

Incorporating Subjective Characteristics in Product Design and Evaluations

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Web Appendix

A. STEPS IN MARKOV CHAIN MONTE CARLO SIMULATION

Our MCMC procedure is carried out by sequentially generating draws from the following distributions:

1. Generate the loading matrix Λ

The loading matrix is a patterned matrix with both fixed and free elements. Some of the fixed elements are zero and others are one, depending on the model setup and the identification requirement. Let λ_j denote the j^{th} column vector of the free elements in the loading matrix, \tilde{v}_{is} be the vector of indicator variables that correspond to these factor loadings, and $\tilde{\theta}_{ij}$ be the corresponding sub-matrix of the measurement errors. The full conditional distribution of λ_j is given by:

$$f(\lambda_j | *) \propto \prod_{i=1}^N \prod_{s=1}^S f(\tilde{v}_{is} | *) f(\lambda_j) \quad (W1)$$

Where $\tilde{v}_{is} | * \sim MVN(\lambda_j(\delta_{ij} + b_{ij}x_{is}), \lambda_j \Delta_{ij} \lambda_j' + \tilde{\theta}_{ij})$ and the prior distribution of λ_j is given as follows. For each element λ_{kj} in λ_j , we let $\lambda_{kj} = 2 \times ld_{kj} - 1$ with $ld_{kj} \sim Beta(\iota, o)$. The transformed Beta distribution has an $[-1, 1]$ interval for the factor loadings. We set $\iota = 2$ and $o = 2$ to ensure diffuse but proper priors.

2. Generate the factor scores z_{is}

$$f(z_{is} | *) = MVN(\omega_{z_{is}}, \Delta_{z_{is}}) \quad (W2)$$

The mean $\omega_{z_{is}}$ and the variance-covariance matrix $\Delta_{z_{is}}$ come from two data sources. The first data source is the measurement equation $v_{is} = \Lambda_i z_{is} + \varepsilon_{is}$. The second data source is from the structural equation $z_{is} = \delta_i + B_i x_s + \mu_{is}$. Therefore, the full conditional distribution for z_{is} can be written as $MVN(\omega_{z_{is}}, \Delta_{z_{is}})$, where $\omega_{z_{is}} = \Delta_{z_{is}} [\Delta_i^{-1}(\delta_i + B_i x_s) + \Lambda_i' \Theta_i^{-1} v_{is}]$ and $\Delta_{z_{is}}^{-1} = \Delta_i^{-1} + \Lambda_i' \Theta_i^{-1} \Lambda_i$.

3. Generate the measurement errors ε_{is}

$$f(\varepsilon_{is} | *) \propto f(v_{is} | *) f(\varepsilon_{is}) \quad (W3)$$

Where $v_{is} | * \sim MVN(\Lambda(\delta_i + B_i x_{is}), \Lambda \Delta_i \Lambda' + \Theta_i)$

$\varepsilon_{is} \sim MVN(0, \Theta_i)$

4. Generate δ_i

$$f(\delta_i | *) \propto \prod_{s=1}^S f(v_{is} | *) f(\delta_i) \quad (W4)$$

Where $\delta_i \sim MVN(\kappa, \Sigma)$

5. Generate $B_i = \{b'_{i1}, b'_{i2}, \dots, b'_{iJ}\}$

$$f(\mathbf{B}_i|*) \propto \prod_{s=1}^S f(\mathbf{v}_{is}|*)f(\mathbf{B}_i) \quad (\text{W5})$$

Where $\mathbf{b}_{ij} \sim \text{MVN}(\beta_j, \mathbf{D}_j)$ for $j = 1, \dots, J$

6. Generate μ_{is}

$$f(\mu_{is}|*) \propto f(\mathbf{v}_{is}|*)f(\mu_{is}) \quad (\text{W6})$$

Where $\mu_{is} \sim \text{MVN}(0, \Delta_i)$

7. Generate $\eta_i = \{\mathbf{A}_i, \gamma_i\}$

$$f(\eta_i|*) \propto \prod_{s=1}^S f(y_{is}|*)f(\mathbf{v}_{is}|*)f(\eta_i) \quad (\text{W7})$$

Where $y_{is}|* \sim \text{MVN}(\gamma_i \delta_i + (\mathbf{A}_i + \gamma_i \mathbf{B}_i) \mathbf{x}_{is}, \gamma_i \Delta_i \gamma_i' + \sigma_e^2)$

