57	1.43	0.29	1.41	1.60
Hours	0.45	0.17	0.41	0.37	0.14	0.35	0.42
Employment	0.67	0.15	0.68	0.44	0.09	0.44	0.99
Job creation/Employ	5.23	5.45	5.22	5.00	5.06	5.02	4.62
Job destruction/Employ	5.45	6.17	5.40	5.19	6.17	5.13	6.81
Price index	1.86	1.03	1.57	1.77	1.10	1.41	1.44
Inflation	1.10	0.64	0.94	1.08	0.62	0.87	0.56
Correlations							
Inflation/Stock ret.	-0.61	0.98	-0.85	-0.68	0.94	-0.92	-0.15
Mon. growth/Stock ret.	0.14	0.87		0.09	0.86		0.16

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 200 periods and repeating the simulation 100 times. The statistics are averages over these 100 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4.

ever, the model generates the negative correlation between inflation and the stock market return and the positive correlation between the growth rate of money and the stock market returns as in Marshall (1992). The negative correlation between inflation and stock market return is due to the quantitative predominance of the technology shock over the monetary shock. Both real and monetary shocks induce an increase in the stock market return but the impact of the real shock is quantitatively more important. Consequently, fluctuations in stock market returns are dominated by technology shocks. Because real shocks have also an immediate deflationary impact, then stock market returns are negatively correlated with inflation. The positive correlation between money growth and stock market returns is due to the fact that an increase in the growth rate of money decreases the interest rate, and allows firms to increase their profits. This increases the market valuation of a firm.

Examining now the economy with a larger value of $\eta = 0.10$, we find that an increase in the sharing parameter decreases the volatility of output, employment and jobs flows. (See second column of Table 3). Therefore, according to the model, the bargaining power of workers and firms seems to be important for the amplitude of employment fluctuations.

Consider now again the economy with a decreasing returns to scale matching technology. The statistics are reported in table 4. It is interesting to note that now job destruction is more volatile than job creation, although the volatility is smaller than in the data. It is also noteworthy that with this alternative

Table 4
Business cycle properties of **Model Economy B** ($\zeta = 0.1$) and U.S. economy.

	$\eta = 0.01$			$\eta = 0.10$			U.S.
	M&R	Money	Real	M&R	Money	Real	Economy
Standard deviations							
Output	1.63	0.58	1.49	1.05	0.33	0.99	1.60
Hours	0.46	0.20	0.38	0.31	0.14	0.27	0.42
Employment	0.94	0.32	0.88	0.40	0.14	0.37	0.99
Job creation/Employ	2.16	1.93	2.18	1.95	1.46	2.04	4.62
Job destruction/Employ	2.23	2.35	2.25	2.76	3.33	2.75	6.81
Price index	1.72	0.86	1.49	1.45	1.04	0.99	1.44
Inflation	0.91	0.58	0.69	0.86	0.59	0.65	0.56
Correlations							
Inflation/Stock ret.	-0.38	0.98	-0.75	-0.60	0.90	-0.98	-0.15
Mon. growth/Stock ret.	0.19	0.86		0.15	0.84		0.16

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 200 periods and repeating the simulation 100 times. The statistics are averages over these 100 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4.

matching technology the responses of the economy to both monetary and real shocks are more persistent as is shown in Figure 3, but less volatile.

8 The model with risk-averse agents and physical capital

In this section we extend the model by assuming risk averse agents and by introducing a second factor of production, that is, physical capital. Agents maximize the life time utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c, \ell, \chi), \qquad u(c, \ell, \chi) = \frac{\left[c + (1-\chi)a - \chi\frac{\ell^\gamma}{\gamma}\right]^{1-\sigma}}{1-\sigma} \tag{31}$$

Because agents face idiosyncratic risk in their earnings, this economy behaves very differently from a representative agent economy. In order to reduce the complexity of the model, we follow Andolfatto (1996) and Merz (1995) and we assume that workers insure themselves against earnings uncertainty and unemployment. This allows us to treat the model as a representative agent model. We will also follow Andolfatto (1996) and Merz (1995) in assuming that wages are determined by splitting the surplus of the match according to a constant proportion η.[8] This sharing rule does not derive from a Nash bargaining process as in the case in which agents are risk neutral, but it is simply assumed. This is because with risk averse agents, the Nash bargaining outcome is difficult to derive and it does not necessarily results in a fixed share of the surplus generated by the match.

The wage received by an employed worker, net of the insurance contribution, will be denoted by $\hat{w}_\chi$. Because employed workers face disutility from working and a reduction in homework production, the income they receive is not necessarily equal to the income received by unemployed workers. To differentiate the income received by employed workers from the income received by unemployed workers, we use the subscript χ.

