Financial Contracting and the Allocation of Skill and Talent: The Case of NIL in College Sports*

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Abstract

We study how compensating student-athletes via NIL affects the allocation of player talent and coaching skill. A general concern is that allowing such compensation will *decrease* league competitiveness. Using a dynamic model of the two-sided selection process between student-athletes and schools, we show that NIL can *increase* competitiveness, but the effect depends critically on both the quality of the incoming recruiting class and differences in coaching skill across schools. A transfer portal can reverse the positive effect of NIL on competitiveness. However, competitiveness is not synonymous with social welfare because of inefficient pairing of athlete talent and coaching skill.

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1 Introduction

Compensating student-athletes for their Name, Image, and Likeness (NIL) has greatly expanded the financial contracting opportunities among schools, sponsors, boosters, and athletes. Under the settlement agreement in the House v. NCAA lawsuit approved in June of 2025, players can be compensated for use of their NIL either directly by the schools they play for or by third parties such as corporate sponsors.[1] A key unanswered question is whether this innovation will ultimately make college sports more or less competitive. It is logical to think that allowing schools to pay players to sign with them should benefit "elite" schools, enabling them to attract an even larger number of promising prospects away from lesser-known "aspiring" schools. This could result in further concentration of talent among elite schools.

At first blush this argument appears compelling. However, it assumes that just because elite schools have the *ability* to pay to attract prospects away from aspiring schools, they are always *willing* to do so. We show that this is not the case, because elite schools respond differently to the introduction of NIL than aspiring schools. Aspiring schools have a strong incentive to attract promising prospects because that gives them an opportunity to improve their reputation and capture future rents from being recognized as elite. With the introduction of NIL, they are able to make aggressive offers by sharing some of these rents with the players. Elite schools are not always willing to match these offers for two reasons. First, since they already have an elite reputation, they don't have an incentive to share their future rents with players. Second, their reputation makes it easier for them to find and attract other highly talented players, which leads to a high opportunity cost of committing to a given prospect.

At the same time, incoming prospects consider not only a school's reputation and any NIL on offer, but also the amount of playing time they can expect at an elite school versus

[1]The House v. NCAA 2025 settlement allows schools to directly compensate players for use of their NIL, institutional brand promotion, and other rights. For simplicity, we use the umbrella term NIL throughout the paper to represent all such compensation either paid directly from the school or NIL collectives associated with the schools, as well as any third parties payments that are (explicitly or implicitly) made contingent on playing for a particular school. Third party NIL deals that are not in any way contingent on playing for a particular school will not affect a player's marginal school choice decision and thus are not relevant to our analysis.

an aspiring school. Since elite schools are likely to be more attractive to superstar players, the possibility of getting benched in favor of a more talented recruit is higher at elite schools. Playing time is particularly important for "unproven" prospects who have promise but require time to develop their talents to have a chance at joining the professional leagues.

To understand these tradeoffs, we develop a dynamic model of the two-sided selection process between schools and players, with a new recruiting class arriving each year. Elite schools are able to attract the superstars in each class, as they are known to have skilled coaches who can help players improve their pro salary prospects. Aspiring schools find it difficult to compete for superstars, but can be a good fit for unproven prospects who could turn out to be pro material if given time to develop. Since the supply of superstars is constrained, elite and aspiring schools compete directly to attract promising unproven prospects. Our dynamic model allows us to characterize how different players value the advantages and disadvantages associated with different schools, and how schools value the benefits created by each type of player. We derive a steady state equilibrium in which the value of being known as an elite school impacts the incentive of aspiring schools to become elite, which feeds back into determining the value of being elite and the overall level of competitiveness in the league.

It turns out that the effect of NIL on competitiveness depends critically on the expected quality of incoming recruits and the expected contribution of coaching skill in improving a player's pro salary prospects. When the proportion of superstars in incoming classes is low, even unproven prospects who need playing time prefer to go with elite schools, as the probability of their getting benched in favor of a superstar is low. In these cases, the introduction of NIL changes the distribution of rents away from schools toward players, but does not affect the allocation of talent across schools. However, when the proportion of superstars is higher, unproven prospects are more likely to be concerned about playing time and thus explore the benefits of signing with aspiring schools. The introduction of NIL intensifies this effect because aspiring schools are now able to make offers to these prospects that share their future expected rents from potentially becoming elite. As noted above, elite schools may not be willing to match these offers because of their higher opportunity cost of taking on unproven prospects and their unwillingness to share their future rents. Thus, over

a significant parameter range NIL can have a strong positive effect on league competitiveness, as a greater number of aspiring schools become elite and are able to compete for all levels of talent.

The above result holds most strongly when the expected contribution of coaching skill in improving a player's pro salary is low. In this circumstance, players perceive little benefit from going with an elite school, so the schools are competing largely on the basis of NIL. However, when coaching skill creates a larger expected pro salary differential, elite schools do not have to fully match the NIL payments offered by aspiring schools. This holds as long as the players perceive a low probability of being benched. It turns out that even with a greater supply of superstars, getting benched is less of a concern when NIL is allowed, since the cost to elite schools of attracting a more talented replacement increases. Thus, in such cases the introduction of NIL can actually *decrease* league competitiveness by concentrating talent among a smaller group of elite schools.

While our main results do not allow for the operation of a transfer portal, we consider an extension that allows elite schools to make NIL offers to induce prospects to transfer from aspiring schools after their first season. We find that there sometimes exists a "Full Poaching" equilibrium, in which unproven prospects always sign initially with aspiring schools, but then are often enticed away by elite schools if they perform well and become "proven." This equilibrium is more likely to exist when the proportion of superstars is low, exactly where, without the transfer portal, unproven players would have been more likely to sign with elite schools. Thus, the introduction of a transfer portal entails a significant movement away from elite schools at the initial signing date, but then a large movement toward those schools by proven players via the transfer portal. This equilibrium exhibits particularly low league competitiveness, as the poaching by elite schools makes it difficult for aspiring schools to become elite. Thus, the combination of NIL with a transfer portal can lead to even lower league competitiveness than a No NIL regime. This result is intensified when the expected contribution of coaching skill in improving a player's pro salary is high, as this makes it cheaper for elite schools to poach proven players and the Full Poaching equilibrium thus exists over a larger parameter space.

While the effect on league competitiveness is a key issue, there is also a question of how

NIL affects social welfare. Our model allows us to identify whether coaching skill and player talent are better utilized in equilibria with or without NIL and with or without a transfer portal. It should be noted, though, that the model focuses only on rents enjoyed directly by players and schools, and potentially ignores some relevant externalities. In addition, the welfare function is not analytically tractable, so we can only provide examples over specific parameter spaces. We find that NIL sometimes induces equilibria with greater league competitiveness but lower social welfare, because some measure of coaching skill or player talent is not being fully exploited. For example, when the introduction of NIL causes unproven prospects to start preferring aspiring schools, league competitiveness increases but welfare can be reduced because the skill of elite school coaches is underutilized. When elite coaches lose out on a player, they are forced to play walk-ons until they can successfully attract another suitable pro prospect. In other situations, greater competitiveness goes hand in hand with higher welfare because it prevents inefficient benching. There are also cases where NIL reduces competitiveness but increases welfare, as NIL can reduce the benching of good unproven players by elite schools, and allow these players to take greater advantage of coaching skill.

We also consider the effect of the transfer portal on welfare. Since elite schools are able to poach many of the proven players between seasons, league competitiveness is often lower, as discussed above. However, this substantially improves the matching between player talent and coaching skill, which can have a more than offsetting positive welfare effect. Thus, when the Full Poaching equilibrium exists, the combination of NIL and the transfer portal can improve welfare despite an overall reduction in competitiveness.

Related Literature

Two recent empirical papers directly study how the introduction of NIL has affected league competitiveness. Li and Derdenger (2025) examine the distribution of three, four, and five star recruits at college football programs before and after the introduction of NIL, and find an increase in the dispersion of talent. They also find that competitiveness improves post-NIL, as sports-books set smaller point differentials even after controlling for talent, performance, and the transfer portal. On the other hand, Owens, Rennhoff, and Roach (2024) find evidence

that post-NIL, elite programs are able to target higher quality recruits, while lesser-known programs reduce the threshold for recruit quality. They conclude that NIL is likely to result in talent being more concentrated at elite schools. Our theoretical results provide important context to these conflicting results, and provide new testable hypotheses for future work.

Several earlier papers studied the effect of free agency in professional sports, which like NIL gave greater bargaining power to athletes. These papers generally tested Rottenberg's (1956) 'Invariance Principle,' which suggests that the allocation of talent across teams will not be affected by the rules governing how players move between teams. For example, Besanko and Simon (1985) document that competitiveness in baseball was not significantly affected by the 1977 introduction of free agency, while Maxcy and Mondello (2006) find similar non-effects of free agency in the NBA, NHL and NFL.

Similar to the advent of free agency in baseball, the 1940s saw a major change in the relationships between actors and movie studios after anti-trust challenges ended the practice of studios locking actors into permanent contracts. In this context, Hanssen and Raskovich (2020) show that after the change, studios reduced the level of experimentation with new actors.

There are also a number of related strands of research in the labor literature. For example, in our setting, unproven student-athletes sometimes sign with aspiring schools even when elite schools are willing to accept them. This is similar to models of competitive assignment in labor markets, such as Gabaix and Landier (2008) and Tervio (2008), in which lower quality firms end up employing less skilled workers since the wages for higher skill workers are bid up by higher quality firms. However, there is no underutilization of skill in these models, which occurs in our setting because of the inability of elite schools to credibly commit not to bench a talented player.

Our paper is also related to Tervio (2009), in which employee talent gets discovered through on-job performance, and then becomes publicly known. In that setting, the paper shows that there is sometimes under-experimentation with uncertain new talent and over-dependence on revealed talent, which results in sub-optimal discovery of new talent and higher salaries for known talent. In our model, by contrast, there is skill or talent discovery on both sides of the relationship, and while we sometimes have a similar under-experimentation

effect, where aspiring schools find it difficult to become elite and league competitiveness suffers, at other times we have over-experimentation, which results in underutilization of known coaching skill.[2]

Strobl and Van Wesep (2013) study an equilibrium where firms have a reputation for publicizing employee talent. Such firms lose talent to better competitors and, thus, are unlikely to become the most productive. However, they gain by paying less to these employees during their stay. This is similar to what happens in our poaching equilibria, where aspiring schools lose their talented athletes to elite schools after their first season, which reduces the probability that aspiring schools will become elite. While such schools gain from having a talented player for at least one season, competitiveness suffers as fewer aspiring schools can become elite and elite schools become stronger.

There is also a literature on employee turnover when firms privately learn about employee talent through their on-the-job performance. In the seminal work of Waldman (1984) and Greenwald (1986), competitors do not learn this information, but can infer it by observing the jobs to which these employees are subsequently assigned. Once informed, they can make offers to employees they find attractive. To be able to retain talented employees, firms attempt to obfuscate through inefficient task assignments. Subsequent papers build on this framework and identify frictions that dampen talent loss. See Bernhardt and Scoones (1993), Burdett and Coles (2010), Berk, van Binsbergen, and Liu (2017), Haltiwanger, Hyatt, and McEntarfer (2018) and Ferreiro and Nikolowa (2023) among others. Unlike these papers, in our setting athlete talent is revealed through their game-time performance, so it is publicly available to other schools and obfuscation is unlikely to succeed.

Axelson and Bond (2015) and Ohlendorf and Schmitz (2012) identify a "talent scorned" effect whereby high-skill workers are sometimes passed over for high-wage jobs they would prefer. In both cases, this effect is driven by dynamic moral hazard considerations, which we do not consider. However, it is similar in flavor to our result that high-skill parties (i.e., elite schools) are harder to partner with since they have better outside options.