$\eta_i \sim \text{MVN}(\varphi, \Omega)$

8. Generate e_{is}

$$f(e_{is}|*) \propto f(y_{is}|*)f(\mathbf{v}_{is}|*)f(e_{is}) \quad (\text{W8})$$

Where $e_{is} \sim N(0, \sigma_e^2)$

9. Generate $\theta_{ik} = \text{diag}(\Theta_i)$ for $k = 1, \dots, K$

Let v_{isk} denote the corresponding indicator variable, $\tilde{\lambda}_k$ be the corresponding factor loading, and z_{isk} represent the corresponding factor score. The full conditional distribution for θ_{ik} ($k = 1, \dots, K$) is:

$$f(\theta_{ik}|*) = IG(\varsigma_k + \frac{S}{2}, [\psi_k + \frac{1}{2} \sum_{s=1}^S (v_{isk} - \tilde{\lambda}_k z_{isk})^2]^{-1}) \quad (\text{W9})$$

10. Generate Δ_i

$$f(\Delta_i|*) = W(\rho + S, [R^{-1} + \sum_{s=1}^S \mu_{is} \mu_{is}']^{-1}) \quad (\text{W10})$$

11. Generate σ_e^2

$$f(\sigma_e^2|*) = IG(\varpi_e + \frac{N \times S}{2}, [\psi_e^{-1} + \frac{1}{2} \sum_{i=1}^N \sum_{s=1}^S (y_{is} - \mathbf{A}_i \mathbf{x}_s - \gamma_i z_{is})^2]^{-1}) \quad (\text{W11})$$

Where $\varpi_e = 2$ and $\psi = 1$ are the priors of the Inverse Gamma distribution.

12. Generate the hyper-parameter ς_k for $k = 1, \dots, K$

$$f(\varsigma_k|*) \propto \prod_{i=1}^N f(\theta_{ik}|*)f(\varsigma_k) \quad (\text{W12})$$

Where the prior distribution is defined as $\log(\varsigma_k) \sim N(0, \tau_k)$. We use the log-transformation to ensure a positive sign of ς_k . We set $\tau_k = 100$.

13. Generate the hyper-parameter ψ_k for $k = 1, \dots, K$

$$f(\psi_k|*) = IG(g_k + N\varsigma_k, [h_k^{-1} + \sum_{i=1}^N \theta_{ik}^{-1}]^{-1}) \quad (\text{W13})$$

Where the prior distribution is defined as $\psi_k \sim IG(g_k, h_k)$. We set $g_k = 0.5$ and $h_k = 1$ to ensure diffuse but proper priors.

14. Generate the hyper-parameter ρ

$$f(\rho|*) \propto \prod_{i=1}^N f(\Delta_i^{-1}|*) f(\rho) \quad (W14)$$

Where the prior distribution is defined as $\log(\rho) \sim N(0, \tau_\rho) | \rho > J$. This prior distribution is selected based on two criteria: 1) ρ has to be a positive number; and 2) ρ needs to be greater than the dimension of the matrix J . We set $\tau_\rho = 100$.

15. Generate the hyper-parameter R

$$f(R^{-1}|*) \propto \prod_{i=1}^N f(\Delta_i^{-1}|*) f(R^{-1}) \quad (W15)$$

Where the prior distribution is defined as $R^{-1} \sim W(\rho_0, (\rho_0 R_0)^{-1})$ with $\rho_0 = 5$ and $R_0 = I$ (I is a $J \times J$ identity matrix).

16. Generate the hyper-parameter κ

$$f(\kappa|*) = MVN(\mathcal{G}_\kappa, \Gamma_\kappa) \quad (W16)$$

$$\text{Where } \mathcal{G}_\kappa = \Gamma_\kappa \left[\left(\frac{\Sigma}{N-1} \right)^{-1} \frac{\sum_{i=2}^N \delta_i}{(N-1)} + \Gamma_\kappa^{-1} r_\kappa \right] \text{ and } \Gamma_\kappa = (\Gamma_{\kappa 0}^{-1} + \left(\frac{\Sigma}{N-1} \right)^{-1})^{-1}. \text{ We set the priors}$$

as $r_\kappa = 0$ and $\Gamma_{\kappa 0} = 100I$ (I is a $J \times J$ identity matrix).