In addition to owning deposits and shares of firms, households also accumulate physical capital which depreciates at rate δ. Capital is rented to firms at the rental rate r_k. The rents from capital are paid at the end of the period and the purchase of new capital goods requires cash.

The production function is now given by $y = A\bar{k}^\xi \ell^\nu - \varphi$, where $\bar{k}$ is the input of capital. The bar over k is used to distinguish the firm's input of capital from the household's holding of capital. The dividends paid to the shareholders are:

$$\pi(s, \varphi) = A\bar{k}^\xi \ell^\nu - \varphi - w(1 + r) - r_k \bar{k} \tag{32}$$

All the equations derived for the previous model can easily be extended to this more general model with very small adaptations. Equations (4) and (7) become:

$$J(s, \varphi) = \tilde{\pi}(s, \varphi) + E\beta(s, s') \int^{\bar{\varphi}(s')} J(s', \varphi')dF(\varphi') \tag{33}$$

$$\kappa = q(s)E\beta(s, s') \int^{\bar{\varphi}(s')} J(s', \varphi')dF(\varphi') \tag{34}$$

For notational convenience we have defined $\beta(s, s') = \beta u_{\hat{c}}(\hat{c}(s'))/u_{\hat{c}}(\hat{c}(s))$, where $\hat{c}(s)$ is consumption net of the disutility from working and $u_{\hat{c}}(.)$ is the derivative of the utility function with respect to $\hat{c}$. Because agents insure themselves against earnings uncertainty, $\hat{c}$ is the same for all agents. We have also defined the function $\tilde{\pi}(s, \varphi) = E[\beta(s, s')p(s)\pi(s, \varphi)/p(s')]$, where $\pi(s, \varphi)$ are the dividends paid by the firm to the shareholders at the end of the period. The function expresses the current value in terms of consumption for the shareholder of the dividend paid by the firm. Because dividends are paid in

[8] However, we do not restrict this proportion to be the one that implements the social optimum.

cash by the firm at the end of the period, in order to transform this cash in consumption, the shareholder needs to wait until the next period. This implies that the current value in terms of consumption of one unit of cash received at the end of the period is equal to $\beta u_{\hat{c}}(\hat{c}(s))p(s)/u_{\hat{c}}(\hat{c}(s'))p(s')$.

The values of being employed and unemployed (again, in terms of current consumption) are:

$$
\begin{aligned}
W(s,\varphi) = {} & w(s,\varphi) - \frac{\ell^\gamma}{\gamma} + E\beta(s,s') \int^{\bar{\varphi}(s')} [W(s',\varphi') - U(s')]dF(\varphi') \\
& + E\beta(s,s')U(s')
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
U(s) = {} & a + h(s)E\beta(s,s') \int^{\bar{\varphi}(s')} [W(s',\varphi') - U(s')]dF(\varphi') \\
& + E\beta(s,s')U(s')
\end{aligned}
\tag{36}
$$

Using the η-sharing rule and equation (34), we derive the surplus of the match which is equal to:

$$
S(s,\varphi) = \tilde{\pi}(s,\varphi) + w(s,\varphi) - a - \frac{\ell^\gamma}{\gamma} + \frac{(1-\eta h(s))\kappa}{(1-\eta)q(s)} \tag{37}
$$

Moreover, by equating $W(s,\varphi) - U(s)$ to $\eta S(s,\varphi)$ and using (32), we can derive the wage $w(s,\varphi)$:

$$
w(s,\varphi) = \frac{\eta(A\bar{k}^{1-\nu}\ell^\nu - \varphi - r_k\bar{k})E\left(\frac{\beta(s,s')p(s)}{p(s')}\right) + (1-\eta)\left(a + \frac{\ell^\gamma}{\gamma}\right) + \frac{\eta h(s)\kappa}{q(s)}}{\left[1-\eta+\eta(1+r)E\left(\frac{\beta(s,s')p(s)}{p(s')}\right)\right]} \tag{38}
$$

The inputs of capital and labor that maximize the surplus of the match are determined by differentiating (37) with respect to $\bar{k}$ and ℓ, and are given by:

$$
\ell(s) = \left(\frac{\nu A}{1+r}\right)^{\frac{1-\xi}{\gamma(1-\xi)-\nu}} \left(\frac{\xi A}{r_k}\right)^{\frac{\xi}{\gamma(1-\xi)-\nu}} \tag{39}
$$

$$
\bar{k}(s) = \left(\frac{\nu A}{1+r}\right)^{\frac{\nu}{\gamma(1-\xi)-\nu}} \left(\frac{\xi A}{r_k}\right)^{\frac{\gamma-\nu}{\gamma(1-\xi)-\nu}} \tag{40}
$$