[2]The latter result arises because of the two-stage nature of the relationship between schools and athletes, which makes it costly for elite schools to take on unproven players as it can delay their return to the market to search for superstars. This feature of our model also allows us to demonstrate the possibility of wasted athlete talent due to benching, and study the effects of a transfer portal.

There is also a large literature on two-sided matching models for applications like college admissions and school choice (see, e.g., Gale and Shapley, 1962, Roth and Sotomayor, 1992, and Abdulkadiroglu and Sonmez, 2003). However, this literature does not generally consider issues like dynamic talent or skill discovery, and is thus less relevant to the current paper. One notable exception is MacLeod and Urquiola (2015), which shows that students' desire to signal their skills to employers leads to a stratification in college reputations based on the average quality of their students. More closely related is Khanna and Mathews (2022), which uses a two-sided matching framework to study how frictions between reputation and commitment impact the choices of startups and financiers in venture capital markets. However, they do not consider how the results depend on the contracting environment, which is the focus of the current paper.

2 Model

Relationships between schools and players last up to two periods. Schools are represented by their current coaching staff, which makes all player-related decisions. We assume that coaches' contracts with the school are fixed, and coaches consider only the benefits that they are able to capture via their contracts when making player-level decisions. This is meant to capture as simply as possible the complicated dynamics between schools and athletic departments as to how the benefits of successful athletic programs (enhanced reputation, student recruiting, fundraising, etc) are divided between the various stakeholders (coaches, administrators, faculty, staff, students, alumni, etc), and how this division of benefits will impact recruiting decisions at the player level.[3] From here forward, whenever we refer to a school making a decision or realizing a benefit, it refers to the decisions and benefits realized by the current coaching staff.

The model focuses on the market for a single position (i.e., starting quarterback, point guard, etc), and we assume each school can have at most one player at that position at

[3]There is a clear agency problem to the extent that coaching staff with short-lived careers make decisions that could impact schools' long run well-being. It is beyond the scope of this paper to study how contracts between schools and coaching staff are designed to help manage or mitigate such problems, particularly given that administrators who design such contracts are themselves short-lived and potentially myopic.

any given time. Each period a unit mass of "pro prospects" enters the market for the position—these are players who have the potential to play professionally after their college career. There is also an unlimited supply of other, "walk-on" players available. The benefits that walk-ons can generate for schools is normalized to zero. Among the pro prospects, a proportion γ are "superstar" prospects, while the remaining $1-\gamma$ are "unproven" prospects. A superstar prospect is certain to provide a base benefit of 1 per period to any school, while an unproven prospect is able to provide a benefit of 1 per period only with probability g; with probability $1-g$ they cannot produce any benefits, and are thus equivalent to walk-ons. A prospect who plays two periods at a school and generates benefits for the school can go pro and receive a pro salary.

A prospect's pro salary depends on the skill of their school's coaching staff. Every coaching staff is either skilled or unskilled. If a prospect plays for a school that has a skilled coaching staff in both periods, the player's pro salary is m_s. If the school they played for did not have a skilled coaching staff for both periods, the player's pro salary is $m_u < m_s$. School benefits and pro salaries are publicly observed when they are paid/earned.

At any given period, the skill of a particular school's coaching staff can be known or unknown. There is no asymmetric information in the model, so coaching staffs with unknown skill do not know their own skill. A staff is known to be skilled if they have ever had one of their prospects realize a pro salary of m_s. On the other hand, if a school's staff ever has a prospect go pro and receive a salary of m_u, then it is known that they are unskilled, and we assume they are replaced at that point with a new staff of unknown skill. Any newly hired coaching staff is skilled with known probability q. There is also a probability φ per period that an incumbent coaching staff departs for exogenous reasons at the end of the period (e.g., a health episode or career change for the head coach) and is replaced by a staff with unknown skill before the next period. From here forward, we refer to a school that is known to have a skilled coaching staff as an "elite school," and a school whose coaching staff has unknown skill as an "aspiring school." The mass of elite schools looking for a new player in a given period t is denoted by η_t. We restrict our attention to cases where $\eta_t > \gamma$, so that some elite schools will have to consider attracting unproven prospects (as otherwise all elite schools always end up with superstars every period and there is no meaningful competition

between elite and aspiring schools). There is always a large mass of aspiring schools looking for new players in every period.

When the incoming class of prospects enters the market at the beginning of the period, each superstar is matched one-to-one to a "best fit" elite school. If they play for that school for two periods, they will generate an additional school benefit of $\Delta > 0$ in the second period (over and above the base benefit of 1 per period), assuming the coaching staff does not depart before the second period. This is to reflect as simply as possible the idea that superstar prospects on the one hand, and elite schools on the other hand, receive heightened exposure in the media such that their unique characteristics become well known before the class enters the market.[4] Because of this informational advantage, the compatibility between school and player characteristics is more easily identified, leading to additional potential benefits from optimal matching. When NIL is allowed, all schools other than the set of best fit elite schools publicly post an NIL offer consisting of an up-front payment at which they are willing to sign any superstar. The best fit school for each superstar can then make an NIL offer directly to the superstar, and the superstar chooses whether to accept this offer or approach another school to take their posted offer. When NIL is not allowed, superstars can sign with their best fit school or another available school with no payments allowed between the parties.

After the superstar market clears, the market for unproven prospects opens. When NIL is allowed, all remaining schools post a price at which they are willing to sign any unproven prospect. The unproven prospects then decide which school to sign with. In equilibrium, all schools of the same type will make the same offer. Since there is always a large mass of aspiring schools available, if unproven prospects prefer to sign with aspiring schools they can all do so. Similarly, if $\eta_t > 1$ and they prefer to sign with elite schools they will all be able to do so. However, if $\eta_t < 1$, some of them will end up signing with aspiring schools even if they prefer elite schools. As above, when NIL is not allowed players can sign with any available school without receiving any payment.

After both markets clear, the season is played and benefits are realized. Coaches decide

[4]For example, sites like 247Sports.com, Rivals.com, and ESPN along with college coaches closely follow promising prospects' in-game performance through their high school careers and rank them on a one to five star scale. They are also likely to suggest which schools would be good matches for those ranked four and five stars.

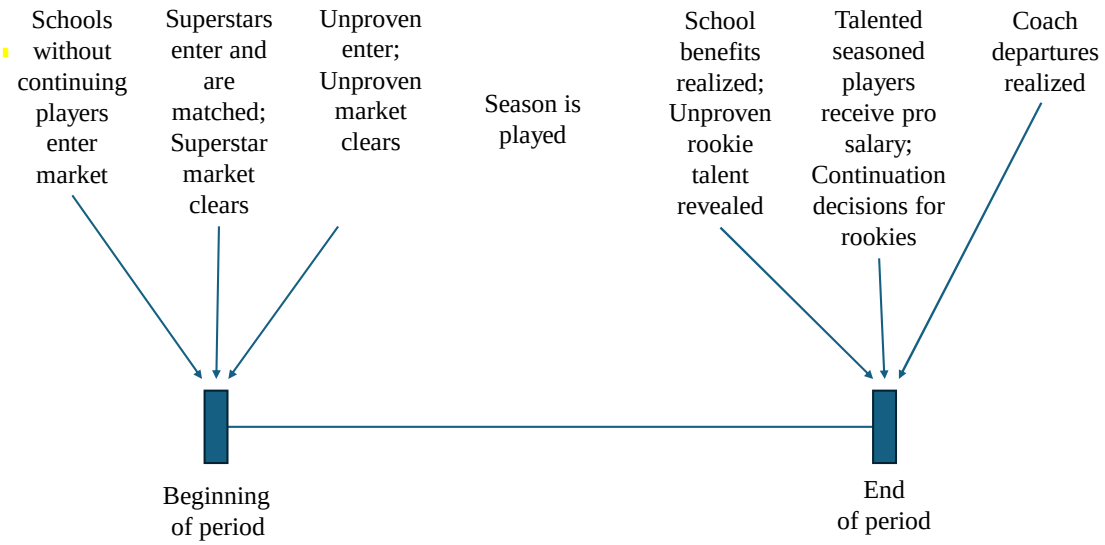

Figure 1: Timeline of the Model

whether to continue a rookie player (play them in their second year, i.e., not bench them) if the player produces a first period benefit. If they do not produce a first period benefit, they are not continued. The continuation decision is taken before knowing whether the coaching staff has to depart for exogenous reasons between periods.

Figure 1 provides an illustration of the timeline for a representative period in the model. Players who are in their first period are referred to as "rookies" while those who are in their second period are referred to as "seasoned" players.

When we allow for NIL, we assume that the cost of NIL, whatever its source, is borne directly by the school, as represented by its coaching staff. In reality, some NIL money may come from third parties such as corporate sponsors or NIL collectives funded by alumni and other boosters. Our assumption is valid to the extent that, in the absence of NIL payments, these funds would have flowed directly to the school via sponsorship deals or donations, and be captured by the coaching staff in the form of higher salaries, improved facilities or working conditions, or other private benefits.

3 Solution for a Simplified Two-Period Game

First we solve a simplified two-period version of the model focused on a single recruiting class, assuming that the value a school will realize when it re-enters the market to find another player is exogenous. In particular, we assume that the expected present value of future rents for an elite (aspiring) school upon reentering the market to recruit a new player is exogenously given as V_e (V_a). Given these values, schools make recruiting decisions taking into account how the decisions are expected to affect the school's status as elite or aspiring and thus its future expected rents. We endogenize these values when we solve the fully dynamic model in Section 4.

3.1 No NIL

Here we solve for the equilibrium of the two-period model when NIL is not allowed. In this case, if an elite school will always continue a prospect if "good" (i.e., if the prospect is a superstar or is an unproven player who reveals their talent by producing a first period benefit), then all prospects will choose an elite school if possible because of the salary benefits offered by the skilled coaching staff. To see this, note that a superstar's discounted expected salary with an elite school is $\beta^2 \left(m_u + (1-\varphi)(m_s - m_u)\right)$, where the skill benefit of $(m_s - m_u)$ is realized only if the coaching staff does not depart between the player's two periods (with probability $(1-\varphi)$). On the other hand, their expected salary with an aspiring school is $\beta^2 \left(m_u + (1-\varphi)q(m_s - m_u)\right)$, where the addition of q accounts for the possibility that the aspiring school does not have a skilled coaching staff. Since this is clearly lower, a superstar will always want to sign with an elite school. For an unproven player, both equations are just multiplied by g, the probability that the player is good and can produce benefits, so the same conclusion holds.

For players to choose an aspiring school over an elite school, there must be some possibility of non-continuation (i.e., benching) of players by elite schools even when they are good. An elite school will continue a best fit superstar if

$$\beta(1-\varphi)(1+\Delta) + \beta(1-\varphi)^2 V_e \geq (1-\varphi)V_e,$$

i.e., if the discounted second period benefit of $1 + \Delta$ plus a delayed realization of V_e (i.e, $\beta(1-\varphi)^2 V_e$) exceeds the payoff to benching the player for the second season and reentering the market right away to try to find another best fit superstar (i.e., $(1-\varphi)V_e$). Note that all of these benefits are conditioned on the coaching staff not departing (which occurs with probability $(1-\varphi)$ each period). This condition simplifies to

$$V_e \leq \frac{\beta(1+\Delta)}{1-\beta(1-\varphi)}$$

which clearly holds for $V_e = 0$, but can be violated for sufficiently high V_e. However, since best fit superstars offer the highest possible benefits when played in both periods, such a high value of V_e is clearly not supportable in a dynamic equilibrium. Thus, for the remainder of this section we assume $V_e \leq \frac{\beta(1+\Delta)}{1-\beta(1-\varphi)}$ holds, so that superstars are always willing to sign with their best fit elite school.

