



Investment with restricted stock and the value of information

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Abstract

In most public companies in China, there are two thirds of shares that cannot be traded freely in the secondary market. These illiquid shares, however, may be allowed to circulate unexpectedly one day. This paper delves into the investor's financial decision-making with restricted stock in a continuous-time framework. Accordingly, this paper assumes that removal of trade restriction arrives as a Poisson process. In the spirit of [Rev. Econom. Statist. 51 (1969) 247; J. Econom. Theory 3 (1971) 373], an analytical solution to the investor's optimal portfolio problem is derived and the price (or cost) of illiquidity can be calculated using numerical method. Furthermore, the value of information is discussed in this framework. Numerical simulation shows that illiquidity has an important influence on the investor's optimal strategy. This model may provide a theoretical framework to assess the cost of state-owned equities (SOEs).

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Keywords: Restricted stock; SOEs; Illiquidity; Continuous-time finance; Optimal portfolio strategy

1. Introduction

Statistics show that among 1000-odd listed companies in China, non-circulating shares held by the Administration of State-Owned Property and

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State-Owned Corporation bodies account for more than 60% of all the shares, while circulating shares held by general investors stand at only 30%. Such an ownership structure results in a thin stock market. What makes things even worse, in our view, is the possibility that those illiquid shares may be circulated unexpectedly one day, which we call a kind of event risk. So, we delve into the investor's financial decision-making with restricted stocks in a continuous-time framework. You may imagine this investor as the state who owns the restricted stocks. Our main purpose henceforth is to show the cost of holding these restricted shares, and in another view, to price the illiquid shares among many other works in this field. Accordingly, we can postulate that the relaxing arrives as a Poisson process. In the spirit of Merton [10,11], we then derive an analytical solution to the investor's optimal portfolio problem. Under our specifications, the price (or cost) of illiquidity can be calculated using numerical method. Furthermore, we discover the value of information on the event using this framework. Our numerical simulation illustrates that illiquidity has an important influence on the investor's optimal strategy. And we do think this model provides us a theoretical framework to assess the cost of state-owned equities (SOEs).

In other related works, Liu et al. [8] investigate the investor's dynamic asset allocation using the event risk framework of Duffie et al. [3]. At the first sight, our work seemingly resembles theirs; our basic structure, however, is greatly different from that, although we are both in the spirit of Merton [10,11]. In their specification, all the shares are the same, and an exogenous shock brings to an event risk. So, they specify that the price dynamics follow a Brown process with drift plus an Poisson process. That is to say, the event risk is formulated in the stochastic differential equation of the stock price dynamics. In contrast, we take a new perspective of the illiquid shares and the occurrence of event risk, similar to that of Kahl et al. [6]. Suppose that we do hold these illiquid shares in our portfolio, it means that we are restricted to sell them until the arrival of event risk. Under this specification, we can calculate the implied value of restricted stock as a fraction of its unrestricted market value using the method of Longstaff [9]. Probably our structure is much closer to that of Kahl et al. [6]. The time horizon, however, in our model is stochastic and follows a Poisson process, which makes our model fit for the pricing of SOEs since we do not know when these shares can be freely traded for certainty. And we think the structure in Liu et al. [8] may be more suitable for the pricing of default bonds, where the price follows an affine jump-diffusion (henceforth AJD). We cannot see much reason in specifying that stock price follow an AJD in our circumstance.

The remainder of this paper is organized as follow: Section 2 presents the basic model; Section 3 models the dynamic portfolio choice with restricted stock where the time of removing such restriction follows a Poisson process; in Section 4, we examine the effects of liquidity restrictions on welfare and opti-

mal portfolio decision using numerical simulation; and within this framework, we investigate the value of information. Section 5 concludes.

2. The basic model

Throughout this article we are assuming a probability space (Ω, \mathcal{F}, P) and a filtration $\{\mathcal{F}_t\}$. Uncertainty in this model is generated by two standard one dimensional Brownian motions B_1 and B_2 defined on the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$. The correlation coefficient between dB_1 and dB_2 is ρ and $-1 < \rho < 1$.

There are three types of assets in our portfolio choice framework. The first asset (“the bond”) is a money market account growing at a continuously compounded, constant rate r . Let β_t denote the value at time t of a riskless bond or money market fund with dynamic given by

$$d\beta_t = r\beta_t dt. \tag{1}$$

The second asset is a stock index fund. Let M_t denote the value of this risky asset which can be viewed either as the stock market or a share in a stock index fund. The dynamics of M_t are given by

$$dM_t = (r + \mu)M_t dt + \sigma M_t dB_1, \tag{2}$$

where μ is the market risk premium and σ is the volatility of returns. Both μ and σ are positive constants.