The equivalent of equation (14) is:

$$\frac{\kappa}{q(s_t)} = E_t\beta(s_t, s_{t+1})\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j-1}\beta(s_{t+i}, s_{t+i+1})F(\bar{\varphi}(s_{t+i}))\right) \int^{\bar{\varphi}(s_{t+j})} \tilde{\pi}(s_{t+j}, \varphi)dF(\varphi) \tag{41}$$

This expression is equivalent to equation (14) once we use the proper marginal rate of substitution to discount future dividends. While in the model with risk neutral agents the marginal rate of substitution in consumption is equal to β, in the economy with risk averse agents the marginal rate of substitution is given by $\beta(s, s') = \beta u_{\hat{c}}(\hat{c}(s'))/u_{\hat{c}}(\hat{c}(s))$. Equation (41) shows that an increase in the expected sum of future dividends (properly discounted) induces a reduction in q. Without endogenous separation, this would imply an increase in the number of vacancies which increases the next period employment. With endogenous separation the impact on the employment rate is more complex. However, the simulation of the calibrated model shows that this mechanism is still at work in this more complicated framework.

The household problem now includes another state and choice variable, that is, the current and next period stock of capital. Denoting with s the vector of aggregate states and with $\hat{s}$ the vector of individual states, the recursive formulation of the household's problem is:

$$\Omega(s, \hat{s}) = \max_{m', d', k', v, c} \{u(\hat{c}) + \beta\, E\, \Omega(s', \hat{s}')\} \tag{42}$$

subject to

$$\hat{c} = c + (1-\chi)a - \chi\,\frac{\ell^{\gamma}}{\gamma} \tag{43}$$

$$c = \hat{w}_{\chi} + \frac{m - d'}{p} - \tau(d, d') - v\kappa - k' + (1-\delta)k \tag{44}$$

$$n' = nF(\bar{\varphi}) + vq \tag{45}$$

$$m' = (1+r)d' + p\, n\, \bar{\pi} + pr_k k - rB \tag{46}$$

Taking first order conditions with respect to d', k' and v we obtain the Euler equations:

$$1 = (1+r)E\left(\frac{\beta u'(\hat{c}')p}{u'(\hat{c})p'}\right) - p\left[\tau_2(d, d') + E\left(\frac{\beta u'(\hat{c}')\tau_1(d', d'')}{u'(\hat{c})}\right)\right] \tag{47}$$

$$1 = E\left(\frac{\beta u'(\hat{c}')(1-\delta)}{u'(\hat{c})}\right) + \beta\, E\left(\frac{\beta u'(\hat{c}'')p' r_k'}{u'(\hat{c})p''}\right) \tag{48}$$

$$1 = (1-\lambda)\, E\left(\frac{\beta u'(\hat{c}')qF(\bar{\varphi}')}{u'(\hat{c})q'}\right) + \beta\, E\left(\frac{\beta u'(\hat{c}'')p'\bar{\pi}'}{u'(\hat{c}')p''}\right)\frac{q}{\kappa} \tag{49}$$

8.1 Properties of the model with capital

To calibrate this model we use the same conditions we used to calibrate the economy without capital. In addition we have three extra parameters: σ, δ and ξ. The coefficient σ is set equal to 2, but different values have little effect on the quantitative properties of the model. The depreciation parameter is set to 0.02 and the parameter ξ is determined by imposing that the quarterly steady state capital-output ratio equals 10. As for the previous model, the standard deviation of the technology shock is set to the value for which the standard deviation of output is close to the empirical one.

Table 5 reports the correlations at different lags and leads of employment, job creation, and job destruction, with inflation and the price level. Table 6 reports the cross-correlation of employment, job creation and job destruction. These tables are for the economy with $\eta = 0.01$ and $\zeta = 0.1$ which best fit the data.

There are some improvements compared to the economy without capital. In particular, the magnitude of the correlation of employment with inflation is bigger, and there is a small positive correlation of employment with one period lagged inflation. Job creation is now negatively correlated with the price level as in the data. In general the statistics generated by the model match the same statistics for the data quite well. The impulse responses of this economy to real and monetary shocks are very similar to the response of the economy without capital.