The condition for an elite school to continue playing an unproven prospect who turns out to be good (or a superstar for which they are not the best fit) is similar but does not include a Δ:

$$\beta(1-\varphi) + \beta(1-\varphi)^2 V_e \geq (1-\varphi)V_e,$$

which simplifies to

$$V_e \leq \frac{\beta}{1-\beta(1-\varphi)}.$$

Clearly, this condition will be violated at a lower V_e than the condition for a superstar. Whenever this condition holds, unproven players will prefer to sign with elite schools. When it does not hold, they will prefer to sign with aspiring schools unless even those schools would not continue them if good.

For an aspiring school, continuation of an unproven player who turns out to be good is optimal if

$$\beta(1-\varphi) + \beta(1-\varphi)^2 q V_e \geq (1-\varphi)V_a.$$

Here, V_e is realized only if they continue playing the prospect, and the coaching staff turns out to be skilled and stays for two periods. The alternative is to go back to the market right

away as an aspiring school, yielding V_a. This condition simplifies to

$$V_e \geq \frac{V_a - \beta}{\beta(1-\varphi)q}.$$

Similar to the above condition for continuation of superstars by elite schools, this condition should always hold in a dynamic model—if aspiring schools will not continue good players their skill cannot be discovered, and they cannot enjoy any rents. Thus, we assume $V_e \geq \frac{V_a-\beta}{\beta(1-\varphi)q}$ holds for the remainder of this section. Throughout the paper, we also assume that superstars will choose their best fit elite school when they are indifferent (which is Pareto optimal). We therefore have

Proposition 1. *With no NIL, superstars always sign with their best fit elite schools, while unproven prospects sign with elite schools to the extent they are available iff* $V_e \leq \frac{\beta}{1-\beta(1-\varphi)}$. *Otherwise, all unproven prospects sign with aspiring schools.*

For low values of V_e, unproven players will always sign with elite schools to the extent they are available since they can expect to be continued and receive the highest possible pro salary if they turn out to be good. That is, there is no experimentation by unproven prospects—aspiring schools can only attract unproven prospects to the extent that such prospects outnumber the available elite schools. However, as V_e rises, it becomes more likely that elite schools will choose to bench good unproven players in order to go back to the market to look for a best fit superstar one period earlier. This causes unproven players to consider experimentation by choosing aspiring schools who will continue them.

3.2 NIL

Since there is always an excess supply of aspiring schools, they will compete aggressively when NIL is allowed, making up-front NIL offers for both superstars and unproven prospects such that they are indifferent between recruiting or not recruiting the prospect. This amounts to paying out the entire expected surplus they get from that relationship, which we refer to as their "Net Relationship Surplus," or "NRS." This surplus includes the expectation of any rents they might be able to capture in the future if recognized as skilled. Note that this

implies that with NIL, V_a will be zero. For an aspiring school with a superstar player, the NRS thus equals

$$NRS_a^s \equiv \beta + \beta^2(1-\varphi) + \beta^2 q(1-\varphi)^2 V_e.$$

The first two terms are the discounted value of the benefit of 1 that the coaching staff will receive for sure each period from playing the player (conditional on the coaching staff not departing), and the third term is the present value of the future rents they will realize if they turn out to be skilled. Similarly, for an aspiring school with an unproven player, their NRS and therefore their NIL offer is

$$NRS_a^u \equiv g\beta + g\beta^2(1-\varphi) + g\beta^2 q(1-\varphi)^2 V_e,$$

where the only difference is that all terms are multiplied by the probability that the player is good. Also note that since $V_a = 0$, an aspiring school will always want to continue a good player for the chance to realize V_e.

Since superstars offer the highest possible benefits, including a Δ for their best fit school if played for both periods, it must be the case that elite schools always want to attract and continue best fit superstars in any equilibrium. Thus, the key question is whether it is worthwhile for them to compete for unproven prospects when not matched to a best fit superstar. For the moment, we assume that the mass of unmatched elite schools is lower than the mass of unproven players. When elite schools are relatively scarce, to attract an unproven prospect they must make an NIL offer that makes the player indifferent between their offer and that from an aspiring school. Since aspiring schools are willing to pay out their expected future rents from being elite, this can be quite costly for the elite school. However, they do not need to offer the same NIL as an aspiring school, since they also deliver ex post benefits through a higher expected pro salary. The expected salary differential an unproven prospect gets from joining an elite school (assuming they will continue playing an unproven prospect who turns out to be good) is

$$d \equiv g\beta^2\left(m_u + (1-\varphi)(m_s - m_u)\right) - g\beta^2\left(m_u + (1-\varphi)q(m_s - m_u)\right)$$

$$= g\beta^2(1-\varphi)(1-q)(m_s - m_u).$$

Thus, to attract an unproven player an elite school must offer NIL equal to $NRS_a^u - d$.

Paying this amount to attract an unproven player will be optimal for an elite school instead of the alternative—playing a walk-on this period and hoping to be matched to a superstar next period—only if their own Net Relationship Surplus with the unproven player exceeds the required payment. An elite school's NRS with an unproven prospect equals

$$NRS_e^u = g\left(\beta + \beta^2(1-\varphi) + \beta^2(1-\varphi)^2 V_e\right) + (1-g)\beta(1-\varphi)V_e - \beta(1-\varphi)V_e$$

$$= g\beta + \beta^2(1-\varphi)g + \beta(1-\varphi)g(\beta(1-\varphi)-1)V_e.$$

If the player turns out to be good (with probability g), the school gets both a first and second period benefit of 1 and returns to the market to realize V_e after the second period (the first term in the top equation). If the player turns out to be bad (with probability $1-g$), they get no benefits from the player and return to the market to realize V_e after the first period (the second term in the top equation). The final term, $-\beta(1-\varphi)V_e$, represents the value of the alternative they are giving up by choosing to pay the NIL required to attract the unproven prospect, i.e., playing a walk-on this period and hoping to be matched to a superstar next period. Overall, the fact that taking on the player causes an expected delay in returning to the market to realize V_e implies that the NRS is decreasing in V_e, as illustrated by the negative coefficient on V_e in the bottom equation.

An elite school is thus willing to pay the required NIL to attract an unproven player if $NRS_e^u - (NRS_a^u - d) \geq 0$ when elite schools are scarce. Now consider the case in which elite schools are not scarce relative to unproven players. In this case, if the above condition holds, the elite schools will compete away their remaining rents and offer to pay out their entire NRS, NRS_e^u, resulting in greater rents going to unproven players. However, if the above condition does not hold, elite schools will not make offers sufficient to attract unproven players, and aspiring schools will sign them. Thus, in either case, the condition $NRS_e^u - (NRS_a^u - d) \geq 0$ determines whether unproven players end up signing with elite or aspiring schools.

Replacing the NRS expressions with their definitions and simplifying yields the condition

$$V_e \leq \frac{d}{(1-\varphi)g\beta(1-(1-\varphi)(1-q)\beta)}.$$

From the point of view of an aspiring school, a higher V_e means a stronger incentive to become elite and a higher willingness to pay NIL, while for an elite school a higher V_e means a larger cost to taking on an unproven player and potentially staying out of the matching market for an extra period. Thus, in order for the elite school to be willing to pay up front to attract an unproven player, the salary differential d must be sufficiently large to offset this effect.

This analysis assumes that an elite school will continue an unproven player who turns out to be good. However, as noted in the previous section, as V_e increases an elite school eventually will not be willing to do so. Specifically, this is true when V_e exceeds $\frac{\beta}{1-\beta(1-\varphi)}$. In this case the unproven player knows that a pro salary can never be realized if they sign with an elite school (because they will be benched), and it will be prohibitively costly for an elite school to attract them (to make up for that loss). We thus have the following result.

Proposition 2. *With NIL, superstars always sign with their best fit elite schools, while unproven prospects sign with elite schools to the extent they are available iff* $V_e \leq Min[\frac{\beta}{1-\beta(1-\varphi)}, \frac{d}{(1-\varphi)g\beta(1-(1-\varphi)(1-q)\beta)}]$. *Otherwise, all unproven prospects sign with aspiring schools.*

This result is similar to Proposition 1 in that a larger V_e will tend to result in more experimentation by unproven players through signing with aspiring schools. However, the effect of the introduction of NIL on this tradeoff depends on the size of the expected pro salary differential attainable with elite schools. If the salary differential is small, the introduction of NIL clearly leads to more experimentation—unproven players will sign with aspiring schools for almost any V_e, and since V_a is always zero it is even more imperative for aspiring schools to become elite. However, if the salary differential is large it is not clear whether the effect of NIL is to generate more experimentation. It is possible that at large levels of the salary differential, there will actually be less experimentation when NIL is introduced, i.e., NIL will further cement the advantage of elite schools in such cases. To determine the likelihood of such outcomes, we need to endogenize both the size of the elite population and the value of

being elite.

4 Steady State Equilibria of the Dynamic Game

In steady state equilibria of the dynamic game, the mass of elite schools looking for players each period must be constant. We denote this mass by η, which does not require a time subscript in steady state. As noted previously, we only consider equilibria where $\eta > \gamma$ throughout, i.e., where the mass of elite schools exceeds the mass of new superstars entering the market, since otherwise all elite schools are matched with superstars and there is no margin for experimentation by unproven players (they have to sign with aspiring schools). As before, we let V_e represent the present value of expected future rents for the coaching staff of an elite school as they enter the market for a new player, and V_a likewise for an aspiring school. However, these are now endogenous quantities that must be determined in equilibrium.

For our analytical results, we focus on pure strategy, subgame perfect steady state equilibria in which: (1) superstars always sign with their best fit elite school, (2) schools always continue unproven prospects that they sign if they turn out to be good, and (3) unproven prospects either never sign with elite schools, or sign with them to the full extent possible (subject to their available supply). We thus analyze four different pure strategy equilibria depending on whether or not NIL is allowed, and whether or not unproven players sign with elite schools. Later we discuss the existence of mixed strategy equilibria where elite schools attract unproven players with positive probability less than 1, and may bench them with positive probability even when they turn out to be good.

4.1 No NIL

4.1.1 Unproven Prospects Sign With Elite Schools

In this equilibrium, unproven prospects always sign with elite schools to the extent they are available, and both elite and aspiring schools always continue unproven players that turn

out to be good.[5] We must have $\eta < 1$ in such an equilibrium, as otherwise no aspiring schools can become elite (since they would have no pro prospects to sign), which is required to maintain a steady state. The steady-state dynamics of η are governed by the following equation:

$$\eta = \gamma(1-\varphi)^2 + (1-\varphi)^2 g(\eta - \gamma) + (1-\varphi)(1-g)(\eta-\gamma) + (1-\varphi)^2 qg(1-\eta).$$

The terms, in order, represent: elite schools matched to superstars two periods ago, elite schools that signed unproven prospects two periods ago that turned out to be good, elite schools that signed unproven prospects one period ago that turned out to be bad, and finally aspiring schools that signed unproven players two periods ago that turned out to be good, and whose coaching staff also turned out to be skilled (all conditional on the coaching staff not departing). Solving this for η yields

$$\eta^e \equiv \frac{\gamma(1-\varphi)^2 - (1-\varphi)(\gamma - \gamma\varphi g - (1-\varphi)gq)}{(1-\varphi)^2 gq + \varphi(1-\varphi g + g)}.$$

This equilibrium is feasible if $\eta^e \in (\gamma, 1)$ given our restriction to equilibria with $\eta > \gamma$. It is straightforward to show:

Lemma 1. $\eta^e \in (\gamma, 1)$ holds as long as $\gamma \in [0, \frac{(1-\varphi)^2 gq}{(2-\varphi)\varphi(1-gq)+gq}]$.