The third asset is a restricted stock of one firm. Our investor is not allowed to trade his shares in this firm, but the shares of this firm can be traded by others who are not subjected to the restriction. Let S_t denote the market value of a share of the firm’s stock. We assume that the dynamics of S_t are given by

$$dS_t = (r + \eta)S_t dt + \nu S_t dB_2, \tag{3}$$

where η is the excess expected return for the firm and ν is its volatility. Furthermore, we make the simplified assumption that the risk premium η is given by the Capital Asset Pricing Model, implying that $\eta = \frac{\mu\nu}{\sigma}$. This financial market is originated in the paper by Kahl et al. [6].

2.1. The investor’s problem

In our specification, the time of removal of trade restriction is random and follows a Poisson process, while in that of Kahl et al. [6], it is a deterministic point of time. As we have emphasized in Section 1, this generalization enables us to tackle the pricing of SOEs, so its economic implication is intuitive.

The investor has an invest horizon of $T < +\infty$, and at time zero, is given N shares of restricted stock in the firm. Our investor is not allowed to trade his

shares in this firm until time $\tau \leq T$. After time τ , the investor can trade his shares in the firm without restriction. Let $X_t = \frac{NS_t}{W_t}$ denote the portfolio weight for his illiquid stockholding, where W_t denotes his total wealth at time t . Let ϕ_t denote the portfolio weight for the stock index fund. So, the portfolio weight for the riskless asset is $1 - \phi_t - X_t$. Following Merton [10,11] and recently Kahl et al. [6], the investor's wealth follows the dynamic process:

$$dW_t = (r + \mu\phi + \eta X)W_t dt + \sigma\phi W_t dB_1 + \nu X W_t dB_2. \quad (4)$$

We assume the utility of an investor only depends on the market value of his portfolio at time T just as earlier models in the literature (e.g., Brennan et al. [1], Dumas and Luciano [4], Liu and Loewenstein [7]). The investor's problem is to choose trading strategy ϕ so as to maximize $E(U(W_T))$. We assume that the investor has CRRA preference, that is, $U(W_t) = \frac{W_t^{1-\gamma}}{1-\gamma}$ for $\gamma > 0$, and $\gamma \neq 1$. To solve this problem, we define the value function at time t as

$$J(W, X, t) = \max_{\phi} E \left[\frac{W_t^{1-\gamma}}{1-\gamma} \middle| \mathcal{F}_t \right] \rightarrow J(W, X, t) = \max_{\phi} E \left[\frac{W_T^{1-\gamma}}{1-\gamma} \middle| \mathcal{F}_t \right]. \quad (5)$$

2.2. Optimal policies of an unconstrained investor

Due to the assumptions in Section 2.1, the unconstrained investor would want to hold the firm's stock only to the extent that it appears in the stock index. So, the investor's problem is just as Merton [11] when there are no restricted stock ($N = 0$). Now we present results in this case for the purpose of comparison. The investor's problem can be written as

$$J(W, 0, 0) = \max_{\phi} E \left[\frac{W_T^{1-\gamma}}{1-\gamma} \right]$$

subject to

$$dW_t = (r + \mu\phi)W_t dt + \sigma\phi W_t dB_1. \quad (6)$$

In his seminal paper, Merton [11] solved this problem and the optimal portfolio selection rules is

$$\phi(t) = \frac{1}{r^* + 1} \quad (7)$$

for all $0 < t < T$, where the "Merton line" r^* is given by

$$r^* = \frac{\sigma^2 \gamma}{\mu} - 1. \quad (8)$$

The lifetime expected utility is

$$J(W, 0, 0) = e^{\rho T} \frac{W^{1-\gamma}}{1-\gamma}, \tag{9}$$

where

$$\rho = (1-\gamma) \left(r + \frac{\kappa}{\gamma} \right) \tag{10}$$

and

$$\kappa = \frac{\mu^2}{2\sigma^2}. \tag{11}$$

Obviously, these calculation simply works as a benchmark for the solution of optimal policies with restricted stocks.