17. Generate the hyper-parameter Σ

$$f(\Sigma^{-1}|*) = W((N-1) + \rho_{\Sigma 0}, \sum_{i=2}^N (\delta_i - \kappa)(\delta_i - \kappa)' + R_{\Sigma 0}) \quad (W17)$$

Where the priors are set as $\rho_{\Sigma 0} = 5$ and $R_{\Sigma 0} = 5I$ (I is a $J \times J$ identity matrix).

18. Generate the hyper-parameter β_j for $j = 1, \dots, J$

$$f(\beta_j|*) \propto MVN(\mathcal{G}_{\beta_j}, \Gamma_{\beta_j}) \quad (W18)$$

$$\text{Where } \mathcal{G}_{\beta_j} = \Gamma_{\beta_j} \left[\left(\frac{D_j}{N} \right)^{-1} \left(\frac{\sum_{i=1}^N b_{ij}}{N} \right) + \Gamma_{\beta_j 0}^{-1} r_{\beta_j 0} \right] \text{ and } \Gamma_{\beta_j} = \left(\Gamma_{\beta_j 0}^{-1} + \left(\frac{D_j}{N} \right)^{-1} \right)^{-1}. \text{ We set the}$$

priors as $r_{\beta_j 0} = 0$ and $\Gamma_{\beta_j 0} = 100I$ (I is a $M \times M$ identity matrix).

19. Generate the hyper-parameter D_j for $j = 1, \dots, J$

$$f(D_j^{-1}|*) = W(\rho_{D_j}, R_{D_j}) \quad (W19)$$

Where $\rho_{D_j} = N + \rho_{D_j 0}$ and $R_{D_j} = \sum_{i=1}^N (b_{ij} - \beta_j)(b_{ij} - \beta_j)' + R_{D_j 0}$. We set the diffuse but proper priors as $\rho_{D_j 0} = 10$ and $R_{D_j 0} = 10I$ (I is a $M \times M$ identity matrix).

20. Generate the hyper-parameter ϕ

$$f(\phi|*) = MVN(\mathcal{G}_\phi, \Gamma_\phi) \quad (W20)$$

$$\text{Where } \mathcal{G}_\phi = \Gamma_\phi \left[\left(\frac{\Omega}{N} \right)^{-1} \frac{\sum_{i=1}^N \eta_i}{N} + \Gamma_{\phi 0}^{-1} \mathbf{r}_{\phi 0} \right] \text{ and } \Gamma_\phi = \left(\Gamma_{\phi 0}^{-1} + \left(\frac{\Omega}{N} \right)^{-1} \right)^{-1}. \text{ We set the priors as}$$

$\mathbf{r}_{\phi 0} = 0$ and $\Gamma_{\phi 0} = 100\mathbf{I}$.

21. Generate the hyper-parameter Ω

$$f(\Omega^{-1}|*) = W(\rho_{\Omega 0} + N, \sum_{i=1}^N (\eta_i - \phi)(\eta_i - \phi)' + \mathbf{R}_{\Omega 0}) \quad (W21)$$

Where the priors are set at $\rho_{\Omega 0} = 15$ and $\mathbf{R}_{\Omega 0} = 15\mathbf{I}$.

B. SUPPLEMENTAL INFORMATION ON STUDY ONE: THE DESIGN OF A HANDHELD POWER TOOL

In this study, the objective attributes were selected based on: 1) their importance to the end users, and 2) their relevance to the subjective characteristics. For example, motor type was not included because its selection does not influence the emotional or physical appeal of the tool. However, the limitation of excluding motor power as an objective attribute is that the proposed model's incremental goodness-of-fit and predictive power over the benchmark conjoint model may be inflated to some extent.

Our experimental design among the calibration profiles provides a D-efficiency of 74.34. The D-efficiency index is calculated using the formula $\frac{100}{S|(X'X)^{-1}|^{1/M}}$ with S representing the number of calibration profiles, X being the design matrix using effects-type dummy variable coding, and M being the dimension of the design matrix X (Kuhfeld, Tobias, and Garratt 1994).

The average rating of each prototype on perceived power ranged from 3.491 (prototype #5) to 5.292 (prototype #2) and, on perceived comfort, it varied from 2.768 (prototype #3) to 5.289 (prototype #9) on a 7-point scale. This indicates that there was a considerable amount of variation in perceived power and perceived comfort across these prototypes.

The factor loadings from the proposed model are presented in the third column of Table W1. In order to evaluate how well the values of the objective attributes and the posterior distribution of the relevant model parameters can predict the actual subjective ratings, we calculated the Pseudo R^2 measures (last column of Table W1). In general, our findings suggest that the indicator variables were reliable measures of the underlying latent constructs and our model was able to explain a reasonable amount of variance in the subjective ratings.