Without the possibility of NIL, unproven players will always prefer elite schools if those schools are willing to continue them when they turn out to be good because of the benefit of a higher expected salary. Thus, we just need to find parameters within the constraints of Lemma 1 under which continuation of good unproven players is optimal.

As noted above, an elite school will continue a good unproven player if $\beta(1-\varphi) + \beta(1-\varphi)^2 V_e \geq (1-\varphi)V_e$, i.e, if $V_e \leq \frac{\beta}{1-\beta(1-\varphi)}$. We thus need to solve for the endogenous value of V_e to see when this will hold. We have the following Bellman equation:

$$V_e = \frac{\gamma}{\eta^e}\left[\beta + \beta^2(1-\varphi)(1+\Delta) + \beta^2(1-\varphi)^2 V_e\right]$$

[5]It is straightforward to show that aspiring schools will continue good unproven players anytime elite schools will do so, as otherwise the aspiring schools will not achieve a positive expected value from recruiting players.

$$+\frac{\eta^e-\gamma}{\eta^e}\left[\begin{array}{c}\beta g+g\beta^2(1-\varphi)\\ +g\beta^2(1-\varphi)^2V_e+(1-g)\beta(1-\varphi)V_e\end{array}\right]$$
$$+\left(1-\frac{\gamma}{\eta^e}-\frac{\eta^e-\gamma}{\eta^e}\right)\beta(1-\varphi)V_e.$$

The first line is the expected value from finding a superstar (with probability $\frac{\gamma}{\eta^e}$), in which case the player is continued for sure and will create a benefit for the school of 1 in both periods, and the additional benefit of Δ in the second period. The coaching staff then realizes V_e if they stay. The second line analogously is the expected value from unproven players (which they sign with probability $\frac{\eta^e-\gamma}{\eta^e}$, and continue with probability g), and the third line is the expected one-period delayed realization of V_e when the school does not sign a prospect at all. Since V_e is clearly increasing in the available mass of superstars, γ, if there is a feasible range of existence for this equilibrium it will be at lower γ up to some cutoff. Analyzing this expression gives the following result.

Proposition 3. *A pure strategy equilibrium with No NIL in which unproven players always sign with elite schools to the extent they are available and $\eta^e \in (\gamma, 1)$ exists iff*

$$\gamma \leq \frac{(1-\varphi)^2(1-g)gq}{\beta(1-\varphi)\Delta\left((1-\varphi)^2gq+\varphi(1-\varphi g+g)\right)+(1-g)((2-\varphi)\varphi(1-gq)+gq)}.$$

When the probability of finding a superstar is low, V_e is naturally low and elite schools are willing to continue good unproven players despite their inability to generate an extra payoff of Δ in the second period. However, as γ and therefore V_e increases, elite schools are more likely to go back to the market early in hopes of being matched to a best fit superstar, and thus will not continue good unproven players. As a result, those players will prefer to sign with aspiring schools who will reliably continue them. It turns out that this constraint on γ is always tighter than the constraint given in Lemma 1, so there is only a single cutoff in the equilibrium condition. This result provides a dynamic verification of Proposition 1, and illustrates that the proportion of superstars is a major driver of the endogenous value of V_e.

4.1.2 Unproven Players Sign With Aspiring Schools

In this equilibrium, unproven players always sign with aspiring schools because elite schools will bench them even if good. The dynamics of η in such an equilibrium are governed by the following equation:

$$\eta = \gamma(1-\varphi)^2 + (1-\varphi)(\eta-\gamma) + (1-\gamma)(1-\varphi)^2 gq.$$

The terms, in order, represent: elite schools that were matched to superstars two periods ago, elite schools that were not matched to superstars one period ago, and aspiring schools that signed unproven players two periods ago that turned out to be good, and whose coaching staff also turned out to be skilled (all conditional on the coaching staff not departing). Solving this for η yields

$$\eta^a \equiv \frac{(1-\varphi)(gq(1+\gamma\varphi-\gamma-\varphi)-\gamma\varphi)}{\varphi}.$$

This equilibrium is feasible as long as $\eta^a > \gamma$ (it does not have to be less than 1 since some aspiring schools will become elite given that all unproven players sign with aspiring schools). We have the following result.

Lemma 2. *$\eta^a > \gamma$ holds as long as $\gamma \in [0, \frac{(1-\varphi)^2 gq}{(2-\varphi)\varphi(1-gq)+gq}]$.*

Notably, this equilibrium is feasible for the same range of γ as the equilibrium derived in section 4.1.1.

Within this range, this equilibrium will exist for parameters where elite schools are unwilling to continue good unproven players, but aspiring schools are willing to do so. As noted above, aspiring schools must always be willing to continue good unproven players as otherwise they cannot achieve a positive expected value from recruiting players. Thus, we just need to find all parameters where the equilibrium is feasible and $\beta(1-\varphi)+\beta(1-\varphi)^2 V_e \leq (1-\varphi)V_e$, i.e., $V_e \geq \frac{\beta}{1-\beta(1-\varphi)}$ holds. Since V_e is increasing in γ, we thus look for the γ at which this equality just holds. For this equilibrium we have

$$V_e = \frac{\gamma}{\eta^a}\left[\beta + \beta^2(1-\varphi)(1+\Delta) + \beta^2(1-\varphi)^2 V_e\right]$$

$$+\left(1-\frac{\gamma}{\eta^a}\right)\beta(1-\varphi)V_e.$$

The expression is simpler here than in section 4.1 because elite schools are only able to sign superstars. Analyzing this expression and combining it with Lemma 2 gives the following result.

Proposition 4. *A pure strategy equilibrium with No NIL in which unproven players always sign with aspiring schools and $\eta^a > \gamma$ exists iff*

$$\gamma \in \left[\frac{(1-\varphi)^2 gq}{\beta(1-\varphi)\varphi\Delta + (2-\varphi)\varphi(1-gq) + gq}, \frac{(1-\varphi)^2 gq}{(2-\varphi)\varphi(1-gq)+gq}\right].$$

This result is again consistent with Proposition 1, and dynamically verifies that when the parameters result in a high enough endogenous value of V_e (because superstars are relatively plentiful), unproven players will experiment by signing with aspiring schools. It is straightforward to show that the existence range for this equilibrium does not overlap with the range for the previous equilibrium, i.e., there is an intermediate range of γ for which neither pure strategy equilibrium exists. We later discuss the types of mixed strategy equilibria that can exist over this range. For values of γ above the range given in the proposition, the equilibria have $\eta < \gamma$ and unproven players sign with aspiring schools since all elite schools are matched to superstars, and there is thus no scope for experimentation.

4.1.3 Numerical Example

Here we illustrate our results using a numerical example. For all numerical examples in the paper we use the following base parameters: $\beta = 0.95$, $g = q = 0.5$, $\varphi = 0.05$, and $\Delta = 0.5$. Figure 2 presents an equilibrium "map" that illustrates the areas of existence for the equilibria derived in the previous subsections, as γ varies on the y-axis and d varies on the x-axis. Note that while d depends on several of the base parameters, it is also directly proportional to the difference $m_s - m_u$, which is independent of everything else in the model. Thus, we can treat d as an independent parameter for this purpose as well.

The solid blue rectangle below $\gamma = 0.38$ gives the area where a pure strategy equilibrium exists in which unproven players always sign with elite schools to the extent they are available,

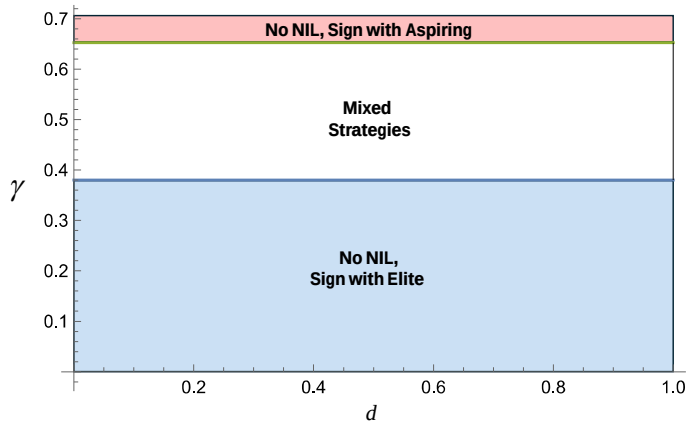

Figure 2: Equilibrium Map with No NIL For Varying Levels of γ and d

corresponding to Proposition 3. The solid red rectangle above $\gamma = 0.65$ gives the area where a pure strategy equilibrium exists in which unproven players always sign with aspiring schools, corresponding to Proposition 4. These areas do not vary with d since their existence is independent of the pro salary advantage—as long as the advantage is positive, players will always prefer elite schools when those schools will continue them if they turn out to be good, and will always prefer aspiring schools when elite schools will bench them even if good. Over the area in between the two rectangles no pure strategy equilibrium exists. Where needed for numerical examples below we construct a mixed strategy equilibrium where elite schools continue good unproven players with some probability if they sign them, and unproven players choose to sign with elite schools with some probability.

4.2 NIL

4.2.1 Unproven Prospects Sign With Elite Schools

In this equilibrium, unproven prospects always sign with elite schools to the extent they are available, and both elite and aspiring schools always continue unproven players that turn out to be good. As before, aspiring schools will compete away all of their rents when NIL is allowed, so $V_a = 0$ will hold. We also must have $\eta < 1$ in such an equilibrium, as otherwise no aspiring schools can become elite. The dynamics of η are identical here to section 4.1.1, so we have $\eta = \eta^e$ and Lemma 1 provides the feasible range of γ for this equilibrium as well.

To attract an unproven player, elite schools have to make them indifferent between their offer and an offer from an aspiring school that gives the player all of the aspiring school's surplus, i.e., aspiring schools offer to pay out NIL equal to their NRS. However, elite schools also offer an expected pro salary differential, so their NIL offer has to just equal the aspiring school's NRS minus the expected salary advantage of the elite school, which is $NRS_a^u - d$ for an unproven player, as derived in section 3.2. Since, as noted above, d is proportional to $(m_s - m_u)$, from here on we refer to the value and changes in d rather than changes in $(m_s - m_u)$ for simplicity.

To attract a superstar, its best fit elite school must beat the better offer between an unmatched elite school and an aspiring school. Given the expected salary differential, the best offer from an aspiring school would require an NIL offer from the best fit elite school of $NRS_a^s - \frac{d}{g}$, where $\frac{d}{g} = \beta^2(1-q)(1-\varphi)(m_s - m_u)$ is the expected pro salary differential for a superstar with an elite school. An unmatched elite school, on the other hand, is willing to offer its full NRS, which is its surplus from attracting a non-best-match superstar less its surplus if it instead attracts an unproven player, or

$$NRS_e^s = \beta + \beta^2(1-\varphi) + \beta^2(1-\varphi)^2 V_e$$

$$- \left[\beta g + g\beta^2(1-\varphi) + g\beta^2(1-\varphi)^2 V_e + (1-g)\beta(1-\varphi)V_e - \left(\beta g + g\beta^2(1-\varphi) + g\beta^2 q(1-\varphi)^2 V_e - d\right)\right].$$

In the proof of Proposition 5 we show that $NRS_e^s > NRS_a^s - \frac{d}{g}$ within the relevant parameter space, so potential offers from unmatched elite schools determine the required NIL payment for best fit elite schools in this equilibrium, which is therefore NRS_e^s.

To find parameters where this equilibrium exists, we need to show that both types of schools are willing to continue good unproven players, and that elite schools are willing to pay the required NIL to attract both best fit superstars and unproven players. Aspiring schools are always willing to continue good unproven players since their expected rents are zero if they bench such players. We thus need to ensure that elite schools are actually willing to continue good unproven players ex post after recruiting them, and that NIL payments are always positive (i.e., prospects never have to pay to sign with an elite school because of a potentially large expected salary differential). Here we state an assumption on d that is

sufficient to ensure that these conditions hold, and also supports the equilibrium conditions in section 4.4 below. The sufficiency of this assumption is proven in the appendix.