2.3. Optimal policies with restricted stock

In the case where $N > 0$, the problem is considerably more complicated. Here we give a result derived by Kahl et al. [6]. When τ is a determinate number, i.e., the investor’s problem is

$$J(W, X, 0) = \max_{\phi} E \left[\frac{W_{\tau}^{1-\gamma}}{1-\gamma} \right]$$

subject to

$$dW_t = (r + \mu\phi + \eta X)W_t dt + \sigma\phi W_t dB_1 + vXW_t dB_2.$$

In this case, $J(W, X, t)$ can be expressed in the form

$$J(W, X, t) = \frac{W^{1-\gamma}}{1-\gamma} F(X) \tag{12}$$

and the optimal policy of investor is

$$\phi(t) = \frac{-(\mu/\sigma^2)(1-\gamma)F + (\gamma\rho v/\sigma + \mu/\sigma^2)XF_X + (\rho v/\sigma X)^2 F_{XX}}{-\gamma(1-\gamma)F + 2\gamma XF_X + X^2 F_{XX}} - \frac{\rho v}{\sigma} X, \tag{13}$$

where $F(X)$ is the solution to a boundary value problem (see Kahl et al. [6]).

By this modelling, Kahl et al. [6] show that trade restriction (illiquidity) has significant effects on the optimal investment and consumption strategies because of the need to hedge the illiquid stock position and smooth consumption in anticipation of the eventual lapse of the restriction. Their framework is convenient to analyze the real value of restricted stocks usually largely held by entrepreneurs and managers as a mechanism to align their interests with out-

side investors. When it comes to the pricing of SOEs, it does not make sense any more. The reason is straightforward: who knows when these restrictions will be relaxed? For deriving a solution to our problem specified above, we do need go further, and we do need some innovations to their structure.

3. Exponentially distributed horizon

In this section, we consider the situation when the time of removal of restrictions is random and follows a Poisson process. From the perspective of modelling, we can formulate it as our investor has a uncertain horizon. In particular, the investor's problem is now to choose admissible trading strategies so as to maximize $E[W_\tau^{1-\gamma}/(1-\gamma)]$ for an event which occurs at the first jump time τ of a standard, independent Poisson process with intensity λ . τ is thus exponentially distributed with parameter λ , that is

$$P\{\tau \in dt\} = \lambda e^{-\lambda t} dt. \quad (14)$$

3.1. Optimal policies without restricted stocks

Again, for purpose of comparison, let us first consider the case without restricted stocks ($N = 0$). In this case, the investor's problem becomes

$$J(W, 0, 0) = \sup_{\phi} E \left[\int_0^{\infty} \lambda e^{-\lambda t} \frac{W_t^{1-\gamma}}{1-\gamma} dt \right] \quad (15)$$

subject to the self-financing condition

$$dW_t = (r + \mu\phi)W_t dt + \sigma\phi W_t dB_1.$$

The above problem was solved by Merton [10,11] and Liu and Loewenstein [7]. A condition on the parameters is required for the existence of the optimal solution.

Assumption 1. The investor's expected horizon parameter λ satisfies

$$\lambda > (1-\gamma) \left(r + \frac{\kappa}{\gamma} \right),$$

where κ is as defined in Eq. (11).

Now, we give the result without proof.

Lemma 1. Suppose that $N = 0$. Under the Assumption 1, the optimal investment policy is Eqs. (7) and (8). Moreover, the lifetime expected utility is

$$J(W, 0, 0) = \frac{\lambda}{\lambda - \rho} \frac{W^{1-\gamma}}{1 - \gamma},$$

where ρ is as defined in Eq. (10).

3.2. Optimal policies with restricted stocks

Now, suppose that $N > 0$.

In this case, the investor makes investment decision with the restricted stocks. Now the investor’s problem becomes

$$J(W, X, 0) = \sup_{\phi} E \left[\int_0^{\infty} \lambda e^{-\lambda t} \frac{W_t^{1-\gamma}}{1 - \gamma} dt \right] \tag{16}$$

subject to the self-financing condition

$$dW_t = (r + \mu\phi + \eta X)W_t dt + \sigma\phi W_t dB_1 + vXW_t dB_2.$$

The Appendix A shows that $J(W, X, t)$ can be expressed in the form

$$J(W, X) = \frac{W^{1-\gamma}}{1 - \gamma} G(X) \tag{17}$$

and the optimal investment in the stock market ϕ is

$$\phi^* = \frac{-(\mu/\sigma^2)(1 - \gamma)G + (\gamma\rho v/\sigma + \mu/\sigma^2)XG_X + (\rho v/\sigma)X^2G_{XX} - \frac{\rho v}{\sigma}X}{-\gamma(1 - \gamma)G + 2\gamma XG_X + X^2G_{XX}} \tag{18}$$