9 Conclusion

Because the Phillips curve is essentially an empirical relation linking labor markets and monetary policy, any model which is to aid our understanding of the Phillips curve relation should be consistent with the complex and volatile flows in the labor market. In this paper we have explored a model economy where these flows are explicitly captured. We then showed how aggregate monetary and real shocks affect the flows of jobs and workers.

The model greatly simplifies and abstracts from many important features of the economy. Its performance rests on many simplifying assumptions about the behavior of households and firms. In evaluating this model as a description of

Table 5
Correlation with inflation and prices at different lags and leads for the economy with capital

	Correlation of current employment with						
	t-3	t-2	t-1	t	t+1	t+2	t+3
Model Economy							
Inflation	-0.15	-0.07	0.09	0.39	0.31	0.24	0.17
Price index	-0.60	-0.66	-0.60	-0.35	-0.15	0.00	0.11
U.S. Economy							
Inflation	0.08	0.21	0.38	0.51	0.51	0.48	0.44
Price index	-0.72	-0.65	-0.50	-0.30	-0.10	0.10	0.27

	Correlation of current job creation with						
	t-3	t-2	t-1	t	t+1	t+2	t+3
Model Economy							
Inflation	-0.07	-0.29	-0.70	-0.13	-0.08	-0.04	-0.01
Price index	0.53	0.35	-0.11	-0.19	-0.23	-0.26	-0.26
U.S. Economy							
Inflation	-0.71	-0.28	-0.34	-0.14	-0.12	-0.12	0.06
Price index	0.13	0.02	-0.12	-0.17	-0.22	-0.27	-0.25

	Correlation of current job destruction with						
	t-3	t-2	t-1	t	t+1	t+2	t+3
Model Economy							
Inflation	0.18	0.27	0.44	-0.31	-0.25	-0.20	-0.16
Price index	0.19	0.36	0.65	0.45	0.29	0.16	0.06
U.S. Economy							
Inflation	0.39	0.30	-0.10	-0.12	-0.09	-0.11	-0.45
Price index	0.46	0.57	0.56	0.53	0.49	0.45	0.27

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 200 periods and repeating the simulation 100 times. The statistics are averages over these 100 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4.

reality we compared the data generated by this economy to a mixture of plant level and firm level observations and aggregate data. For all of these reasons the match between the model and the data is not perfect. Nevertheless, this economy seems to capture many of the important qualitative features of labor markets and the Phillips curve relation.

A model economy with these features can now form the basis for thinking about the implications of the Phillips curve relation for economic policy. In particular, one could use this framework to compute the optimal monetary

Table 6
Cross-correlation of employment, job creation and job destruction.

	Correlation at lags and leads k						
	t-3	t-2	t-1	t	t+1	t+2	t+3
Model Economy							
corr(Cre_{t+k}, Emp_t)	0.09	-0.05	-0.23	-0.48	-0.82	-0.85	-0.73
corr(Des_{t+k}, Emp_t)	-0.56	-0.68	-0.73	-0.61	-0.19	0.06	0.20
corr(Cre_{t+k}, Des_t)	-0.27	-0.29	-0.31	-0.33	0.32	0.57	0.61
U.S. Economy							
corr(Cre_{t+k}, Emp_t)	0.27	0.15	0.04	-0.19	-0.58	-0.68	-0.60
corr(Des_{t+k}, Emp_t)	-0.63	-0.65	-0.59	-0.35	-0.01	0.29	0.45
corr(Cre_{t+k}, Des_t)	-0.39	-0.44	-0.47	-0.43	-0.14	0.18	0.34

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 200 periods and repeating the simulation 100 times. The statistics are averages over these 100 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4.

policy response to real shocks. The results of this paper suggest that these models hold great promise for understanding the correlations associated with the Phillips curve and their possible relevance for monetary policy.

Acknowledgements

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Figure 1a – Money shock: Nominal interest rate