Assumption 1: $d < Min\left[\frac{\beta^2(1-\varphi)g(1-\beta(1+\varphi q-\varphi-q))}{1-\beta(1-\varphi)}, \frac{\beta g(1-\beta(1-\varphi)(\beta(1-\varphi)(1-\Delta-q)+\Delta))}{1-\beta(1-\varphi)}, \beta g(1+\beta(1-\varphi))\right]$

It is straightforward to show that for much of the parameter space (which covers all of our numerical illustrations), the third expression is the smallest and is thus the operative upper limit on d.

Given the sufficiency of this assumption, we just need to determine parameters where elite schools are willing to pay enough NIL to attract unproven players (which subsumes their willingness to do so for best fit superstars). We first derive V_e as follows:

$$V_e = \frac{\gamma}{\eta^e}\left[\beta + \beta^2(1-\varphi)(1+\Delta) - NRS_e^s + \beta^2(1-\varphi)^2 V_e\right]$$

$$+\frac{\eta^e-\gamma}{\eta^e}\begin{bmatrix} \beta g + g\beta^2(1-\varphi) - (NRS_a^u - d) \\ +g\beta^2(1-\varphi)^2 V_e + (1-g)\beta(1-\varphi)V_e \end{bmatrix}$$

$$+\left(1-\frac{\gamma}{\eta^e}-\frac{\eta^e-\gamma}{\eta^e}\right)\beta\left[(1-\varphi)V_e\right].$$

This is similar to the expression for V_e from section 4.1.1, except that both the first two lines now include a subtraction of the required NIL payment, which is based on the NIL offered by unmatched elite and aspiring schools, respectively. Analyzing this expression and combining it with Lemma 1 gives the following result.

Proposition 5. *A pure strategy equilibrium with NIL in which unproven players always sign with elite schools to the extent they are available and* $\eta^e \in (\gamma, 1)$ *exists iff*

$$\gamma \leq Min\left[\begin{array}{c} \left(\frac{(1-\varphi)^2 gq}{(2-\varphi)\varphi(1-gq)+gq}\right), \\ \left(\frac{(1-\varphi)gq(1-\beta(1-\varphi))d}{\varphi(1-g)(1-\beta(1-\varphi))d+\beta^3(1-\varphi)\Delta g((1-\varphi)^2 gq+\varphi(1-\varphi g+g))(1-\beta(1-\varphi)(1-q))}\right) \end{array}\right].$$

Like in Proposition 2, we cannot rank the two possible upper bounds since d can have any magnitude satisfying Assumption 1. Consistent with Proposition 2, this equilibrium exists when γ and therefore V_e is small, making elite schools willing to pay for and continue good

unproven players rather than go back to the market to find a best fit superstar. For smaller d the elite schools' willingness to pay is the constraint on existence of the equilibrium, while for larger d keeping η greater than γ is the constraint on existence of the equilibrium.

Comparing this to the No NIL case, it is straightforward to show that the upper limit on γ in Proposition 3 is lower than the first expression in the Min function in Proposition 5. Thus, when d is low enough that the second expression in the Min function is the lowest, the upper limit on γ will be higher without NIL than with NIL, implying more experimentation when NIL is introduced. However, when d is high it is possible that the upper limit will actually be higher with NIL, resulting in less experimentation when NIL is introduced. The introduction of NIL has two effects on the conditions for equilibrium existence. First, it allows aspiring schools to pay out their expected surplus to attract players, forcing elite schools to pay more to attract them, resulting in greater experimentation. Second, it leads to a reduction in V_e, which makes elite schools more willing to continue good unproven players (i.e., it reduces the rents to going back to the market hoping to be matched to a superstar). When d is low, the former effect is large relative to the latter effect, so introducing NIL leads to more experimentation. However, when d is high the former effect is attenuated because the large salary differential minimizes the amount of NIL the elite schools have to pay, so the latter effect can dominate, leading to less experimentation.

4.2.2 NIL, Unproven Players Sign With Aspiring Schools

In this equilibrium, unproven players always sign with aspiring schools. For this to be the case, elite schools must be unwilling to pay the NIL necessary to attract them. The dynamics of η are identical here to section 4.2, so we have $\eta = \eta^a$ and Lemma 2 provides the feasible range of γ for this equilibrium as well.

To find parameters where this equilibrium exists, we need to show that elite schools are unwilling to pay for unproven players given that all unproven players are expected to sign with aspiring schools. To do so, we need to know whether the elite school would be willing to continue a good unproven player if they did attract them. Under Assumption 1, they would continue them, so we derive the condition based on this expected behavior. First we

solve for V_e:

$$V_e = \frac{\gamma}{\eta^a}\left[\beta + \beta^2(1-\varphi)(1+\Delta) - \left(NRS_a^s - \frac{d}{g}\right) + \beta^2(1-\varphi)^2 V_e\right]$$

$$+\left(1 - \frac{\gamma}{\eta^a}\right)\beta\left[(1-\varphi)V_e\right].$$

This is similar to the expression for V_e from section 4.2, except that the first line now includes a subtraction of the required NIL payment, which in this case is based on the NIL offered by aspiring schools because, as shown in the proof of Proposition 6, $NRS_a^s - \frac{d}{g} > NRS_e^s$ within the relevant parameter space for this equilibrium (i.e., aspiring schools are willing to make better offers for superstars than unmatched elite schools). Analyzing this expression and combining it with Lemma 2 gives the following result.

Proposition 6. *A pure strategy equilibrium with NIL in which unproven players always sign with aspiring schools and $\eta^a > \gamma$ exists iff*

$$\gamma \in \left[\frac{(1-\varphi)gq(1-\beta(1-\varphi))d}{(1-\beta(1-\varphi))d((1-\varphi)gq+\varphi) + \beta^3(1-\varphi)\varphi\Delta g(1-\beta(1-\varphi)(1-q))}, \frac{(1-\varphi)^2 gq}{(2-\varphi)\varphi(1-gq)+gq}\right].$$

Comparing this to the No NIL case in Proposition 4, the upper limits (based on the $\eta > \gamma$ condition) are the same. When d is small, the range of existence with NIL will be larger. Indeed, at the extreme of $d = 0$, this equilibrium would exist for the entire feasible range of γ, further illustrating the potential for NIL to generate greater experimentation by unproven players. However, for large enough d it is possible for the existence range for this equilibrium to be smaller than with no NIL, which results in less experimentation when NIL is introduced.

4.2.3 Numerical Example

Figure 3 maps the two equilibria for the NIL case given the same base parameter values as used previously. The area with vertical blue lines is where a pure strategy equilibrium exists in which unproven players always sign with elite schools to the extent they are available. The area with horizontal red lines is where a pure strategy equilibrium exists in which unproven

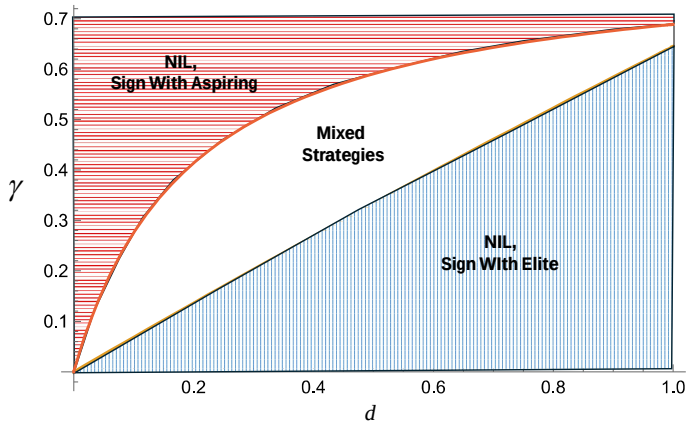

Figure 3: Equilibrium Map with NIL For Varying Levels of γ and d

players always sign with aspiring schools. Consistent with Propositions 5 and 6, the existence thresholds for both of these areas are increasing in d, i.e., the existence range for equilibria where unproven players prefer elite schools expands as the pro salary benefit of coaching skill increases, while the existence range for equilibria where they prefer aspiring schools contracts. As with the No NIL model, no pure strategy equilibrium exists between the two colored areas, so where needed for our numerical examples we construct a mixed strategy equilibrium where elite schools choose to make an acceptable NIL offer to unproven players with some probability.

4.3 Comparison of No NIL and NIL Regimes and Overall Discussion

We now compare the results across the No NIL and NIL regimes using our numerical examples. Figure 4 overlays the two prior figures to demonstrate that, depending on the value of d, the introduction of NIL can result in either an increase or a decrease in experimentation. With low values of d, on the left side of the figure, the relevant thresholds are lower with NIL than without, i.e., the existence range for equilibria where unproven players prefer elite schools is smaller and the existence range where they prefer aspiring schools is larger. This is consistent with NIL increasing experimentation by enabling aspiring schools to more effectively compete for unproven prospects. At the extreme left of the figure, where d and thus the advantage of coaching skill from the player's perspective is very small, this result is

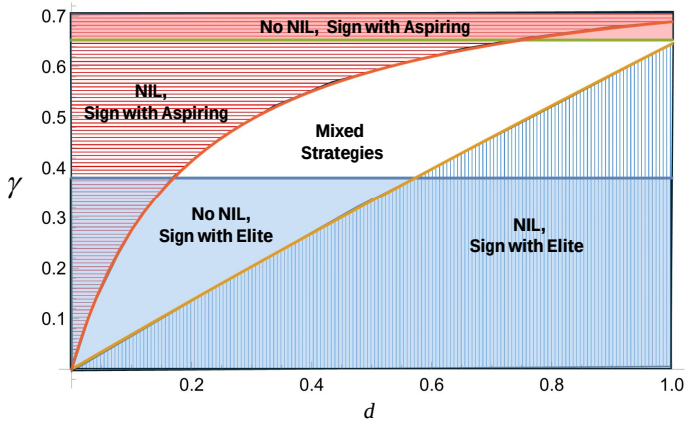

Figure 4: Combined Equilibrium Map For Varying Levels of γ and d

magnified—the equilibrium where unproven players prefer aspiring schools exists over nearly the entire parameter space.

However, on the right side of the figure where d is relatively large, this is reversed, and at the extreme the opposite equilibrium exists over most of the parameter space. This is consistent with NIL decreasing experimentation because it reduces the value of being elite, making elite schools more willing to continue unproven players who turn out to be good.

In Figure 5 we graph the equilibrium mass of elite schools, η, as a function of the proportion of superstars, γ, for the No NIL equilibrium (in red) and for two different values of d for the NIL equilibrium ($d = 0.1$ in blue, and $d = 0.9$ in green), using the same base parameters as above. In our model, η is a critical endogenous variable that represents the extent of competitiveness across schools. A low value of η indicates that few aspiring schools get the chance to become elite, and thus a small number of schools dominate the market, while a high value of η indicates plentiful opportunities for schools to become elite, thereby increasing competitiveness.