where the function $G(X)$ satisfies a Hamilton–Jacobi–Bellman equation:

$$\begin{aligned} & \frac{1}{2}(\rho\sigma v\phi^*X + v^2X^2)(-\gamma(1 - \gamma)G + 2\gamma XG_X + X^2G_{XX}) \\ & + \left(\frac{\rho\sigma v}{2}\phi^*X + v^2X^2 \right)(-\gamma G_X - XG_{XX}) + \frac{v^2X^2}{2}G_{XX} + \left(r + \frac{\mu\phi^*}{2} + \eta X \right) \\ & \times ((1 - \gamma)G - XG_X) + (r + \eta)XG_X + \lambda G + \lambda = 0. \end{aligned} \tag{19}$$

with the condition

$$G(0) = \frac{\lambda}{\lambda - \rho}, \tag{20}$$

$$G_X(0) = 0. \tag{21}$$

To our knowledge, the function of $G(X)$ cannot be solved in closed form. We can, however, use simulation techniques to offer numerical solutions to

$J(W, X)$ and the optimal weight in stock index ϕ^* since this is just a ordinary differential equation.

4. Simulation

In this section, we study the effects of liquidity restriction on the investor through numerical simulation and then derive some implications of our model.

We focus firstly on investor's optimal portfolio strategy in the presence of restricted stocks, and investigate how the intensity of the arrival of relaxing restriction affects his optimal demand for risky assets. Then, we investigate the welfare effects of trade restriction and calculate numerically their economic costs, which we interpret as the illiquidity cost or the price of illiquidity. We further delve into the relationship between the share of illiquid equities X and the discount on illiquid equities. Finally, we discuss the value of information on the relaxation of SOEs.

In our simulation, the riskless rate is 5%, the expected premium on the stock market is 5%, the volatility of returns on the stock market is 20%, and the rate of time preference equals the riskless rate.

4.1. The optimal portfolio strategy

In Eq. (18), the optimal weight in stock index is given analytically wherein the function $G(X)$ has to be solved through numerical simulation. Compared with the situation where there is no restriction, we can observe that the optimal weight invested in stock index is no longer a constant fraction of the investor's wealth which is given in the classic papers of Merton [10,11]. Now, the optimal weight ϕ^* depends in a complicated way on the fraction of his wealth. We perform the numerical simulation on the optimal weight, with the result given in Fig. 1 below.

Fig. 1 shows that the optimal weight in stock index is decreasing on the illiquid fraction, given others constant. This conclusion is easily interpreted in economics. With more weight on illiquid assets, the optimal strategy for the investor to hedge the risk is to invest less in the stock index while keeping more safe assets.

Furthermore, with the rise in the intensity, the optimal weight in stock index also decreases significantly, which can be explained as follows: The later the event will come, the more the investor will choose to invest in the safe assets. Interestingly, in some case, the restriction can lead to the investor taking a short position in the stock index that would not appear in the situation of no restriction. For example, in the case of $\lambda = 5$ and $\beta = 1$, when the illiquidity fraction is close to 1, the optimal weight may be below zero, which means a