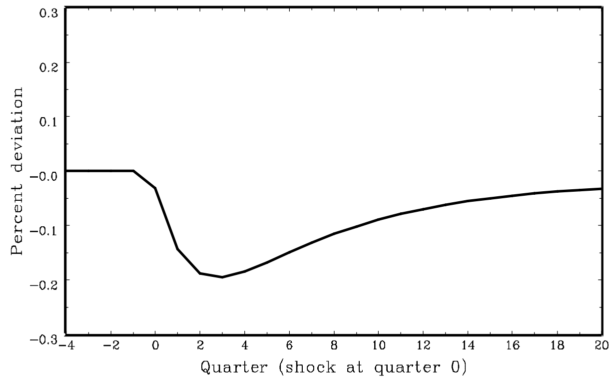

Figure 1b – Money Shock: Hours

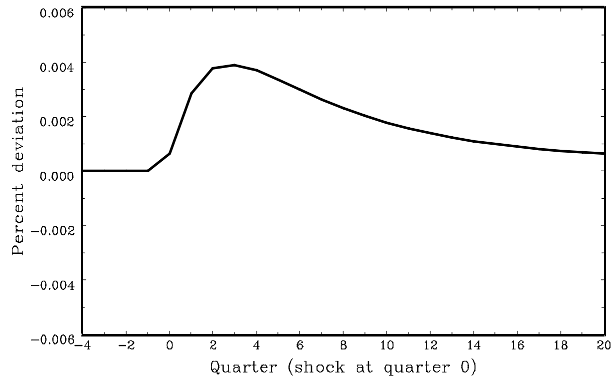

Figure 1c – Money shock: Employment

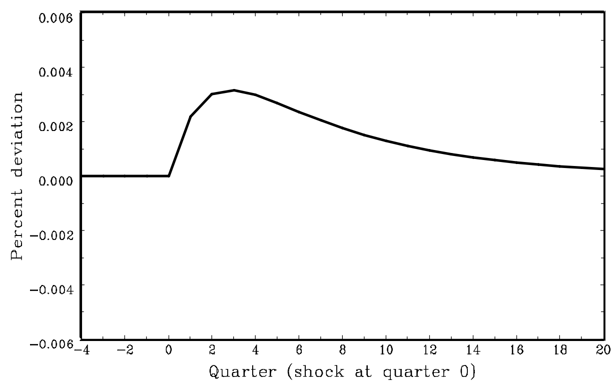

Figure 1d – Money shock: Output

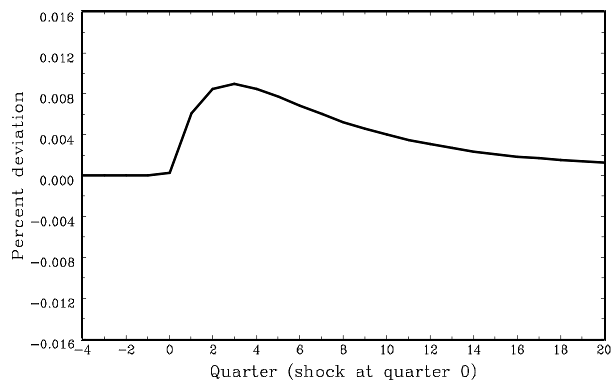

Figure 1e – Money shock: Job matching variables

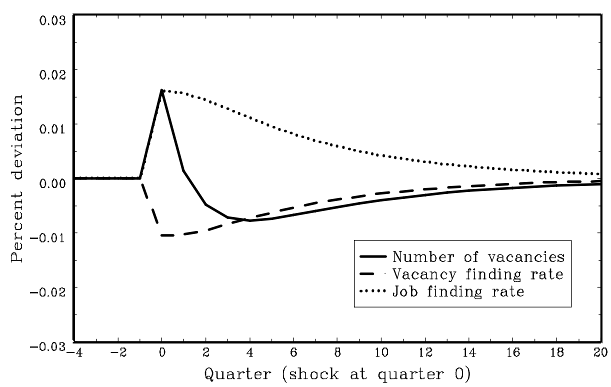

Figure 1f – Money shock: Jobs Flows

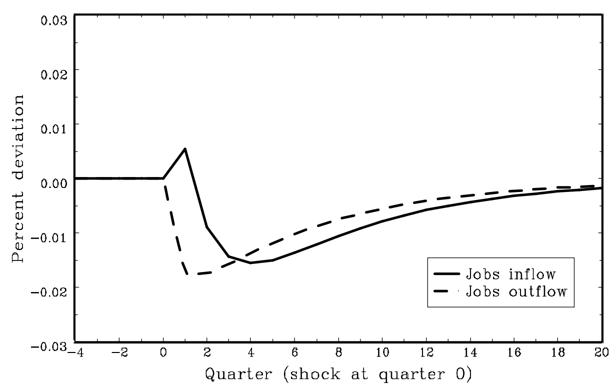

Figure 1g – Money shock: Prices