For the part of the parameter space where a pure strategy equilibrium does not exist, we construct a mixed strategy equilibrium. With No NIL, the mixed strategy involves unproven players mixing over whether they choose to sign with elite schools, and elite schools mixing over whether they continue good unproven prospects who join them. With NIL, the mixed strategy involves the elite schools mixing over whether to offer sufficient NIL to attract unproven prospects. Thus, at both lower and higher extremes of γ, where pure strategy

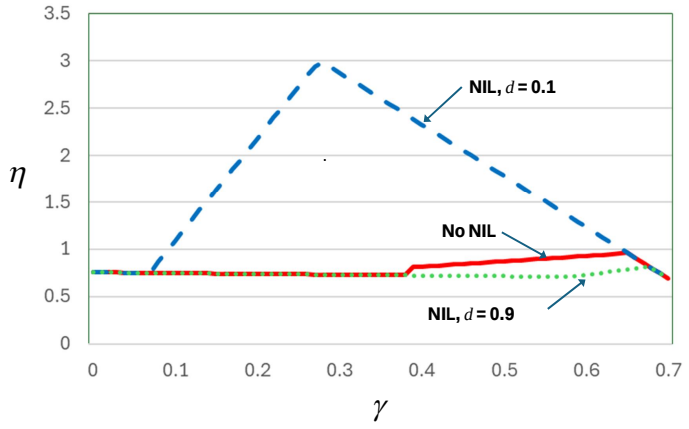

Figure 5: η as a Function of γ

equilibria exist, the value of η is given by either η^e or η^a as appropriate. In the middle, where there are no pure strategies, the value of η is as derived for the corresponding mixed equilibrium.

For the No NIL case, η stays below 1 over the entire range of γ, peaking at $\eta = 0.96$ around $\gamma = 0.65$, where the equilibrium switches from the mixed strategy to a pure strategy in which unproven prospects choose aspiring schools even when elite schools are available. Strikingly, when $d = 0.1$, η becomes substantially higher in the NIL case, peaking at approximately $\eta = 3$ where $\gamma = 0.28$. This clearly illustrates the significant effect of an increase in experimentation by unproven prospects in the NIL case, resulting in a much higher level of competitiveness among schools. However, when $d = 0.9$, η remains low in the NIL case, and is even lower than the No NIL case over a substantial range. Here, NIL leads to greater market concentration and lower competitiveness because the pro salary differential from skilled coaching is much higher.

In Figure 6, we graph the value of being elite, V_e, for the same equilibria and base parameter values as the previous figure. Clearly, the introduction of NIL substantially reduces the rents that elite schools can realize from their coaches' skill, and more so the lower is the pro salary differential. These effects are intuitive—NIL creates a more competitive environment that forces elite schools to pay some of their rents out to both superstar and unproven prospects. However, an increased pro salary differential mitigates this effect. The lowering of the rents to being elite also has a byproduct of making elite schools more willing to continue

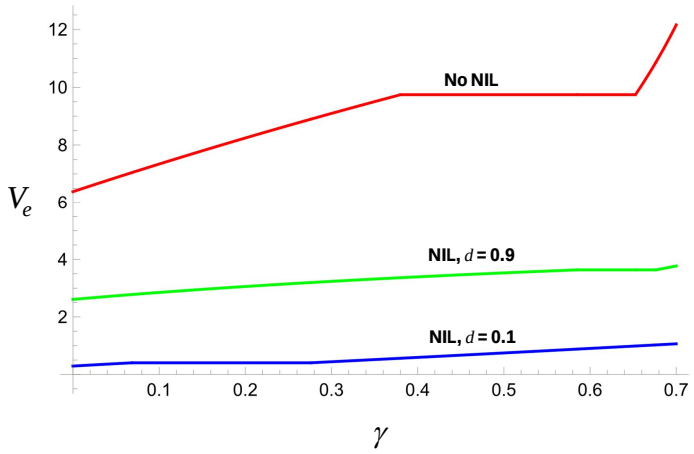

Figure 6: V_e as a Function of γ

good unproven prospects.

5 Social Welfare

We measure social welfare as total discounted expected school surplus plus total discounted expected pro salaries for a given cohort of prospects. This can be calculated as follows for a generic equilibrium (pure or mixed strategy), where p is the probability that an elite school will continue an unproven prospect who turns out to be good and ρ is the probability that unproven prospects sign with elite schools to the extent they are available:

$$\gamma\left(\beta+\beta^2(1+(1-\varphi)\Delta)+\beta^2\left(m_u+(1-\varphi)\left(m_s-m_u\right)\right)\right)$$

$$+\rho(Min(\eta,1)-\gamma)\left(\beta g+\beta^2 gp\left(1+m_u+(1-\varphi)\left(m_s-m_u\right)\right)\right)$$

$$+(1-\rho)(Min(\eta,1)-\gamma)\left(\beta g+\beta^2 g\left(1+m_u+(1-\varphi)q\left(m_s-m_u\right)\right)\right)$$

$$+(1-Min(\eta,1))\left(\beta g+\beta^2 g\left(1+m_u+(1-\varphi)q\left(m_s-m_u\right)\right)\right)$$

The first line of the equation represents superstars, who all sign with their best fit elite schools. They produce certain benefits of 1 per period for the school, plus Δ in the second period (as long as the skilled coaching staff does not depart between seasons), and get pro salaries of m_s (unless the coaching staff departs, in which case they get only m_u). The second

line represents unproven prospects who have the opportunity to sign with elite schools (with mass $(Min(\eta, 1) - \gamma)$) and sign with them (with probability ρ). They produce benefits only with probability g, are continued with probability p, and do not generate a Δ even if continued. The last two lines represent unproven prospects who sign with aspiring schools (either because they chose an aspiring school, or because they did not have an opportunity to sign with an elite school). Their expected benefits differ only by the probability q that the coaching staff in an aspiring school is skilled and can generate the $m_s - m_u$ salary differential.

Since analytical results are not tractable, we numerically evaluate this welfare function for each of the cases and equilibria that were explored in the previous section. Note that since m_u and m_s enter the welfare equation separate from d, we set values for those parameters that are consistent with our chosen values of d. For simplicity, we set $m_u = 0$ for both cases, which implies that $m_s = 0.466$ for the $d = 0.1$ case and $m_s = 4.199$ for the $d = 0.9$ case.

Figure 7 graphs the welfare function for the No NIL and NIL equilibria that exist as γ increases when $d = 0.1$. When both cases simultaneously have equilibria where $\rho = 1$, i.e., unproven prospects always sign with elite schools to the extent they are available, or where $\rho = 0$, i.e., unproven prospects always sign with aspiring schools, their welfare functions coincide. However, over a large parameter space in the middle, they diverge. First, as γ rises to the point where the NIL equilibrium switches to a mixed strategy, the NIL welfare function dips below the No NIL welfare function. This is because some of the coaching skill of elite schools is being wasted—because of NIL, aspiring schools are able to attract unproven prospects away from elite schools, leaving their skill idle for a period. However, when γ gets to the point where the No NIL equilibrium switches to a mixed strategy, the No NIL welfare function quickly drops below the NIL welfare function. This quick drop is due to the fact that, in order to get prospects to mix over their school choice, the probability with which they are benched by elite schools even when good must quickly rise from zero to $1 - p = 1 - q$, so in this case some of the prospects' talent is wasted due to benching.

Figure 8 repeats the above exercise, but with $d = 0.9$. Here, the No NIL and NIL cases have overlapping $\rho = 1$ equilibria over a large parameter space, and thus their welfare functions coincide. However, when the No NIL case switches to a mixed strategy, we have the same discontinuous drop due to an increased probability of good unproven prospects

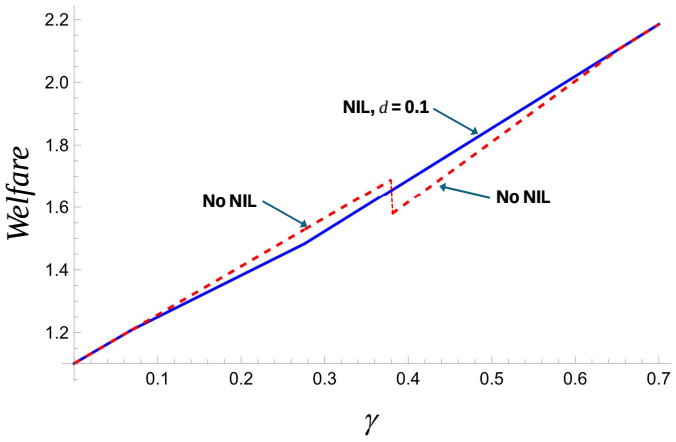

Figure 7: Social Welfare as a Function of γ with $d = 0.1$

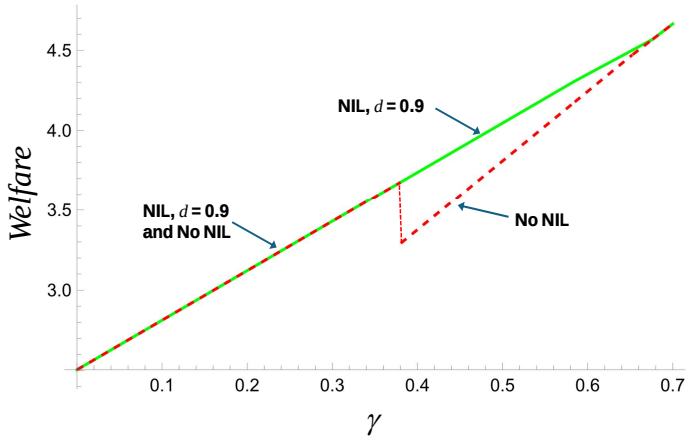

Figure 8: Social Welfare as a Function of γ with $d = 0.9$

getting benched. Thus, in this case the wasting of prospects' talent is relevant for larger values of γ, as above, but coaching skill is not wasted because the introduction of NIL does not induce a switch toward aspiring schools by unproven players. Overall, with a higher pro salary differential ($d = 0.9$), the welfare impact of NIL is more likely to be positive because experimentation by unproven prospects is reduced.

6 Transfer Portal

So far we have assumed that players must stay with the same school for two seasons to go pro. In this section we explore an extension in which a transfer portal operates and players can potentially benefit from moving to a school with a known skilled coaching staff. Specifically, we assume that any school can make a NIL offer to a player that has been at another school for one season. If the offer is accepted, they will play at the new school for their second season and then go pro. For superstar prospects or previously unproven prospects that generated a benefit for their existing school in their first season (who we hereafter refer to as "proven" prospects), the skill of their new coaching staff will determine their professional salary. That is, if the coaching staff of their new school is skilled, they will achieve a professional salary of m_s. However, no transferred player can generate an incremental benefit of Δ for their new school—this benefit is available solely to a superstar's original best fit school.

The addition of the transfer portal also requires us to consider initial NIL offers that are not entirely front-end loaded, because schools know that there may be a transfer offer for the player at the end of the first period that they will have to respond to with additional NIL if they want to keep the player. Thus, initial offers will have to pro-rate NIL over the two periods (or offer a lower up front NIL payment in anticipation that additional NIL will be negotiated for the second period).