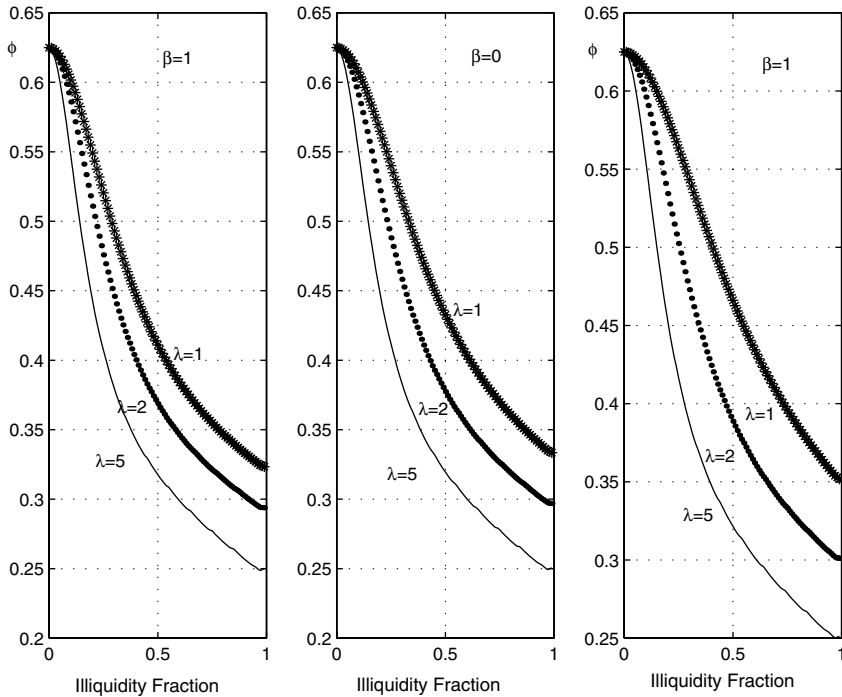


Fig. 1. Optimal stock market portfolio weight.

short position in the stock index. The reason, as has been explained in Kahl et al. [6], is that the investor partially gainsaid the illiquidity effects by taking an offsetting position in the stock index.

4.2. The cost of liquidity restrictions

Perhaps the most important issue for us to address is the welfare effects of the investor with restricted stocks. We calculate the welfare costs of illiquidity by comparing the investor’s derived utility of wealth $J(W, X, t)$ with restricted stocks with that in the case of no such restriction.

The method we use herein is intuitive. Suppose that you have 10 RMB, and if you choose to buy N shares of restricted stocks, then you obtain the maximum utility of A ; now suppose you choose to buy unrestricted shares, to say, X shares with the market price P , and you also achieve the same maximum utility of A . Given these conditions, you can surely derive the price of restricted stocks, to say P_{il} , so $P_{il} = \frac{XP}{N}$. We call this method “Utility Equivalence Theorem”, which is also adopted in Longstaff [9].

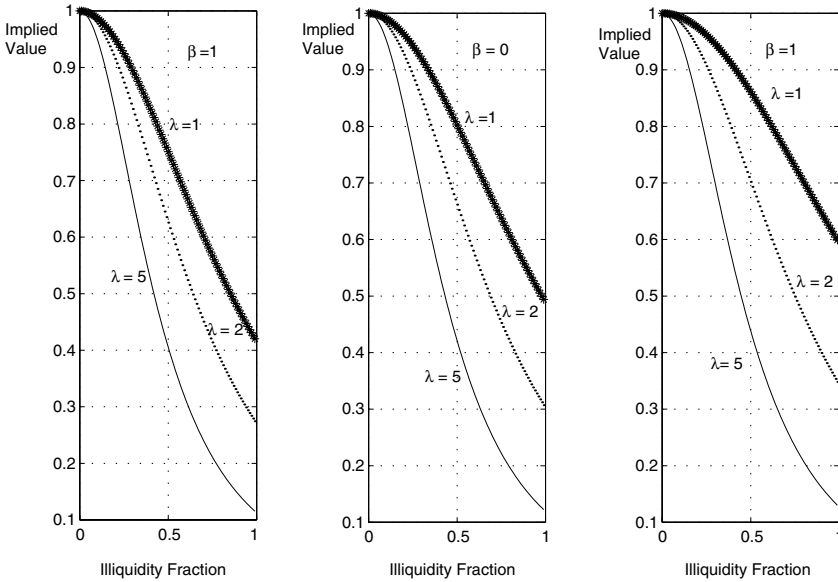


Fig. 2. Implied value of restricted stock.

Using this method, we can calculate the implied value of restricted stock as a fraction of its unrestricted market value. Fig. 2 reports the simulation results for different value of the beta of the firm, which is given by $\beta = \frac{\mu}{\sigma}$, and for different level of X and the intensity of Poisson process.

From Fig. 2, we can see that the implied value of restricted stock to an investor can be significantly less than its market value without restriction. For example, when the fraction of illiquid stocks account for 50% of the total wealth, and the intensity of Poisson process is equal to 5 and $\beta = -1$, then the implied value of restricted stock is only near 40% of its unrestricted market value. Furthermore, as illustrated in Fig. 2, the costs of illiquidity can be greatly larger when the illiquid share account for most of the investor's wealth, that is when X is near 1, as can be observed from the figure above: when $X = 1$ and $\lambda = 5$, $\beta = -1$, the implied value of restricted stocks is almost near 0.1.