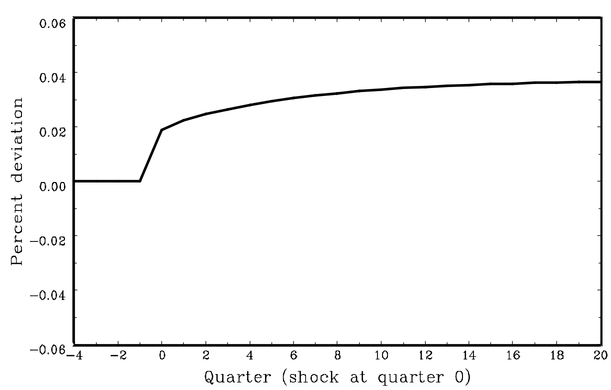

Figure 1h – Money shock: Inflation

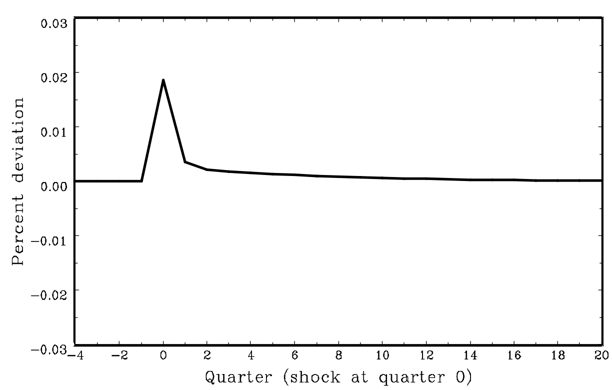

Figure 2a – Real shock: Nominal interest rate

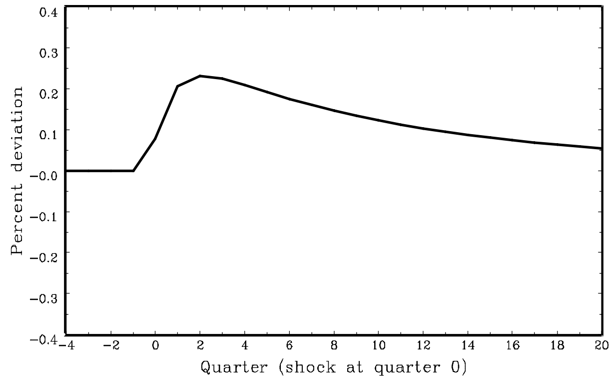

Figure 2b – Real Shock: Hours

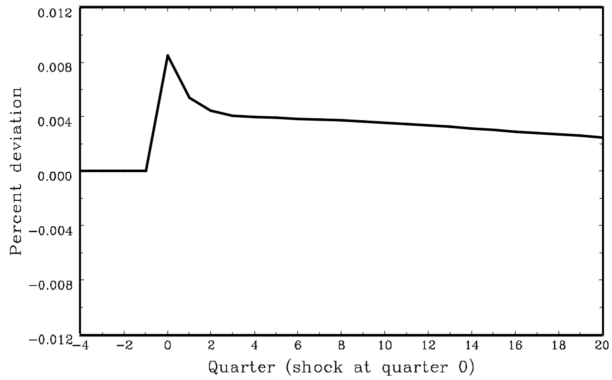

Figure 2c – Real shock: Employment

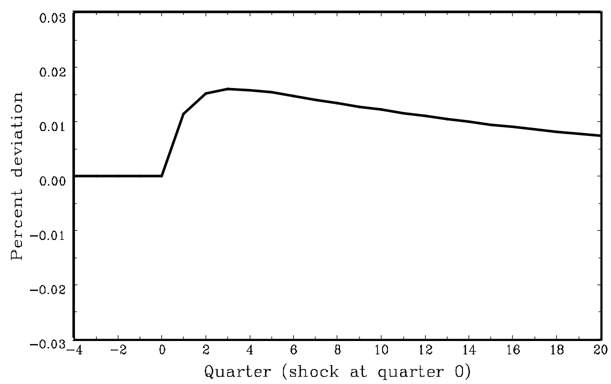

Figure 2d – Real shock: Output

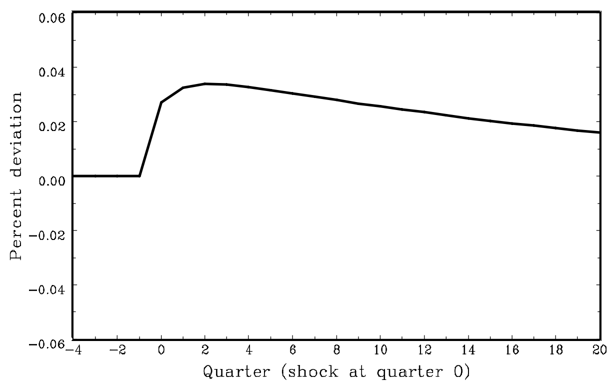