In this setting we can derive for some parameters a "Full Poaching" equilibrium, in which all unproven prospects initially sign with aspiring schools, and all elite schools who are not matched to superstars make successful offers to attract away, or "poach," newly proven prospects. To do so, we first derive NIL offers in the transfer portal. Consider an elite school that considers poaching a newly proven prospect from an aspiring school. Their net relationship surplus is

$$NRS_e^p = \beta + \beta(1-\varphi)V_e - \beta(1-\varphi)V_e = \beta,$$

i.e., the prospect will play their one remaining season and produce a present value benefit of β for the school, and the school will then return to the market the next period, just like they would if they did not poach the prospect. Thus, they are willing to pay up to β in

NIL to attract the proven prospect. However, since the prospect is currently at an aspiring school they also perceive an expected pro salary differential since the elite schools' coaches are known to be skilled, while their current coach is skilled only with probability q. The expected differential is (note that we assume here that at the time of the operation of the transfer portal it is already known that the elite school's coach will not depart between seasons):

$$d^p \equiv \beta\left(m_u + (m_s - m_u)\right) - \beta\left(m_u + q(m_s - m_u)\right)$$

$$= \beta(1-q)(m_s - m_u).$$

Note from the definition of d above that

$$d^p = \beta(1-q)(m_s - m_u) = \frac{d}{g\beta(1-\varphi)}$$

Given this, the player would be willing to stay at their current aspiring school given an offer of β from an elite school if the aspiring school offers them NIL of

$$\beta + \frac{d}{g\beta(1-\varphi)}.$$

Of course, if the aspiring school is not willing to offer this much NIL to keep the player (as is true in a Full Poaching equilibrium), then an elite school can poach them for less than β. The aspiring school's surplus from keeping the player is $\beta + \beta q(1-\varphi)V_e$, so this is their maximum retention offer. The successful poaching offer that elite schools will make in such a case will therefore be

$$\beta + \beta q(1-\varphi)V_e - \frac{d}{g\beta(1-\varphi)}.$$

For the Full Poaching equilibrium to exist, we must have $\beta + \beta q(1-\varphi)V_e - \frac{d}{g\beta(1-\varphi)} < \beta$, i.e., $\beta q(1-\varphi)V_e < \frac{d}{g\beta(1-\varphi)}$. In words, the player's perceived salary differential must exceed the discounted expected reputation benefit for the aspiring school. For a steady state we also need $\eta \in (\gamma, \gamma + g(1-\gamma))$ to ensure that all superstars are matched to an elite school, but also that the number of elite schools poaching newly proven prospects does not exceed the number of available newly proven prospects. The latter is necessary so that some aspiring

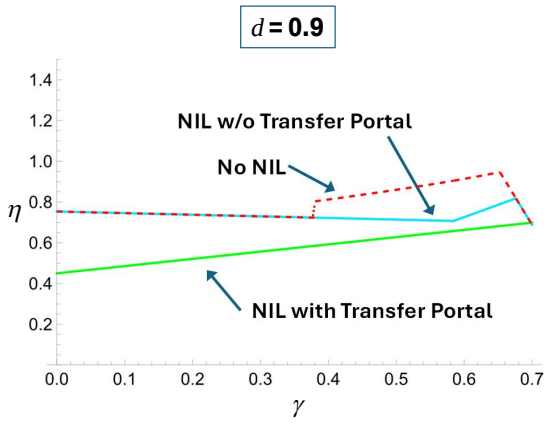

Figure 9: η as a Function of γ

schools can still have their skills discovered to maintain a constant mass of skilled coaches as some skilled coaches depart each period. Finally, we need to show that elite schools not matched to superstars prefer to poach newly proven prospects from aspiring schools rather than compete to attract unproven rookie prospects. Showing the existence range for the equilibrium given these requirements is not analytically tractable, but below we construct examples where we can illustrate its existence numerically, which generally occurs at higher values of d. However, we do have the following analytical result.

Proposition 7. *In any parameter space where the Full Poaching equilibrium exists, it has lower η, and therefore lower league competitiveness, than either of the pure strategy equilibria derived in Section 4.2.*

The reduction in η in this case is because elite schools can "cherry pick" prospects after they have been proven in the aspiring schools, severely limiting the aspiring schools' ability to become elite. In Figure 9 we graph the equilibrium value of η for the case of NIL with a transfer portal together with the values for the NIL and No NIL equilibria derived above. The base parameters are the same as for the earlier numerical examples, and we set $d = 0.9$ (we do not illustrate the $d = 0.1$ case here because the Full Poaching equilibrium does not exist within that parameter space). The green line gives η for the newly derived transfer portal case. With the high value of coaching skill in this example, it is easy for elite schools to poach proven players, so a Full Poaching equilibrium exists everywhere. Consistent with Proposition 7, the Full Poaching equilibrium has the lowest competitiveness throughout the

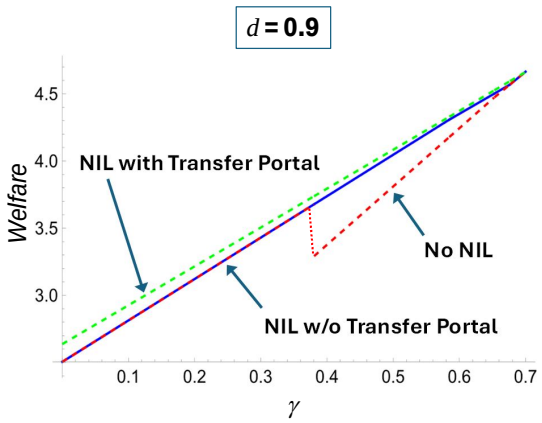

Figure 10: Social Welfare as a Function of γ

entire feasible range of γ. This shows that the introduction of the transfer portal on top of NIL can lead to significantly reduced league competitiveness.

However, as illustrated previously, social welfare and league competitiveness are not always aligned. In Figure 10 we graph social welfare across the three cases using the same numerical example. Here the NIL equilibrium with transfer portal has slightly higher welfare everywhere than both the NIL and No NIL equilibria without transfer portal. This is because poaching proven players and giving them the benefit of known coaching skill has a positive effect that outweighs the lower discovery of skilled coaches from aspiring schools.

It is also worth noting that adding a transfer portal in the absence of NIL does not have a significant effect. If an elite school will bench a proven prospect without the transfer portal (which is the only reason unproven prospects would sign with an aspiring school to begin with), then the elite schools also will not have any incentive to poach such prospects away from aspiring schools. Furthermore, there is no reason for a player who will be continued at an elite school to transfer down to an aspiring school.

7 Conclusion

We study the impact of NIL on the competitiveness of college sports. There is a general concern that allowing schools to pay athletes via NIL will make elite schools even stronger and hurt competitiveness. We show that the effect on competitiveness depends critically on

both the quality of the incoming athlete class and the difference in coaching skills between elite and aspiring schools.

In a dynamic model without NIL, when the supply of superstars in the incoming class is low, promising but unproven prospects also prefer to join elite schools as the probability they will be benched in favor of a superstar is low. However, when the supply of superstars is higher, they may prefer to sign with an aspiring school that is less likely to bench them. An athlete's decision is also impacted by the difference in coaching skill between elite and aspiring schools. If the difference is small, promising prospects are more likely to take a chance with aspiring schools. If the difference is large, these prospects are more willing to stick with an elite school even if their probability of being benched is high.

The introduction of NIL provides aspiring schools an additional tool to compete against elite schools in attracting promising talent. Under reasonable parameter spaces, aspiring schools can make large enough NIL payments that elite schools may be unwilling to match for two reasons. First, getting an unproven prospect is potentially more valuable to an aspiring school because if they turn out to be good, the school can achieve elite status and capture additional rents. Thus, they are willing to pay more than elite schools who are already enjoying such rents. Second, since elite schools enjoy a higher probability of landing a superstar, their incentive to compete for an unproven prospect against an aspiring school is lower. We show this is the case particularly when the supply of superstars in the incoming class is high and/or when the difference in coaching skill is low.

We also show that the addition of a transfer portal dampens the advantage NIL provides to aspiring schools as it allows elite schools to poach away their promising prospects once they are revealed to be talented. This hurts the ability of aspiring schools to become elite and thus can reduce competitiveness between schools. Finally, we show that higher competitiveness is not always synonymous with higher social welfare, as higher competitiveness can result in inefficient utilization of coaching skill and/or athlete talent.

8 Appendix

Proof of Proposition 1: Follows from the preceding discussion in the text. ■

Proof of Proposition 2: Follows from the preceding discussion in the text. ■

Proof of Lemma 1: Setting $\gamma = 0$ gives $\eta^e = \frac{(1-\varphi)^2 gq}{(1-\varphi)^2 gq+\varphi(g+1-\varphi g)}$, which is clearly positive and less than 1. Taking the derivative of η^e with respect to γ yields $\frac{\partial \eta^e}{\partial \gamma} = \frac{\varphi(\varphi+g-1-g\varphi)}{(1-\varphi)^2 gq+\varphi(g+1-\varphi g)} < 0$, so there exists a threshold in gamma below which our equilibrium condition holds. Setting the expression for η^e equal to γ and solving for γ yields $\gamma = \frac{(1-\varphi)^2 gq}{(2-\varphi)\varphi(1-gq)+gq}$. Thus, the feasible range for existence with respect to the equilibrium constraint on η is as given in the result. ■

Proof of Proposition 3: Plugging in the expression for η^e and solving for V_e yields

$$V_e = \frac{\beta\begin{pmatrix}\gamma\varphi(1+\beta(1-\varphi)(\Delta+1)) + (1-\gamma)(1-\varphi)^2 g^2 q(\beta(1-\varphi)+1) \\ +\gamma g\left((\beta(1-\varphi)^2\Delta(\varphi(1-q)+q)) + (\beta(1-\varphi)+1)\left((1-\varphi)^2 q - \varphi\right)\right)\end{pmatrix}}{(1-\varphi)(1-\beta(1-\varphi))\left(\gamma(1-g)\left(\beta\left((1-\varphi)^2 gq+\varphi\right)-\varphi\right) + (1-\varphi)gq(\beta(1-\varphi)g+1)\right)}.$$

It is straightforward to show that this is increasing in γ. Setting the RHS equal to $\frac{\beta}{1-\beta(1-\varphi)}$ and solving for γ yields

$$\gamma = \frac{(1-\varphi)^2(1-g)gq}{\beta(1-\varphi)\Delta\left((1-\varphi)^2 gq+\varphi(1-\varphi g+g)\right)+(1-g)((2-\varphi)\varphi(1-gq)+gq)}.$$

Thus, the feasible range for existence based on this equilibrium condition is

$$\gamma \in [0, \frac{(1-\varphi)^2(1-g)gq}{\beta(1-\varphi)\Delta\left((1-\varphi)^2 gq+\varphi(1-\varphi g+g)\right)+(1-g)((2-\varphi)\varphi(1-gq)+gq)}].$$

Putting the two conditions together, we have a feasible existence range of

$$\gamma \in \left[0, Min\left(\begin{matrix}\frac{(1-\varphi)^2 gq}{(2-\varphi)\varphi(1-gq)+gq}, \\ \frac{(1-\varphi)^2(1-g)gq}{\beta(1-\varphi)\Delta((1-\varphi)^2 gq+\varphi(1-\varphi g+g))+(1-g)((2-\varphi)\varphi(1-gq)+gq)}\end{matrix}\right)\right]$$

It is straightforward to show that both terms in the Min function are positive, and that the second term is lower, so the existence range is given by

$$\gamma \in [0, \frac{(1-\varphi)^2(1-g)gq}{\beta(1-\varphi)\Delta\left((1-\varphi)^2 gq+\varphi(1-\varphi g+g)\right)+(1-g)((2-\varphi)\varphi(1-gq)+gq)}].$$

■

Proof of Lemma 2: At $\gamma = 0$ we have $\eta^a = \frac{(1-\varphi)^2 gq}{\varphi} > 0$. Note that η^a is decreasing in gamma, so there will be a maximum gamma for which the equilibrium is feasible by this condition, which is defined by the γ at which the RHS above equals γ, or $\gamma = \frac{(1-\varphi)^2 gq}{(2-\varphi)\varphi(1-gq)+gq}$.