TABLE W1: INDICATOR VARIABLE ESTIMATES: POWER TOOL STUDY

Latent Construct	Indicator	Factor Loading	Pseudo R^2
Perceived Power	<i>pwr1</i>	1	0.638
	<i>pwr2</i>	0.728 (0.052)	0.478
	<i>pwr3</i>	0.745 (0.051)	0.512
Perceived Comfort	<i>cft1</i>	1	0.548
	<i>cft2</i>	0.657 (0.051)	0.402
	<i>cft3</i>	0.656 (0.053)	0.332
	<i>cft4</i>	0.677 (0.050)	0.430

Posterior standard deviations are in parentheses.

Pseudo R^2 is for regression of indicator on objective attributes (i.e. $\hat{v}_{is} = \Lambda(\delta_i + B_i x_s)$).

In Table W2, we provide a summary of the comparison between our structural equation model (SEM) and the path model. We only report the impact of the subjective characteristics on purchase likelihood in this table because the other parameter estimates from the SEM model and the path model are highly similar.

TABLE W2: COMPARISON BETWEEN SEM MODEL AND PATH MODEL:
POWER TOOL STUDY

	SEM	Path
Perceived Power*	0.207	0.183
Perceived Comfort	0.726	0.568
In-Sample Fit		
- <i>Pseudo R^2</i>	0.615	0.587
- <i>RMSD</i>	0.330	0.348
Predictive Power		
- <i>MAE</i>	10.79%	11.01%
- <i>RMSE</i>	12.13%	12.82%

C. SUPPLEMENTAL INFORMATION ON STUDY TWO: THE DESIGN OF A TOOTHBRUSH

We chose the toothbrush category in our study 2 for the following reasons. First, our pilot study suggested that the vast majority of college students are at least somewhat concerned about both dental hygiene and the perceived comfort of their toothbrushes. Second, we believed that the large variety of toothbrush designs on the market is an indication of consumers' heterogeneous preferences.

The D-Efficiency for our experimental design is 65.63 among the calibration profiles. Based on the data collected in Condition 1, the average ratings of each toothbrush on perceived effectiveness varied from a minimum of 2.507 (toothbrush #5) to a maximum of 5.633 (prototype #12) and on perceived comfort ranging from 3.389 (toothbrush #3) to 5.286 (toothbrush #4).

Table W3 gives the factor loadings estimated from the proposed model. The indicator variables appear to be good measures of the latent constructs. The Pseudo R^2 measures also indicated that a reasonable amount of variance in the subjective ratings is explained by our model.

TABLE W3: INDICATOR VARIABLE ESTIMATES: TOOTHBRUSH STUDY

Latent Construct	Indicator	Factor Loading	Pseudo R^2
Perceived Effectiveness	<i>eff1</i>	1	0.849
	<i>eff2</i>	0.881(0.021)	0.791
	<i>eff3</i>	0.895(0.021)	0.819
	<i>eff4</i>	0.851(0.023)	0.737
Perceived Comfort	<i>cft1</i>	1	0.620
	<i>cft2</i>	0.705(0.034)	0.585
	<i>cft3</i>	0.573(0.039)	0.433

Posterior standard deviations are in parentheses.

Pseudo R^2 is for regression of indicator on objective attributes (i.e. $\hat{v}_{is} = \Lambda(\delta_i + B_i x_s)$)

In Table W4, we provide a summary of the comparison between our structural equation model (SEM) and the path model. We only report the impact of the subjective characteristics on purchase likelihood in this table because the other parameter estimates from the SEM model and the path model are highly similar.

TABLE W4: COMPARISON BETWEEN SEM MODEL AND PATH MODEL:
TOOTHBRUSH STUDY

	SEM	Path
Perceived Effectiveness	0.309	0.246
Perceived Comfort	0.250	0.184
In-Sample Fit		
- <i>Pseudo R^2</i>	0.658	0.590
- <i>RMSD</i>	0.329	0.364
Predictive Power		
- <i>MAE</i>	9.26%	10.25%
- <i>RMSE</i>	11.74%	12.28%

Reference:

Kuhfeld, Warren F., Randall D. Tobias, and Mark Garratt (1994), "Efficient Experimental Design with Marketing Research Applications", *Journal of Marketing Research*, Vol. 31, No. 4, 545-557.