Figure 2e – Real shock: Job matching variables

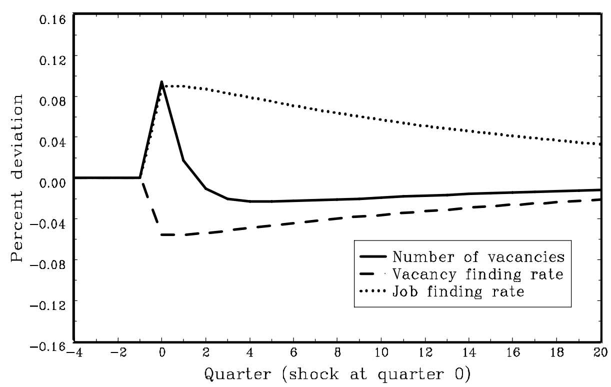

Figure 2f – Real shock: Jobs Flows

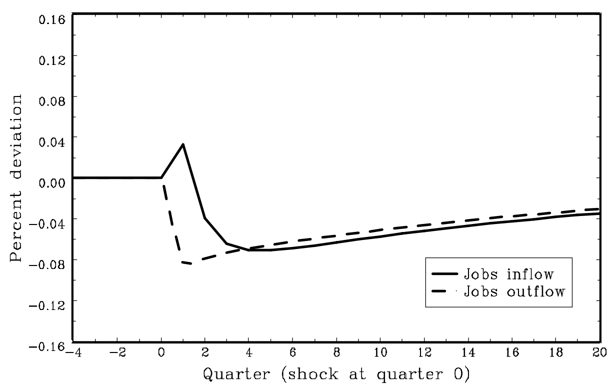

Figure 2g – Real shock: Prices

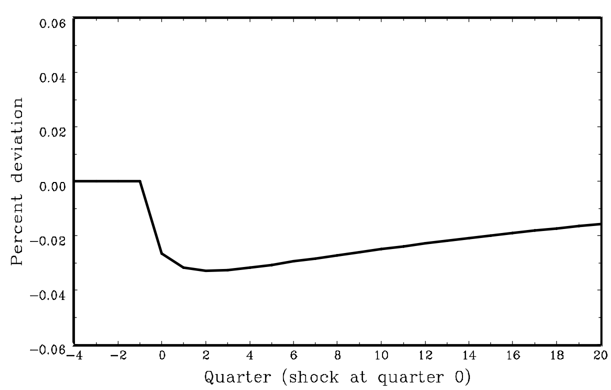

Figure 2h – Real shock: Inflation

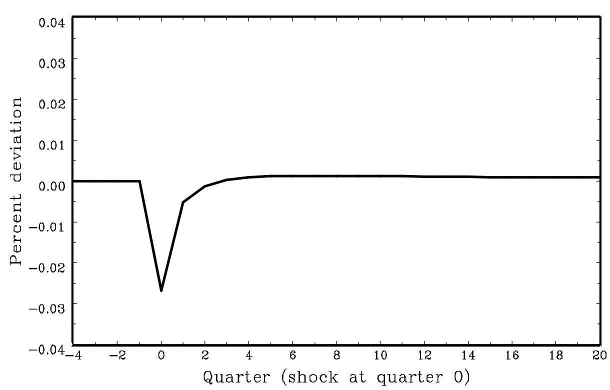

Figure 3a – Money shock: Output

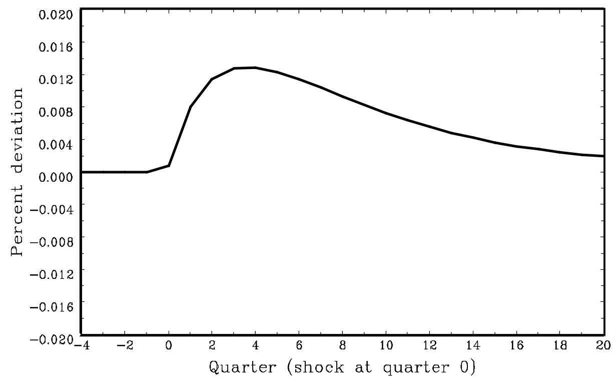

Figure 3b – Money shock: Employment

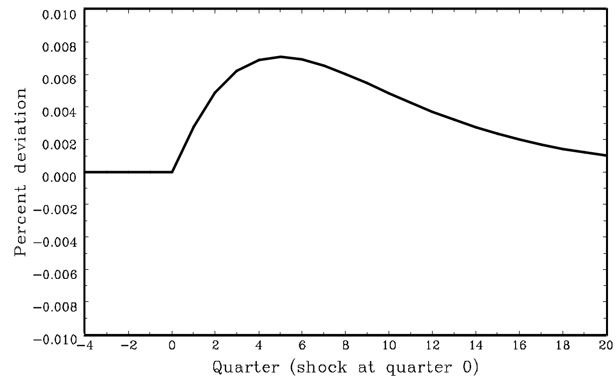