■

Proof of Proposition 4: Plugging in our expression for η^a and solving for V_e yields

$$V_e = \frac{\beta\gamma\varphi\left(\beta(1-\varphi)\Delta + \beta(1-\varphi) + 1\right)}{(1-\varphi)(1-\beta(1-\varphi))((1-\gamma)(1-\varphi)gq - (1-\beta)\gamma\varphi)}.$$

Now we need to find the γ at which this solves $\beta(1-\varphi) + \beta(1-\varphi)^2 V_e = (1-\varphi)V_e$, i.e., $V_e = \frac{\beta}{1-\beta(1-\varphi)}$. This yields $\gamma = \frac{(1-\varphi)^2 gq}{\beta(1-\varphi)\varphi\Delta + (2-\varphi)\varphi(1-gq)+gq}$. Thus, combining with Lemma 2, we have the existence range given in the result. ■

Proof of Sufficiency of Assumption 1: First consider the condition that elite schools must be willing to continue good unproven prospects in an equilibrium in which unproven players always sign with elite schools to the extent they are available (i.e., the equilibrium derived in Section 4.2.1). The condition for this to be true is

$$\beta(1-\varphi) + \beta(1-\varphi)^2 V_e - (1-\varphi)V_e \geq 0. \tag{1}$$

Plugging the value of V_e derived for this equilibrium in the proof of Proposition 5 below into the LHS of (1) and evaluating the expression while setting $\gamma = 0$ gives

$$\frac{(1-\varphi)\left(\beta - (1-\beta(1-\varphi))d - \beta^2(1+g\varphi-\varphi-g) - \beta^3(1-\varphi)^2 g(1-q)\right)}{1-\beta(1-\varphi)(g(\beta(1-\varphi)(1-q)-1)+1)},$$

which can be shown to be positive for $d = 0$ and is decreasing in d. Thus, for sufficiently small d the condition holds at low enough γ. The LHS of (1) obviously decreases in V_e, which can easily be shown to increase in both γ and d. Plugging the maximum gamma for existence of the equilibrium from Proposition 5 (focusing on the second expression in the Min term, which is sufficient given the Min operator) into the LHS of (1) gives $(1-\varphi)\beta - \frac{(1-\beta(1-\varphi))d}{\beta g(1-\beta(1-\varphi)(1-q))}$, which is clearly positive at $d = 0$ and decreasing in d. Solving for the d at which this equals zero yields the first expression on the RHS of Assumption 1. Thus, Assumption 1 is sufficient

for the satisfaction of this condition over the range of existence of this equilibrium.

Next consider the condition that elite schools must be willing to continue unproven prospects who turn out to be good in an equilibrium in which unproven players never sign with elite schools (i.e., the equilibrium derived in Section 4.2.2). As noted in the proceeding text, we assume that this condition is maintained in our derivation of the equilibrium in that section. Similar to the above, we need to show that (1) holds everywhere within the parameter range of existence given in Proposition 6. Plugging the value of V_e derived for this equilibrium in the proof of Proposition 6 below into the LHS of (1) and evaluating the expression while setting $\gamma = 0$ gives $(1-\varphi)\beta > 0$, so (1) always holds for sufficiently small γ. The derivative of the LHS is easily shown to be decreasing in γ, so it suffices to show that the γ at which the LHS equals zero is greater than the upper limit of existence for this equilibrium as given in Proposition 5. It is straightforward to show that the γ at which the LHS equals zero is decreasing in d, and that the d at which this coincides with the upper limit in Proposition 5 equals the second expression on the RHS in Assumption 1. Thus, Assumption 1 is sufficient for the satisfaction of this condition over the relevant range.

Finally, consider the condition that NIL payments always be non-negative (due to limited wealth for prospects). NIL could become negative if d is so large that prospects would be willing to pay to sign with elite schools in order to achieve a higher expected pro salary. Since an aspiring school gets a minimum expected payoff from signing an unproven player of $g\beta(1+\beta(1-\varphi))$ (i.e., their NRS if $V_e = 0$), and they always are willing to pay out their full NRS via NIL in order to attract a prospect, a sufficient minimum NIL that must be required for an elite school to beat the aspiring school given an expected salary differential of d is $g\beta(1+\beta(1-\varphi))-d$. Thus, NIL is always nonnegative in equilibrium if $d \leq g\beta(1+\beta(1-\varphi))$, i.e., the third expression on the RHS in Assumption 1. ■

Proof of Proposition 5: First we must show that the NIL required for a best fit elite school is NRS_e^s rather than $NRS_a^s - \frac{d}{g}$. Since both expressions depend on V_e, we first solve for V_e assuming that $NRS_e^s > NRS_a^s - \frac{d}{g}$ and then confirm that it holds at the calculated V_e. Solving the expression for V_e using the definition of η^e gives

$$V_e = \frac{d\varphi(\gamma + g(q-\gamma)) - dgq - \beta^2\gamma\Delta(g(1-\varphi)((1-q)\varphi+q)+\varphi)}{(1-\beta(1-\varphi)(g(\beta(1-q)(1-\varphi)-1)+1))(\gamma\varphi - \gamma g\varphi - gq(1-\varphi))}.$$

It is then straightforward to show that $NRS_e^s - (NRS_a^s - \frac{d}{g})$ evaluated with this V_e is positive at $\gamma = 0$ and decreasing in γ, and that the γ at which $NRS_e^s - (NRS_a^s - \frac{d}{g}) = 0$ corresponds to the upper limit on γ given in the Proposition (i.e., the second term in the Min function).

Given the above value for V_e, the elite school will pay for the unproven player if their NRS is bigger than the NIL they have to pay to get the player. Their NRS is $NRS_e^u = \beta g + g\beta^2(1-\varphi) + g\beta(1-\varphi)V_e\left[\beta(1-\varphi) - 1\right]$, where the last term is negative due to the one period delay in their realization of V_e. So for them to be willing to pay for an unproven player we need $NRS_e^u - (NRS_a^u - d) \geq 0$, i.e.,

$$\beta g + g\beta^2(1-\varphi) + g\beta(1-\varphi)V_e\left[\beta(1-\varphi) - 1\right] - \left[\beta g + g\beta^2(1-\varphi) + g\beta^2 q(1-\varphi)^2 V_e - d\right] \geq 0,$$

Evaluating the LHS while setting $\gamma = 0$ given the expression for V_e derived above yields $\frac{(1-\beta+\beta\varphi)d}{1-\beta(1-\varphi)(g(\beta(1-q)(1-\varphi)-1)+1)}$, which is positive as long as $d > 0$. It is straightforward to show that the LHS of the inequality is decreasing in γ, so an elite school is willing to pay for an unproven player for low gamma up to some threshold. Setting $NRS_e^u - (NRS_a^u - d) = 0$ and solving for γ gives

$$\gamma = \frac{(1-\varphi)gq(1-\beta(1-\varphi))d}{\varphi(1-g)(1-\beta(1-\varphi))d + \beta^3(1-\varphi)\Delta g\left((1-\varphi)^2 gq + \varphi(1-\varphi g+g)\right)(1-\beta(1-\varphi)(1-q))}.$$

This is clearly positive as long as d is positive. Thus, the feasible existence range based on this requirement is

$$\gamma \in [0, \frac{(1-\varphi)gq(1-\beta(1-\varphi))d}{\varphi(1-g)(1-\beta(1-\varphi))d + \beta^3(1-\varphi)\Delta g\left((1-\varphi)^2 gq + \varphi(1-\varphi g+g)\right)(1-\beta(1-\varphi)(1-q))}].$$

Putting this together with Lemma 1, and noting that there is no definitive ranking between the two upper bounds, we have existence for the range given in the proposition.■

Proof of Proposition 6: Solving for V_e given the expression for η^a yields

$$V_e = \frac{\gamma\varphi\left(\beta^2(1-\varphi)\Delta + \frac{d}{g}\right)}{(1-\varphi)((1-\gamma)(1-\varphi)gq(1-\beta(1-\varphi)) - \gamma\varphi(\beta(\beta(1-\varphi)(1-q)+\varphi-2)+1))}.$$

As above, we need to verify that we are using the correct NIL payment for superstars, i.e.,

that $NRS_e^s < NRS_a^s - \frac{d}{g}$ for this case. It is straightforward to show that $NRS_e^s - (NRS_a^s - \frac{d}{g})$ evaluated with this V_e is positive at $\gamma = 0$ and decreasing in γ, and that the γ at which it equals zero corresponds to the upper limit on γ given in Proposition 5 above. Thus, $NRS_e^s < NRS_a^s - \frac{d}{g}$ at all higher γ, including the entire existence range for the equilibrium under consideration as derived in the proposition.

An elite school is willing to pay for an unproven player conditional on continuing them if good if their NRS is greater than the NIL they have to pay. Their NRS conditional on continuing if good is, as above $NRS_e^u = \beta g + g\beta^2(1-\varphi) + g\beta(1-\varphi)V_e\left[\beta(1-\varphi) - 1\right]$, where the last term is negative due to the one period delay in their realization of V_e. So for them to be willing to pay we need $NRS_e^u - (NRS_a^u - d) \geq 0$. Plugging in our expression for V_e, the LHS equals

$$\frac{\beta\gamma\varphi(1-\beta(1-\varphi)(1-q))\left(\beta^2(1-\varphi)\Delta g + d\right)}{\gamma\varphi(\beta(\beta(1-\varphi)(1-q)+\varphi-2)+1) - (1-\gamma)(1-\varphi)gq(1-\beta(1-\varphi))} + d$$

Evaluating this at $\gamma = 0$ yields $d > 0$, so they are willing to pay at very low $\gamma's$. Taking the derivative with respect to γ gives

$$\frac{\beta(1-\varphi)\varphi gq(1-\beta(1-\varphi))(\beta(1-\varphi)(1-q)-1)\left(\beta^2(1-\varphi)\Delta g + d\right)}{(\gamma\varphi(\beta(\beta(1-\varphi)(1-q)+\varphi-2)+1) - (1-\gamma)(1-\varphi)gq(1-\beta(1-\varphi)))^2} < 0,$$

so they are less likely to pay as γ rises. The threshold γ at which they become just unwilling to pay is where the former expression equals zero. Setting it equal to zero and solving for γ gives a threshold of

$$\gamma = \frac{(1-\varphi)gq(1-\beta(1-\varphi))d}{(1-\beta(1-\varphi))d((1-\varphi)gq+\varphi) + \beta^3(1-\varphi)\varphi\Delta g(1-\beta(1-\varphi)(1-q))}.$$

Thus, based on this existence condition, the equilibrium can exist for

$$\gamma > \frac{(1-\varphi)gq(1-\beta(1-\varphi))d}{(1-\beta(1-\varphi))d((1-\varphi)gq+\varphi) + \beta^3(1-\varphi)\varphi\Delta g(1-\beta(1-\varphi)(1-q))} \geq 0.$$

Putting this together with Lemma 2, the equilibrium exists for the range given in the proposition whenever the range is non-empty. ■

Proof of Proposition 7: First we need to derive the equilibrium η for the Full Poaching equilibrium, which is governed by

$$\eta = \gamma(1-\varphi)^2 + (\eta-\gamma)(1-\varphi) + (1-\varphi)^2 q((1-\gamma)g - (\eta-\gamma)).$$

The terms represent, in order, elite schools matched to superstars two periods ago, elite schools that poached newly proven players one period ago, and aspiring schools with skilled coaches that signed unproven players two periods ago that turned out to be good and were not poached. Solving this for η yields an equilibrium η of $\frac{(1-\varphi)(gq(1-\gamma+\gamma\varphi-\varphi)-\gamma(\varphi-(1-\varphi)q))}{\varphi+(1-\varphi)^2 q}$. Subtracting this from η^e yields

$$\frac{(1-\varphi)(\varphi g-(1-\varphi)(1-g)q)\left(\gamma(2-\varphi)\varphi-(1-\gamma)(1-\varphi)^2 gq\right)}{\left((1-\varphi)^2 q+\varphi\right)\left((1-\varphi)^2 gq+\varphi(1+g-\varphi g)\right)},$$

which is easily shown to be positive as long as $\varphi < \frac{q(1-g)}{g+q(1-g)}$.

Subtracting the equilibrium η for the Full Poaching equilibrium from η^a yields

$$\frac{(1-\varphi)^2 q\left((1-\gamma)(1-\varphi)^2 gq-\gamma(2-\varphi)\varphi\right)}{\varphi\left((1-\varphi)^2 q+d\right)},$$

which is easily shown to be positive for the entire range of γ under consideration. ■

9 References

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