And the price of illiquidity is a increasing function of X for the implies value of restricted stock is decreasing on X . This conclusion is also intuitive, since the more the amount of restricted stocks, the more constraint when you re-balance your portfolio continuously. These conclusion has implication for the SOEs. Based on our model, the more the SOEs, the larger the discount on illiquidity, which means great loss in welfare of the state. In China, because of the great difference in the cost of restricted stocks and unrestricted stocks, however, the state as a matter of fact obtains much from the illiquidity of the SOEs if we

take the state as a self-interest body. When it comes to the social welfare, however, it is a great loss for the illiquid SOEs to exist in the economy according to our simulation results.

Finally, as shown in Fig. 2, when the correlation between the return of the firm and the return of the market vary, the implied value of restricted stock also changes significantly. For example, when $X = 0.5$, $\lambda = 5$, the implied value ranges from 0.62 to 0.7 with the value of β from -1 to 1. Obviously, it reflects the ability for the investor to hedge the risk of restricted stockholding. When the correlation coefficient is equal to 1, the investor can then hedge the risk of illiquidity by investing in the stock index, so the implied value is priced higher than that of β equal 0 or -1 .

The most interesting thing we do think is that when the intensity is given the value of 5 years, the implied value ranges from 0.2 to 0.3, this result is close to the empirically estimated price of SOEs by Chen and Xiong [2].

4.3. *The value of information*

In this section, we try to investigate the value of information. In the field of finance, “information” is always a key word for academics which may date back to the work of Hayek [5] and many others. With prior information, we can profit much in the stock market—of course, we should pay for the information—thus, bringing to an efficient market.

In our specifications herein, we refer the information to news on the time you know exactly when the restricted stocks can be circulated, comparing with the case in which you only know the intensity of the arrival. Obviously, in the former case, you can profit more with the information.

We also perform numerical simulation on the value of information according to our specification above, with the result of Fig. 3 below.

First we shall explain the curves in the Fig. 3. Take the first as example when $\beta = -1$. The real line of $\lambda = 5$ reflect the value of information when we do know that the time of removal of restriction is in 5 years, comparing with the case in which we only know the expected value is 5. Obviously, when we know the exact time, we know more information.

As shown in Fig. 3, the value of information increases with the illiquidity fraction in most time. This result is easily to explain. With more illiquid assets in the portfolio, the investor will encounter more risk, so the information is more valuable for him.

And the longer the restriction lasts, the more the value of information in most cases. These conclusion is economically intuitive. With the time to remove the restriction prolonged, the variance of Poisson process becomes larger, so, the information becomes more important and more valuable for the investor.

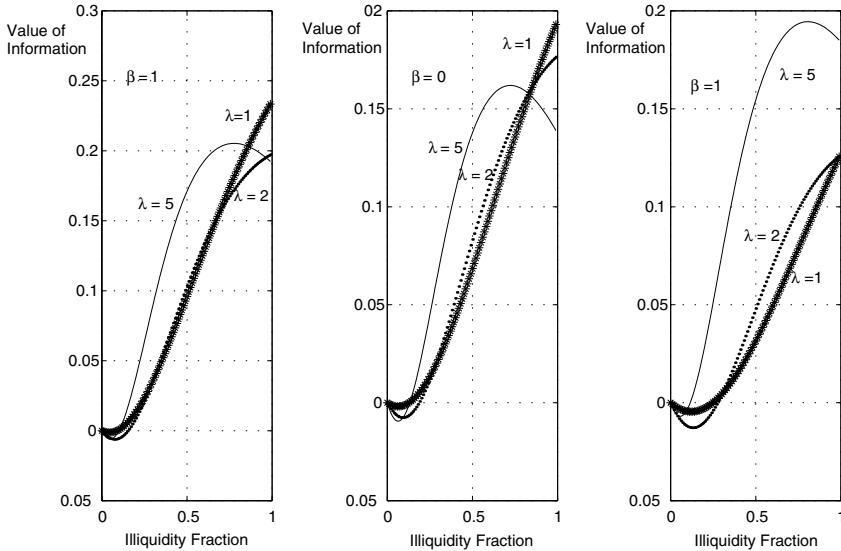


Fig. 3. Value of informations.

5. Conclusion

This paper studies the portfolio choice with restricted stock and the value of information in a continuous-time framework. To address these issues, we model the optimal consumption problem from the perspective of the state which we just take as a normal agent in the economy. In this framework, the price of illiquidity can also be calculated relatively easily which may enrich the literature in pricing liquidity.