Figure 3c – Money shock: Job matching variables

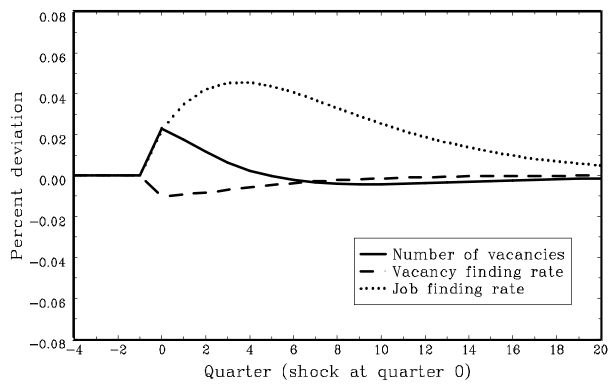

Figure 3d – Money shock: Jobs Flows

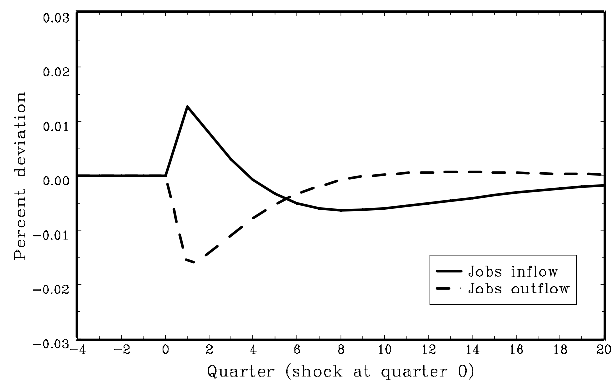

Figure 3e – Money shock: Output

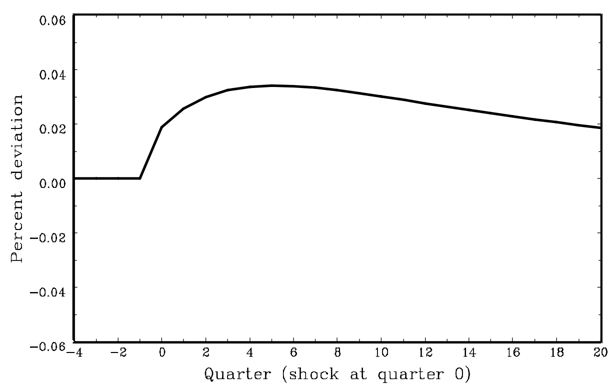

Figure 3f – Money shock: Employment

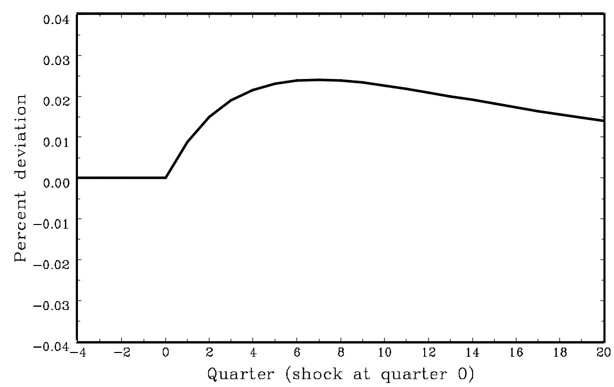

Figure 3g – Money shock: Job matching variables

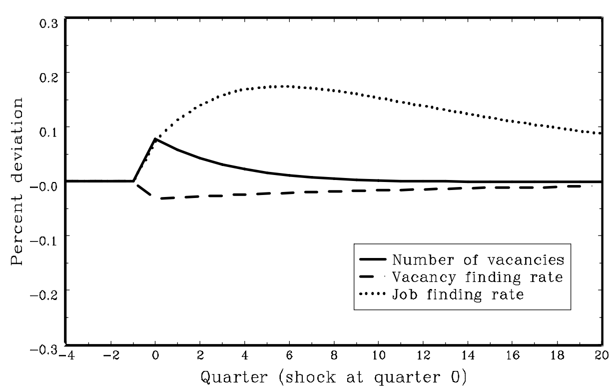

Figure 3h – Money shock: Jobs Flows