can shed any light on how their conditions relate to those in Forni *et al.* (2004) in the context of generalized dynamic factor models.

**Cheng Yong Tang** (*University of Colorado, Denver*) **and Yingying Fan** (*University of Southern California, Los Angeles*)

We most heartily congratulate Fan and his colleagues for their thought-provoking and impactful work on estimating the large covariance matrix, which is pivotal in many contemporary scientific and practical studies. Facilitated by a factor model, a parsimonious structure is proposed for the large covariance matrix by combining a low rank matrix and a sparse covariance matrix. In the authors' framework, a factor model is used to characterize the systematic common components underlying the target large-scale dynamics in various problems, and a sparse covariance matrix is imposed to incorporate the remaining idiosyncratic contributions to the variations and covariations. Our comments are mainly on the treatment for the idiosyncratic component, i.e. the remaining dynamics after identifying and removing the systematic part.

An important assumption of the approach proposed is that a sparse covariance matrix $\mathbf{\Sigma}_u$ is imposed for modelling the idiosyncratic component. One may naturally wonder that, in situations when a sparse $\mathbf{\Sigma}_u$ is inadequate, what alternative approach can be used for modelling the idiosyncratic component. Further, can a similar idea of parsimonious modelling by structural decomposition be extended for solving other problems such as large precision matrix estimation? In the framework of graphical models, Tang and Fan (2013) investigate the problem of large precision matrix estimation by parsimoniously modelling the idiosyncratic component by using a sparse precision matrix $\mathbf{\Omega}_u = \mathbf{\Sigma}_u^{-1}$. They observe that the large-scale precision matrix $\mathbf{\Omega}_u = \mathbf{\Sigma}^{-1}$ depends on the idiosyncratic component only through the precision matrix $\mathbf{\Omega}_u$. Thus a similar idea of structural decomposition can be equally applied for estimating the large precision matrix, with the systematic component being captured by a factor model. Facilitated by the interpretation that 0s in a precision matrix imply conditional independence between the corresponding components, a sparse $\mathbf{\Omega}_u$ can have useful practical implications. For example, in the famous Fama–French factor model (Fama and French, 1993) in finance, a non-diagonal sparse precision matrix for the idiosyncratic component characterizes the interpretable market effects among returns of stocks at different levels, such as the industrial segmentwise connections, and the intrinsic within-industry associations, say, among financial firms. Existence of such effects after removing the dynamics corresponding to the systematic component may result in a non-sparse $\mathbf{\Sigma}_u$, yet sparse modelling can still be valid by exploring the sparse precision matrix $\mathbf{\Omega}_u$.

**Joong-Ho Won** (*Korea University, Seoul*) **and Woncheol Jang and Johan Lim** (*Seoul National University*)

We congratulate Fan and his colleagues for a stimulating paper in which they have made a substantial contribution to challenging problems in large covariance estimation.

As practitioners, we are most interested in finite sample positive definiteness of the estimator proposed by the authors. They suggest using a scaling constant $C$ in the threshold for the idiosyncratic covariance matrix $\mathbf{\Sigma}_{u,K}^{\mathcal{T}}$ and adjusting $C$ to render its minimum eigenvalue positive. This idea leads to the univariate root finding procedure of expression (4.1). Although this procedure looks apparently simple, it requires computing the minimum eigenvalue of a $p \times p$ matrix, which is computationally expensive by itself for even a modest value of $p$, for every value of $C$ tried. Furthermore, altering $C$ means that the thresholding must be recomputed in every iteration, changing the sparsity pattern of the initial $\mathbf{\Sigma}_{u,K}^{\mathcal{T}}$. Thus we are concerned that the resulting cost of solving expression (4.1) may not be so cheap, especially when the target function in it is not smooth (Fig. 1).

Here we consider an alternative procedure that ensures positive definiteness while preserving the initial sparsity pattern. First, project $\mathbf{\Sigma}_{u,K}^{\mathcal{T}}$ onto a space of positive definite matrices. This can be done by solving

$$\text{minimize}\|X - \mathbf{\Sigma}_{u,K}^{\mathcal{T}}\|^2, \qquad \text{subject to } \lambda_{\min}(X) \geqslant \mu, \tag{12}$$

for a matrix variable $X$ and some $\mu > 0$. The solution to problem (12) is given by $X^* = \Sigma_{i=1}^{p} \max\{\lambda_i, \mu\} q_i q_i'$ for the spectral decomposition of $\mathbf{\Sigma}_{u,K}^{\mathcal{T}} = \Sigma_{i=1}^{p} \lambda_i q_i q_i'$ (Boyd and Vandenberghe, 2004). Second, replace the entries of $X^*$ that correspond to the zero-thresholded entries of $\mathbf{\Sigma}_{u,K}^{\mathcal{T}}$ with 0. Repeat these two steps until convergence. This alternating projections procedure is guaranteed to converge, as both steps are convex (Boyd and Dattorro, 2003). The first step (12) requires a spectral decomposition of $\mathbf{\Sigma}_{u,K}^{\mathcal{T}}$ as in the root finding procedure, but the second step is free of comparisons with varying thresholds.