In our model, the state is allowed to invest in the stock index and safe assets to hedge risk and smooth consumption, the cost of illiquidity, however, is still larger as shown in Fig. 1. And in the case of illiquidity fraction equal 1, the discount on illiquidity is almost 90% in our simulation.

In a view of practice in the economy, restricted stock may be helpful in retaining key employees and wifful managers, and it can also be used to alleviate the agency problem in modern companies, it also can, however, bring great cost to those people who receive it as a kind of compensation, thus makes the incentive contract less efficient. For the government of China, SOEs that are not allowed to trade freely in the secondary market keep the state in charge of the national economy. According to our model, this restriction surely brings great costs to the state, making the SOEs less valuable. Furthermore, as has been emphasized in our companion paper, the SOEs should be responsible for the thinness in the market. However, to understand the pros and cons of SOEs

may need more knowledge in politics than we have, so we try to avert this issue as possible as we can in this model.

We do not add a jump in the price of stock when relaxing the restriction for the convenience of modelling. This may be suitable for our analysis herein, if we can adjust the framework in a controllable way. We shall try to do it in further research. And obviously our model is a partial equilibrium model in which the price dynamics is specified in advance. And if we endogenize the price in a meaningful manner, we may get more insight into the illiquidity issue addressed here.

Appendix A. Optimal portfolio

Using the definition of X , the dynamic budget constraint can be expressed as

$$dW_t = (r + \mu\phi + \eta X)W_t dt + \sigma\phi W_t dB_1 + vXW_t dB_2$$

that is

$$dW_t = (rW_t + \mu\phi W_t + \eta NS_t) dt + \sigma\phi W_t dB_1 + vNS_t dB_2.$$

Since W and S form a joint Markov process, the derived utility of wealth $J(W, S, t)$ satisfies the Hamilton–Jacobi–Bellman equation:

$$\begin{aligned} \max_{\phi} \left[\frac{1}{2} (\sigma^2 \phi W^2 + 2\rho\sigma v\phi NSW + v^2 N^2 S^2) J_{WW} + \frac{1}{2} v^2 S^2 J_{SS} \right. \\ \left. + (v^2 NS^2 + \rho\sigma v\phi SW) J_{WS} + (rW + \mu\phi W + \eta NS) J_W \right. \\ \left. + (r + \eta) S J_S + \lambda J + \lambda \frac{W^{1-\gamma}}{1-\gamma} \right] = 0. \end{aligned}$$

Differentiating this formulae with respect to ϕ gives following first-order conditions

$$\phi^* = -\frac{\mu}{\sigma^2} \left(\frac{J_W}{WJ_{WW}} \right) - \frac{\rho v S}{\sigma} \left(\frac{J_{WS}}{WJ_{WW}} \right) - \frac{\rho v NS}{\sigma W}.$$

We conjecture (and then verify) that the derived utility of wealth function is of the form

$$J(W, X) = \frac{W^{1-\gamma}}{1-\gamma} G(X).$$

Differentiating this expression (via the chain rule) with respect to the variables W, S and substituting into the first-order conditions. So, the following equation is just Hamilton–Jacobi–Bellman equation:

$$\begin{aligned} & \frac{1}{2}(\rho\sigma v\phi^*X + v^2X^2)(-\gamma(1-\gamma)G + 2\gamma XG_X + X^2G_{XX}) \\ & + \left(\frac{\rho\sigma v}{2}\phi^*X + v^2X^2\right)(-\gamma G_X - XG_{XX}) + \frac{v^2X^2}{2}G_{XX} \\ & + \left(r + \frac{\mu\phi^*}{2} + \eta X\right)((1-\gamma)G - XG_X) + (r + \eta)XG_X + \lambda G + \lambda = 0. \end{aligned}$$

This equation depends only on $G(X)$ and its derivatives with respect to X . So, our conjecture is verified if we can demonstrate that $G(X)$ is independent of W on the initial values.

Compared to Merton [10], the initial conditions should be

$$\begin{aligned} G(0) &= \frac{\lambda}{\lambda - \rho}, \\ G_X(0) &= 0. \end{aligned}$$

In solving for $G(X)$, we compute the function values numerically using a standard finite difference technique. In particular, we linearize the differential equation for $G(X)$ by evaluating ϕ^* using the estimated values of the function and its derivatives.

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